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EXTENSION OF MEASURES AND INTEGRALS BY THE HELP OF A PSEUDOMETRIC

BELOSLAV RIEČAN

There are various methods of constructing an extension of a measure μ from a ring $\mathcal{R}$ to a σ-ring $\mathcal{S}$ containing $\mathcal{R}$. One of them is the following: An extension $\bar{\mu}$ of μ is constructed ($\bar{\mu}$ need not be a measure, e.g. $\bar{\mu}$ may be the outer measure induced by μ) and a pseudometric is defined by the equality $\varrho(E, F)=\bar{\mu}(E \triangle F)$. Then the family $\mathcal{S}=\mathcal{R}^-$ (the closure with respect to ϱ) is one of the convenient σ-rings. (Of course, some assumptions concerning the finiteness of μ are necessary; see e.g. [6], [9].)

A similar method can be used for integrals (see e.g. [3], [7]).

Here we shall study the method from a general point of view. We shall work with functions $J: S \to \langle -\infty, \infty \rangle$, where S is a sublattice of a given lattice H. If H is a set of sets then the measure extension theory is obtained; if H is a set of real-valued functions then the integral extension theory is obtained. The same idea has been realised (only with different constructions) in papers [1], [8], [11], [12], [13].

Generating function

First we shall construct a function for generating our pseudometric. Its construction and corresponding proofs are known.

Assumptions. 1. *H will denote a lattice with the following properties:*

1.1. *H is relatively σ-complete, i.e. every monotone bounded sequence has the least upper bound and the greatest lower bound.* If $(x_n)_{n=1}^{\infty}$ is an increasing sequence and x is its supremum, then we write $x_n \nearrow x$; the symbol $x_n \searrow x$ has an analogous meaning. We use the symbols also for the lattice R of real numbers.

1.2. *H is σ-continuous, i.e. the relations* $x_n \nearrow x$, $y_n \nearrow y$ ($x_n \searrow x$, $y_n \searrow y$) *imply* $x_n \wedge y_n \nearrow x \wedge y$ ($x_n \vee y_n \searrow x \vee y$).

2. *A is a sublattice of H satisfying the following condition: To every* $x \in H$ *there are* $a_n \in A$ ($n=1, 2, \ldots$) *such that*

$$x \leqq \bigvee_{n=1}^{\infty} a_n$$

$\left(\bigvee_{n=1}^{\infty} a_n\right.$ is the supremum of $\left.(a_n)_{n=1}^{\infty}\right)$.

3. *$J_0: A \to R$ is a real-valued function with the following properties:*

3.1. *J_0 is increasing, i.e. $x \leqq y$, x, $y \in A$ implies $J_0(x) \leqq J_0(y)$.*

3.2. *J_0 is a valuation, i.e. $J_0(x \vee y) + J_0(x \wedge y) = J_0(x) + J_0(y)$ for every $x, y \in A$.*

3.3. *J_0 is upper continuous in the following sense: If $x_n \nearrow x$, $x_n \in A$ $(n = 1, 2, \ldots)$, $x \in A$, then $J_0(x_n) \nearrow J_0(x)$.*

We shall extend the function J_0 in the following two steps.

Lemma 1. *Let $x \in H$, $x_n \in A$, $y_n \in A$ $(n = 1, 2, \ldots)$, $x_n \nearrow x$, $y_n \nearrow x$. Then $\lim_{n \to \infty} J_0(x_n) = \lim_{n \to \infty} J_0(y_n)$.*

Proof. [1, Lemma 1], [11, Lemma 2.4], [12, Lemma 1].

Definition 1. *By B we denote the set of elements $b \in H$ such that there exist $a_n \in A$ $(n = 1, 2, \ldots)$ for which $a_n \nearrow b$. Further we denote by J_1 the mapping $J_1 : B \to R$ defined by the equality*

$$J_1(b) = \lim_{n \to \infty} J_0(a_n),$$

where $a_n \nearrow b$, $a_n \in A$ $(n = 1, 2, \ldots)$.

