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ON SIGNED MAJORITY TOTAL DOMINATION IN GRAPHS

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Abstract. We initiate the study of signed majority total domination in graphs. Let $G = (V, E)$ be a simple graph. For any real valued function $f\colon V \to \mathbb{R}$ and $S \subseteq V$, let $f(S) = \sum_{v \in S} f(v)$. A signed majority total dominating function is a function $f\colon V \to \{-1, 1\}$ such that $f(N(v)) \geqslant 1$ for at least a half of the vertices $v \in V$. The signed majority total domination number of a graph G is $\gamma^{\rm t}_{\rm maj}(G) = \min\{f(V)\colon f$ is a signed majority total dominating function on $G\}$. We research some properties of the signed majority total domination number of a graph G and obtain a few lower bounds of $\gamma^{\rm t}_{\rm maj}(G)$.

Keywords: signed majority total dominating function, signed majority total domination number

MSC 2000: 05C35

1. Introduction

Let $G = (V, E)$ be a simple graph and v a vertex in V. The open neighborhood of v, denoted by $N(v)$, is the set of vertices adjacent to v, i.e., $N(v) = \{u \in V\colon uv \in E\}$. The closed neighborhood of v is the set $N[v] = N(v) \cup \{v\}$. The degree of v in G is $d_G(v) = |N(v)|$, a vertex v is called an even vertex if $d_G(v)$ is even. $\Delta(G)$ and $\delta(G)$ denote the maximum degree and the minimum degree of the vertices of G. When no ambiguity can occur, we often write simply $d(v)$, δ, Δ instead of $d_G(v)$, $\delta(G)$ and $\Delta(G)$, respectively. Let $S \subseteq V$, $G[S]$ denote the subgraph of G induced by S.

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2. Definition of signed majority total domination

For any real-valued function $f\colon V \to \mathbb{R}$ and $S \subseteq V$, let $f(S) = \sum_{v\in S} f(v)$. The weight of f is defined as $f(V)$. A function $f\colon V \to \{-1,1\}$ is said to be a *majority dominating function* on G if $f(N[v]) \geqslant 1$ for at least half the vertices $v \in V$. The *majority domination number* is $\gamma_{\rm maj}(G) = \min\{f(V)\colon f$ is a majority dominating function on $G\}$.

A *signed total dominating function* (STDF) of G is defined in [1] as a function $f\colon V \to \{-1,1\}$, such that $f(N(v)) \geqslant 1$ for every $v \in V$. A STDF f is minimal if no $g < f$ is also a STDF on G. The *signed total domination number* $\gamma_{\rm st}(G) = \min\{f(V)\colon f$ is a STDF on $G\}$.

In this paper we initiate the study of majority domination in graphs. A function $f\colon V \to \{-1,1\}$ is said to be a *signed majority total dominating function* (SMTDF) on G if $f(N[v]) \geqslant 1$ for at least a half of the vertices $v \in V$. The *signed majority total domination number* of G denoted by $\gamma^{\rm t}_{\rm maj}(G)$, is equal to $\min\{f(V)\colon f$ is a SMTDF on $G\}$. A SMTDF f is minimal if no $g < f$ is a SMTDF on G.

To ensure existence of SMTDF, we henceforth restrict our attention to graphs without isolated vertices.

The motivation for studying this variation of the signed majority total domination number is rich and varied. For example, by assigning the value -1 or $+1$ to the vertices of a graph we can model networks of people or organizations in which global decisions must be made (e.g., positive or negative responses or preferences). We assume that each individual has one vote and that each individual has an initial opinion. We assign $+1$ to vertices (individuals) which have a positive opinion and -1 to vertices which have a negative opinion. We also assume, however, that an individual's vote is affected by the opinions of neighboring individuals. In particular, each individual gives equal weight to the opinions of the neighboring individuals (thus individuals of high degree have greater 'influence'). A voter votes 'aye' if there are more vertices in its open neighborhood with positive opinion than those with negative opinion, otherwise the vote is 'nay'. We seek an assignment of opinions that guarantee a majority decision, that is, for which at least a half of the vertices vote aye. We call such an assignment of opinions a positive majority assignment. Among all positive majority assignments of opinions, we are interested primarily in the minimum number of vertices (individuals) who have a positive opinion. The signed majority total domination number is the minimum possible sum of all opinions, -1 for a negative opinion, $+1$ for a positive opinion, in a positive majority assignment of opinions. The signed majority total domination number represents, therefore, the minimum number of individuals which can have positive opinions and in voting so force at least a half of individuals to vote aye.

Theorem 1. *A signed majority total dominating function f of a graph G is minimal only if for every vertex $v \in V$ with $f(v) = 1$ there exists a vertex $u \in N(v)$ with $f(N(u)) \in \{1, 2\}$.*

P r o o f. Let f be a minimal SMTDF and assume that there is a vertex v such that $f(v) = 1$ and $f(N(u)) \notin \{1, 2\}$ for any $u \in N(v)$. Define a new function $g\colon V \to \{-1, 1\}$ by $g(v) = -1$ and $g(w) = f(w)$ for all $w \neq v$. Then for all $u \in N(v)$ either $f(N(u)) \leqslant 0$, in which case $g(N(u)) = f(N(u)) - 2 \leqslant 0 - 2 = -2$, or $f(N(u)) \geqslant 3$, in which case $g(N(u)) \geqslant 1$. For $w \notin N(v)$ we have $g(N(w)) = f(N(w))$. Thus g is a signed majority total dominating function. Since $g < f$, the minimality of f is contradicted. $\square$

3. Graphs with positive or negative signed majority total domination numbers

Obviously, every signed total dominating function is also a signed majority total dominating function. Thus we have the following result.

Theorem 2. *For any graph G we have $\gamma^{\rm t}_{\rm maj}(G) \leqslant \gamma_{\rm st}(G)$.*

Theorem 3. *For any complete graph K_n $(n \geqslant 2)$ we have*

$$\gamma^{\rm t}_{\rm maj}(K_n) = \begin{cases} 3 & \textit{for } n \textit{ odd}, \\ 0 & \textit{for } n \textit{ even}. \end{cases}$$

P r o o f. Let $K_n = (V, E)$. Let f be a minimum SMTDF on K_n. Then there exists at least one vertex $v \in V$, $f(N(v)) \geqslant 1$. Let P and M be the sets of vertices in K_n that are assigned the values $+1$ and -1 under f, respectively. Then $|P| + |M| = n$ and $|P| - |M| = f(V) = f(N(v)) + f(v) \geqslant 1 - 1 = 0$. It follows that $|P| \geqslant \lceil n/2 \rceil$ and $|M| \leqslant \lfloor n/2 \rfloor$. Hence $\gamma^{\rm t}_{\rm maj}(K_n) = |P| - |M| \geqslant \lceil n/2 \rceil - \lfloor n/2 \rfloor$.

Case 1: If n is even, then $\gamma^{\rm t}_{\rm maj}(K_n) \geqslant 0$. Define $g\colon V \to \{-1, 1\}$ by

$$g(x) = \begin{cases} 1 & \text{for } n/2 \text{ vertices } x \text{ in } K_n, \\ -1 & \text{otherwise.} \end{cases}$$

Then g is a SMTDF of K_n of zero weight. So $\gamma^{\rm t}_{\rm maj}(K_n) \leqslant 0$. Consequently, $\gamma^{\rm t}_{\rm maj}(K_n) = 0$.

Case 2: If n is odd, then $\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_n) = |P| - |M| \geqslant 1$, i.e., $|P| \geqslant |M| + 1$. Then there exists one vertex $u \in P$ such that $f(N(u)) \geqslant 1$. Thus $|P| - |M| = f(V) = f(N(u)) + f(u) \geqslant 2$ and $|P| + |M| = n$. It follows that $|P| \geqslant \lceil (n+2)/2 \rceil$ and $|M| \leqslant \lfloor (n-2)/2 \rfloor$. Hence $\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_n) = |P| - |M| \geqslant \lceil (n+2)/2 \rceil - \lfloor (n-2)/2 \rfloor = 3$. Define $g\colon V \to \{-1, 1\}$ by

$$g(x) = \begin{cases} 1 & \text{for } \lceil (n+2)/2 \rceil \text{ vertices } x \text{ in } K_n, \\ -1 & \text{otherwise.} \end{cases}$$

Then g is a SMTDF of K_n of weight 3. So $\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_n) \leqslant 3$. Consequently, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_n) = 3$. □

Theorem 4. *For any complete bipartite graph $K_{m,n}$ $(n \geqslant m \geqslant 1)$,*

$$\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_{m,n}) = \begin{cases} 1 - n & \text{for } m \text{ odd}, \\ 2 - n & \text{for } m \text{ even}. \end{cases}$$