Definition 2. *Let $x \in H$. Then we put*

$$J(x) = \inf\{J_1(b);\, x \leqq b,\, b \in B\}$$

if the set $\{J_1(b);\, x \leqq b,\, b \in B\}$ is non empty; otherwise $J(x) = \infty$.

Theorem 1. *The function J is increasing and it is an extension of J_0. If $x_n \nearrow x$, then $J(x) = \lim_{n \to \infty} J(x_n)$.*

Proof. See [1, Prop. 3.1], [11, Theorem 3.1], [12, Theorem 1].

Of course, E. M. Alfsen does not assume that to any $x \in H$ there are $a_n \in A$ such that $x \leqq \vee a_n$. But then, in the case of $\lim_{n \to \infty} J(x_n) < \infty$, we are not able to prove that $\{J(b);\ x \leqq b \in B\} \neq \emptyset$ although $\{J(b);\ x_n \leqq b \in B\} \neq \emptyset$ $(n = 1, 2, \ldots)$ and hence there are $b_n \in B$, $b_n \geqq x_n$ $(n = 1, 2, \ldots)$. H need not be σ-complete and therefore $\vee b_n$ need not exist. It seems to us that this detail in the Alfsen theory is not correct.

A pseudometric

Now we shall not follow the excellent Alfsen definition $\varrho(x, y) = J(x \vee y) - J(x \wedge y)$, since we want to say a little more about the algebraic structure of investigated lattices. We shall introduce axiomatically two binary operations on H: $\triangle$ and $+$. Further we shall assume that all the elements of H are non-negative and hence H has the least element. If H is a set of sets, then $a \triangle b$ is the symmetric difference of a, b and $a+b$ is the union of a, b. If H is a set of functions, then $a+b$ has the usual sense and $a \triangle b = |a-b|$. The reader can easily verify that in the classical cases all our axioms are satisfied. Recently a similar algebraic structure has been studied in [4] and [5], where two binary operations $+$ and $\setminus$ are given. With respect to the Brehmer system (the so-called C-lattice) our operation $\triangle$ can be defined by the formula $a \triangle b = (a \setminus b) + (b \setminus a)$.

Assumptions. *H has the least element O. On the lattice H there are given two binary operations* $\triangle$, $+$ *satisfying the following identities:*

1.3. $a \triangle a = 0$, $a \triangle 0 = a$.
1.4. $a \triangle b = b \triangle a$.
1.5. $a + b = b + a$.
1.6. $a \leqq b \Rightarrow a + c \leqq b + c$.
1.7. $a_n \nearrow a$, $b_n \nearrow b \Rightarrow a_n + b_n \nearrow a + b$.
1.8. $a \triangle b \leqq (a \triangle c) + (b \triangle c)$.
1.9. $(a \vee b) \triangle (c \vee d) \leqq (a \triangle c) + (b \triangle d)$.
1.10. $(a \wedge b) \triangle (c \wedge d) \leqq (a \triangle c) + (b \triangle d)$.
1.11. $(a + b) \triangle (c + d) \leqq (a \triangle c) + (b \triangle d)$.
1.12. $a \leqq (a \triangle b) + b$.

A is closed under the operation $+$.
J_0 *has moreover the following properties:*

3.4. $J_0(0) = 0$.
3.5. $J_0(a + b) \leqq J_0(a) + J_0(b)$.

Lemma 2. *For any* $x, y \in H$ *it is* $J(x + y) \leqq J(x) + J(y)$.

Proof. Take first $a, b \in B$ and $a_n \in A$, $b_n \in A$ $(n = 1, 2, \ldots)$ such that $a_n \nearrow a$, $b_n \nearrow b$. Then by 1.7 also $a_n + b_n \nearrow a + b$, hence $a + b \in B$ and

$$J_1(a + b) = \lim_{n \to \infty} J_0(a_n + b_n) \leqq \lim_{n \to \infty} J_0(a_n) + \lim_{n \to \infty} J_0(b_n) =$$

$$= J_1(a) + J_1(b).$$

Finally let $x, y \in H$, $J(x) < \infty$, $J(y) < \infty$. Then to every $\varepsilon > 0$ there are $a, b \in B$ such that $x \leqq a$, $y \leqq b$ and

$$J(x) + \frac{\varepsilon}{2} > J_1(a), \quad J(y) + \frac{\varepsilon}{2} > J_1(b).$$

By 1.5 and 1.6 we have $x + y \leqq a + b$, hence

$$J(x + y) \leqq J_1(a + b) \leqq J_1(a) + J_1(b) < J(x) + J(y) + \varepsilon.$$

Lemma 3. *Let* $H_1=\{x\in H;\ J(x)<\infty\}$. *Then* $a+b\in H_1$, $a\triangle b\in H_1$ *for every* a, $b\in H_1$.

Proof. It follows from Lemma 2 and 1.8.

Definition 3. *Let* $H_1=\{x\in H;\ J(x)<\infty\}$. *We define a mapping* $\varrho\colon H_1\times H_1\to R$ *by the equality* $\varrho(x, y)=J(x\triangle y)$.

Lemma 4. ϱ *is a pseudometric on* H_1.

Proof. It follows from 1.3, 1.4, 1.8 and Lemma 2.

Now we can finish our extension process.

Definition 4. *Let* (H_1, ϱ) *be the pseudometric space defined in Definition 3. Since* J *is an extension of* J_0 *and* J_0 *is finite on* A, *we have* $A\subset H_1$. *Therefore we can define* $S=A^-$ *(the topological closure) and* $\bar{J}=J|S$ *(the restriction of* J *to* S).

Lemma 5. *For all* x, $y\in H_1$ *it holds* $|J(x)-J(y)|<J(x\triangle y)$.

Proof. By 1.12 and Lemma 2 we have $J(x)\leqq J(x\triangle y)+J(y)$ and similarly $J(y)\leqq J(x\triangle y)+J(x)$.

Theorem 2. *S is closed under the operations* $+$, $\vee$, $\wedge$; $\bar{J}$ *is a valuation on* S.

Proof. Evidently $x\in S$ if and only if to every $\varepsilon>0$ there is such an $a\in A$ that $J(a\triangle x)<\varepsilon$. Then the first three assertions follow from this fact, 1.9, 1.10, 1.11 and Lemma 2.

Now we prove that $\bar{J}$ is a valuation. Take x, $y\in S$. Let ε be an arbitrary positive number. Then there are such a, $b\in A$ that $J(x\triangle a)<\varepsilon$, $J(y\triangle b)<\varepsilon$, hence by Lemma 5

$$|J(x)-J(a)|<\varepsilon,\quad |J(y)-J(b)|<\varepsilon.$$

Further, by 1.9

$$\begin{aligned}|J(x\vee y)-J(a\vee b)|&\leqq J((x\vee y)\triangle(a\vee b))\leqq\\ &\leqq J(x\triangle a)+J(y\triangle b)<2\varepsilon.\end{aligned}$$

Analogously we have by 1.10

$$\begin{aligned}|J(x\wedge y)-J(a\wedge b)|&\leqq J((x\wedge y)\triangle(a\wedge b))\leqq\\ &\leqq J(x\triangle a)+J(y\triangle b)<2\varepsilon.\end{aligned}$$

Finally

$$\begin{gathered}|J(x\vee y)+J(x\wedge y)-J(x)-J(y)|\leqq\\ \leqq|J(x\vee y)-J(a\vee b)|+|J_0(a\vee b)+J_0(a\wedge b)-J_0(a)-\\ -J_0(b)|+|J_0(a)-J(x)|+|J_0(b)-J(y)|+\\ +|J(x\wedge y)-J_0(a\wedge b)|\leqq\\ \leqq 2\varepsilon+0+\varepsilon+\varepsilon+2\varepsilon=6\varepsilon.\end{gathered}$$

Since ε was arbitrary, we have $|J(x\vee y)+J(x\wedge y)-J(x)-J(y)|=0$.