Proof. Let f be a minimum SMTDF on $K_{m,n}$. Let U and W be the partite sets of $K_{m,n}$ with $|U| = m$ and $|W| = n$. Let U^+ and U^- be the sets of vertices in U that are assigned the value $+1$ and -1 under f, respectively. Let W^+ and W^- be defined analogously. Then $\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_{m,n}) = f(V) = |U^+| - |U^-| + |W^+| - |W^-|$. If $m = n$, then, by relabelling the sets U and W if necessary, we may assume that $f(N(w)) \geqslant 1$ for at least one vertex w in W. If $m < n$, then, since at least a half of the open neighborhood sums under f are positive, it is evident that $f(N(w)) \geqslant 1$ for at least one vertex w in W. Hence, $f(U) = f(N(w)) \geqslant 1$.

We now show that $W = W^-$, that is to say, each vertex of W is assigned the value -1 under f. Assume the contrary, i.e., there exists at least one vertex $v \in W^+$. Let $g\colon V \to \{-1, 1\}$ be defined as follows: $g(v) = -1$ and $g(u) = f(u)$ for $u \neq v$. Then $g(N(w)) = g(U) \geqslant 1$ for each $w \in W$. Since $|W| \geqslant |U|$, it follows that g is a SMTDF on $K_{m,n}$ of weight less than that of f, a contradiction. Therefore, we have $W = W^-$.

Since $|U^+| - |U^-| = f(U) \geqslant 1$ and $m = |U^+| + |U^-|$, it follows that $|U^+| \geqslant \lceil (m+1)/2 \rceil$ and $|U^-| \leqslant \lfloor (m-1)/2 \rfloor$. Hence, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_{m,n}) = |U^+| - |U^-| + |W^+| - |W^-| \geqslant \lceil (m+1)/2 \rceil - \lfloor (m-1)/2 \rfloor - n$. However, if we assign to $\lceil (m+1)/2 \rceil$ vertices of U the value $+1$, and to the remaining $n + \lfloor (m-1)/2 \rfloor$ in $K_{m,n}$ the value -1, then we produce a SMTDF of $K_{m,n}$ of weight $\lceil (m+1)/2 \rceil - \lfloor (m-1)/2 \rfloor - n$. Thus $\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_{m,n}) \leqslant \lceil (m+1)/2 \rceil - \lfloor (m-1)/2 \rfloor - n$. Consequently, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_{m,n}) = \lceil (m+1)/2 \rceil - \lfloor (m-1)/2 \rfloor - n$. □

Corollary 1. *For any positive integer k, there exists a complete bipartite graph G with $\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) \leqslant -k$.*

Theorem 5. *For any path P_n $(n \geqslant 2)$ we have*

$$\gamma^{\mathrm{t}}_{\mathrm{maj}}(P_n) = \begin{cases} -1 & \text{for } n \text{ odd,} \\ 0 & \text{for } n \text{ even.} \end{cases}$$

P r o o f. Let $P_n\colon v_1, v_2, \ldots v_n$ be a path on n vertices, and let f be a minimum SMTDF on P_n. Let $C_f = \{v \in P_n\colon f(N(v)) \geqslant 1\}$, $P = \{v \in P_n\colon f(v) = 1\}$ and $M = \{v \in P_n\colon f(v) = -1\}$, then $|C_f| \geqslant \lceil n/2 \rceil$. Since $|N(v)| \in \{1, 2\}$ for any $v \in C_f$, we have $N(v) \subseteq P$. It follows that $|P| \geqslant \lfloor n/2 \rfloor$ and $|M| \leqslant \lceil n/2 \rceil$. Hence, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(P_n) \geqslant \lfloor n/2 \rfloor - \lceil n/2 \rceil$.

On the other hand, define a function $g\colon V \to \{-1, 1\}$ by

$$g(v_i) = \begin{cases} -1 & \text{for } i \text{ odd,} \\ 1 & \text{for } i \text{ even.} \end{cases}$$

Then g is a SMTDF of P_n of weight $\lfloor n/2 \rfloor - \lceil n/2 \rceil$. So $\gamma^{\mathrm{t}}_{\mathrm{maj}}(P_n) \leqslant \lfloor n/2 \rfloor - \lceil n/2 \rceil$. Consequently, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(P_n) = \lfloor n/2 \rfloor - \lceil n/2 \rceil$. The result now follows. □

Theorem 6. *For any cycle C_n $(n \geqslant 3)$ we have*

$$\gamma^{\mathrm{t}}_{\mathrm{maj}}(C_n) = \begin{cases} 3 & \text{for } n \text{ odd,} \\ 0 & \text{for } n \text{ even.} \end{cases}$$