A quasilinear structure

From the point of view of the applications it is useful to have some identity like $J(x+y)=J(x)+J(y)$ or $J(x-y)=J(x)-J(y)$. Since no such identity holds for measures, we shall work only with the implication $x \leqq y \Rightarrow J(y)=J(x)+J(y\backslash x)$. This implication holds for measures as well as for integrals. In the first case $y\backslash x$ is the set-theoretic difference and in the second case it is the difference of functions. Now we shall axiomatically introduce a binary operation \. However, in the case of functions we must be careful. Namely, we work only with non-negative functions and the difference of two non-negative functions need not be non-negative. Therefore we interprete $a\backslash b$ in this case as $a\backslash b=a-(a\wedge b)=a-\min(a, b)$ (see [4], [5], [10], [14]).

Assumptions. *On the lattice H there is given a binary operation \ satisfying the following conditions:*

1.13. $(a\backslash b)\triangle(c\backslash d)\leqq(a\triangle c)+(b\triangle d)$.

1.14. *If* $a\leqq b$, *then* $a\triangle b=b\backslash a$.

1.15. *If* $a\leqq b$, *then* $a=b\backslash(b\backslash a)$.

1.16. *If* $a_n\nearrow a$, *then* $a_n\backslash b\nearrow a\backslash b$.

1.17. *If* $a_n\searrow a$, *then* $a_1\backslash a_n\nearrow a_1\backslash a$.

The set A is closed under the operation \.

J_0 *has moreover the following property:*

3.6. $J_0(b)=J_0(a\wedge b)+J_0(b\backslash a)$.

Theorem 3. *S is closed under the operation \. For every* $x, y\in S$ *we have* $J(y)=J(x\wedge y)+J(y\backslash x)$.

Proof. The first assertion follows from 1.13 and Lemma 2. Let $x, y\in S$, $\varepsilon>0$. Then there are $a, b\in A$ such that $J(x\triangle a)<\varepsilon$, $J(y\triangle b)<\varepsilon$. Further

$$\begin{aligned}|J(y)-J(x\wedge y)-J(y\backslash x)|&\leqq|J(y)-J(b)|+\\+|J(b)-J(a\wedge b)-J(b\backslash a)|&+|J(a\wedge b)-J(x\wedge y)|+\\+|J(b\backslash a)-J(y\backslash x)|&\leqq\\ \leqq J(y\triangle b)+0+J((a\wedge b)\triangle(x\wedge y))&+J((b\backslash a)\triangle(y\backslash x))\leqq\\ \leqq J(y\triangle b)+J(a\triangle x)+J(b\triangle y)&+J(b\triangle y)+J(x\triangle a)<5\varepsilon.\end{aligned}$$

Limit theorems

Now let all the assumptions 1.1—1.17 and 3.1—3.6 be satisfied.

Theorem 4. *Let* $x_n\in S$ $(n=1, 2, \ldots)$, $x_n\nearrow x$, $\lim_{n\to\infty} J(x_n)<\infty$. *Then* $x\in S$ *(and, of course,* $J(x)=\lim_{n\to\infty} J(x_n)$ *by Theorem* 1).

Proof. By Theorem 3 we have

$$J(x_m) = J(x_m \wedge x_n) + J(x_m \backslash x_n).$$

Since $x_m \nearrow x$, then by 1.2 and 1.16 $x_m \wedge x_n \nearrow x \wedge x_n$, $x_m \backslash x_n \nearrow x \backslash x_n$, hence by Theorem 1

$$J(x) = J(x \wedge x_n) + J(x \backslash x_n) = J(x_n) + J(x \backslash x_n).$$

We know (Theorem 1) that $J(x) = \lim_{n\to\infty} J(x_n) < \infty$. Since $J(x) < \infty$, $J(x_n) < \infty$ and also $J(x \backslash x_n) < \infty$, we have

$$J(x \backslash x_n) = J(x) - J(x_n),$$

and therefore

$$\lim_{n\to\infty} J(x \backslash x_n) = 0.$$

Hence to every $\varepsilon > 0$ there is n such that $J(x \backslash x_n) < \varepsilon/2$. By 1.14 we have $x \triangle x_n = x \backslash x_n$, hence

$$\varrho(x, x_n) = J(x \triangle x_n) < \frac{\varepsilon}{2}.$$

But $x_n \in S$, hence there is $a \in A$ such that

$$\varrho(x_n, a) < \frac{\varepsilon}{2}$$

and therefore

$$\varrho(x, a) < \varepsilon.$$

We see that $x \in A^- = S$.

Theorem 5. *Let $x_n \in S$ $(n = 1, 2, \ldots)$, $x_n \searrow x$.*) Then $x \in S$ and $J(x) = \lim_{n\to\infty} J(x_n)$.*

Proof. First we prove that $x \in S$. By 1.17 $x_1 \backslash x_n \searrow x_1 \backslash x$. But

$$J(x_1 \backslash x_n) = J(x_1) - J(x_n)$$

by Theorem 3, hence $\lim_{n\to\infty} J(x_1 \backslash x_n) < \infty$. Hence by Theorem 4 $x_1 \backslash x \in S$ and

$$J(x_1 \backslash x) = \lim_{n\to\infty} J(x_1 \backslash x_n) = J(x_1) - \lim_{n\to\infty} J(x_n).$$

*) $\lim_{n\to\infty} J(x_n) > -\infty$ automatically, because J is a non-negative function.

Now 1.15 and Theorem 3 imply that

$$x = x_1 \setminus (x_1 \setminus x) \in S.$$

Moreover by Theorem 3

$$\lim_{n\to\infty} J(x_n) = J(x_1) - J(x_1 \setminus x) = J(x \wedge x_1) = J(x).$$

Linear case

In this section we shall deal with lattice ordered groups and we adopt the terminology used in [2].

Theorem 6. *Let G be an Abelian lattice ordered group, which is σ-complete (i.e. every non-empty countable bounded subset of G has the supremum and the infimum). Let F be a subgroup of G closed under the lattice operations. Let there to every $x \in G$ exist $a_n \in F$ $(n = 1, 2, \ldots)$ such that $x \leqq \vee a_n$. Finally let $I_0 : F \to R$ be a linear positive operator such that $x_n \searrow x$, $x_n \in F$ $(n = 1, 2, \ldots)$, $x \in F$ implies $I_0(x_n) \searrow I_0(x)$.*

Then there are a subgroup T containing F and closed under the operations $x \to x^+$, $x \to x^-$ and a linear positive operator I: $T \to R$, which is an extension of I_0 and is continuous in the following sense: If $x_n \nearrow x$ $(x_n \searrow x)$, $x_n \in T$ for all n and $(I(x_n))_{n=1}^{\infty}$ is bounded, then $x \in T$ and $I(x) = \lim_{n\to\infty} I(x_n)$.

Proof. Put $H = \{x \in G; x \geqq 0\}$, $A = F \cap H$, $J_0 = I_0 | A$. Further let $+$ be the group operation, $a \setminus b = a - (a \wedge b)$, $a \triangle b = |a - b|$. Evidently all assumptions 1.1—1.17, 3.1—3.6 are satisfied and hence all assertions of Theorems 2—5 hold. Of course, S need not be a subgroup and we do not know whether J is linear.

First we prove that J is linear on S. Let $f, g \in S$. Evidently $f, g \geqq 0$. Put $h = f + g$. Then $h \geqq f$, hence

$$J(f+g) = J(h) = J(f) + J(h \setminus f) =$$
$$= J(f) + J(h - f) = J(f) + J(g).$$

Now we define the set $T = \{x \in G;\ x = y - z,\ y \in S,\ z \in S\}$. Evidently T is a subgroup. If $x \in T$, Then $x = y - z$, where $y, z \in S$. Hence

$$x^+ = x \vee 0 = (y - z) \vee 0 = (y - z) \vee (z - z) =$$
$$= (y \vee z) - z \in T$$

and

$$-x^- = x \wedge 0 = (y - z) \wedge 0 = (y \wedge z) - z \in T.$$

Hence we can define I: $T \to R$ by the equality

$$I(x) = J(x^+) - J(x^-).$$

If $x = y - z$, where $y, z \in S$, then $y - z = x^+ - x^-$, hence $y + x^- = x^+ + z$ and by the linearity of J