P r o o f. Let $C_n\colon v_1, v_2, \ldots v_n$ be a cycle on n vertices and f a minimum SMTDF on C_n. Let $C_f = \{v \in C_n\colon f(N(v)) \geqslant 1\}$, $P = \{v \in C_n\colon f(v) = 1\}$ and $M = \{v \in C_n\colon f(v) = -1\}$. If n is even, then similarly to the proof of Theorem 5 we have $\gamma^{\mathrm{t}}_{\mathrm{maj}}(C_n) = 0$. Therefore, in the following proof we assume that n is odd. Then $|C_f| \geqslant (n+1)/2$. Since $|N(v)| = 2$ for every $v \in C_f$, we have $N(v) \subseteq P$. It follows that $|P| \geqslant (n+1)/2$ and $|M| \leqslant (n-1)/2$. Then there exists at least one vertex in $P \cap C_f$, i.e., there exist at least three consecutive vertices in $C_n[P]$.

Case 1: $M = \emptyset$.

In this case, $P = n \geqslant 3$, $|M| = 0$. So $\gamma^{\mathrm{t}}_{\mathrm{maj}}(C_n) = |P| - |M| = n \geqslant 3$.

Case 2: Every vertex is an isolated vertex in $C_n[M]$.

Since there exist at least three consecutive vertices in $C_n[P]$, we have $|P| \geqslant 3 + (n-3)/2 = (n+3)/2$. Thus $|M| \leqslant (n-3)/2$. So $\gamma^{\mathrm{t}}_{\mathrm{maj}}(C_n) = |P| - |M| \geqslant 3$.

Case 3: There exist some vertices which are not the isolated vertices in $C_n[M]$.

Without loss of generality, we can assume that $f(v_1) = f(v_2) = \ldots = f(v_k) = -1$, $f(v_{k+1}) = f(v_n) = 1$ $(k \geqslant 2)$. Let $U = C_n - \{v_1, v_2, \ldots, v_k\}$. Then $C_f \subseteq U$. Note that $\sum\limits_{v \in P} d_{C_n[U]}(v) \geqslant 2|C_f| \geqslant 2 \times (n+1)/2 = n+1$. Since $d_{C_n[U]}(v_{k+1}) = d_{C_n[U]}(v_n) = 1$, we have $|P| \geqslant 2 + (n-1)/2 = (n+3)/2$. Thus $|M| \leqslant (n-3)/2$. So $\gamma^{\mathrm{t}}_{\mathrm{maj}}(C_n) = |P| - |M| \geqslant 3$.

On the other hand, define a function $g\colon V \to \{-1, 1\}$ by

$$g(v_i) = \begin{cases} 1 & \text{for } i = 2 \text{ or } i \text{ odd}, \\ -1 & \text{for } i \geqslant 4 \text{ and } i \text{ even}. \end{cases}$$

Then g is a SMTDF of C_n of weight 3. So $\gamma^{\mathrm{t}}_{\mathrm{maj}}(C_n) \leqslant 3$. Consequently, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(C_n) = 3$. The result now follows. $\square$

In general, $\gamma^{\mathrm{t}}_{\mathrm{maj}}$ and γ_{maj} are not comparable. On the one hand, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_{m,n}) = 1 - n < \gamma_{\mathrm{maj}}(K_{m,n}) = 3 - n$ for m odd $(1 < m \leqslant n)$, on the other hand, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(K_n) = 3 > \gamma_{\mathrm{maj}}(K_n) = 1$ for n odd.

For m an integer, let $P(m, \gamma^{\mathrm{t}}_{\mathrm{maj}})$ be the smallest order of a connected graph with signed majority total domination number equal to m.

Theorem 7. *For $m \geqslant 0$ an integer, $P(-m, \gamma^{\mathrm{t}}_{\mathrm{maj}}) = m + 2$.*

Proof. Let $G = (V, E)$ be a connected graph with $\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) = -m$, and consider a minimum SMTDF f on G. P and M be the sets of vertices in G that are assigned the values $+1$ and -1 under f, respectively. Then $-m = f(V) = |P| - |M|$, so $|M| = |P| + m$. Obviously, $|P| \geqslant 1$, thus $|V| = |P| + |M| = 2|P| + m \geqslant m + 2$. On the other hand, by Theorem 2, we note that star $K_{1,m+1}$ is a connected graph of order $m + 2$ with signed majority total domination number equal to $-m$. So $P(-m, \gamma^{\mathrm{t}}_{\mathrm{maj}}) \leqslant m + 2$. Consequently, $P(-m, \gamma^{\mathrm{t}}_{\mathrm{maj}}) = m + 2$. $\square$