$$J(y) + J(x^-) = J(x^+) + J(z),$$

whence

$$J(y) - J(z) = J(x^+) - J(x^-) = I(x).$$

If $x \in F$, then $x^+, x^- \in A \subset S$, hence $x = x^+ - x^- \in T$. Moreover,

$$I(x) = J(x^+) - J(x^-) = J_0(x^+) - J_0(x^-) = \\ = I_0(x^+) - I_0(x^-) = I_0(x),$$

hence I is an extension of I_0.

If $x_1, x_2 \in T$, $x_1 = y_1 - z_1$, $x_2 = y_2 - z_2$, $y_1, y_2, z_1, z_2 \in S$, then $x_1 + x_2 = = (y_1 + y_2) - (z_1 + z_2)$ and

$$I(x_1 + x_2) = J(y_1 + y_2) - J(z_1 + z_2) = \\ = J(y_1) - J(z_1) + J(y_2) - J(z_2) = I(x_1) + I(x_2),$$

I is linear. I is also positive, since $x = y - z \geqq 0$, $y, z \in S$ implies $y \geqq z$, hence $J(y) \geqq J(z)$ and $I(x) = J(y) - J(z) \geqq 0$.

Finally, let $x_n \nearrow x$, $x_n \in T$, $(I(x_n))_{n=1}^{\infty}$ is bounded. Then $x_n^+ \nearrow x^+$, $x_n^- \searrow x^-$. Moreover,

$$0 \leqq J(x_n^+) = I(x_n) + J(x_n^-) \leqq I(x_n) + J(x_1^-),$$

$$0 \leqq J(x_n^-) \leqq J(x_1^-),$$

hence both sequences $(J(x_n^+))_{n=1}^{\infty}$ and $(J(x_n^-))_{n=1}^{\infty}$ are bounded. By Theorems 4 and 5, $x^+, x^- \in S$ and $J(x^+) = \lim_{n\to\infty} J(x_n^+)$, $J(x^-) = \lim_{n\to\infty} J(x_n^-)$, hence $x \in T$ and

$$I(x) = J(x^+) - J(x^-) = \lim_{n\to\infty} I(x_n).$$

The dual assertion follows easily by the linearity of I.

Remark. In any Abelian lattice ordered group the two definitions of pseudometric

$$\varrho(x, y) = J(x \triangle y)$$

and

$$\varrho_1(x, y) = J(x \vee y) - J(x \wedge y)$$

coincide. Indeed, in the case

$$|x - y| = (x \vee y) - (x \wedge y)$$

(see [2], ch. XIV., § 4., Th. 8; of course, the proof is not very difficult). Since J is linear, we obtain

$$J(|x-y|)=J(x\vee y)-J(x\wedge y).$$

Measure

For measures we do not obtain any new result.

Theorem 7. *Let H be a relatively σ-complete Boolean algebra, $A\subset H$ be a Boolean ring and $J_0\colon A\to R$ a finite measure. Then there is a measure J' defined on the δ-ring D generated by A that is an extension of J_0.*

Proof. We again apply Theorems 2—5. Here $a+b=a\vee b$, $a\backslash b=a\wedge b'$ (b' is the complement of b), $a\triangle b=(a\backslash b)\vee(b\backslash a)$. The theorem will be proved if we show that $D\subset S$. But S is a ring closed under countable infimums, hence S is a δ-ring over A. Therefore S contains the least δ-ring over A.

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ПРОДОЛЖЕНИЕ МЕР И ИНТЕГРАЛОВ ПРИ ПОМОЩИ ПСЕВДОМЕТРИКИ

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Резюме

В работе продолжается действительная функция J_0 определенная на некоторой подструктуре R данной структуры H. При помощи подходящей псевдометрики на H продолжается J_0 на замыкание R^- множества R. Если в качестве H взять некоторую структуру множеств, то возможно получить теорему о продолжении меры, если взять структуру функций, то возможно получить теорему о продолжении интеграла.