4. Lower bounds on signed majority total domination number

Theorem 8. *For any graph G of order n and maximum Δ, minimum degree $\delta \geqslant 1$,*

$$\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) \geqslant \frac{\delta - 2\Delta + 1}{\Delta + \delta} n.$$

Proof. Let f be a minimum SMTDF on G. Let P and M be the sets of vertices in G that are assigned the values $+1$ and -1 under f, respectively. Let $P = P_\Delta \cup P_\delta \cup P_\Theta$ where P_Δ and P_δ are sets of all vertices of P with degree equal to Δ and δ, respectively, and P_Θ contains all other vertices in P, if any.

Let $M = M_\Delta \cup M_\delta \cup M_\Theta$ whereP_Δ, P_δ and P_Θ are defined similarly. Further, for $i \in \{\Delta, \delta, \Theta\}$, let V_i be defined by $V_i = P_i \cup M_i$. Thus $n = |V_\Delta| + |V_\delta| + |V_\Theta|$.

Since for at least a half of the vertices $v \in V$, $f(N(v)) \geqslant 1$, we have

$$\sum_{v\in V} f(N(v)) \geqslant \lceil n/2\rceil - \Delta(n - \lceil n/2\rceil) = (\Delta+1)\lceil n/2\rceil - \Delta n \geqslant (1-\Delta)n/2.$$

The sum $\sum\limits_{v\in V} f(N(v))$ counts the value $f(v)$ exactly $d(v)$-times for each vertex $v \in V$, i.e., $\sum\limits_{v\in V} f(N(v)) = \sum\limits_{v\in V} f(v)d(v)$. Thus, $\sum\limits_{v\in V} f(v)d(v) \geqslant (1-\Delta)n/2$. Breaking the sum up into the six summations and replacing $f(v)$ with the corresponding value of 1 or -1 yields

$$\sum_{v\in P_\Delta} d(v) + \sum_{v\in P_\delta} d(v) + \sum_{v\in P_\Theta} d(v) - \sum_{v\in M_\Delta} d(v) - \sum_{v\in M_\delta} d(v) - \sum_{v\in M_\Theta} d(v) \geqslant (1-\Delta)n/2.$$

We know that $d(v) = \Delta$ for all v in P_Δ or M_Δ, and $d(v) = \delta$ for all v in P_δ or M_δ. For any vertex v in either P_Θ or M_Θ, $\delta + 1 \leqslant d(v) \leqslant \Delta - 1$. Therefore, we have

$$\Delta|P_\Delta| + \delta|P_\delta| + (\Delta-1)|P_\Theta| - \Delta|M_\Delta| - \delta|M_\delta| - (\delta+1)|M_\Theta| \geqslant (1-\Delta)n/2.$$

For $i \in \{\Delta, \delta, \Theta\}$, we replace $|P_i|$ with $|V_i| - |M_i|$ in the above inequality. Therefore, we have

$$\Delta|V_\Delta| + \delta|V_\delta| + (\Delta-1)|V_\Theta| \geqslant (1-\Delta)n/2 + 2\Delta|M_\Delta| + 2\delta|M_\delta| + (\Delta+\delta)|M_\Theta|.$$

It follows that

$$\begin{aligned}
(3\Delta-1)n/2 &\geqslant 2\Delta|M_\Delta| + 2\delta|M_\delta| + (\Delta+\delta)|M_\Theta| + (\Delta-\delta)|V_\delta| + |V_\Theta| \\
&= 2\Delta|M_\Delta| + 2\delta|M_\delta| + (\Delta+\delta)|M_\Theta| + (\Delta-\delta)(|P_\delta| + |M_\delta|) \\
&\quad + (|P_\Theta| + |M_\Theta|) \\
&= 2\Delta|M_\Delta| + (\Delta+\delta)|M_\delta| + (\Delta+\delta+1)|M_\Theta| + (\Delta-\delta)|P_\delta| + |P_\Theta| \\
&\geqslant (\Delta+\delta)|M_\Delta| + (\Delta+\delta)|M_\delta| + (\Delta+\delta)|M_\Theta| = (\Delta+\delta)|M|.
\end{aligned}$$

Therefore,

$$|M| \leqslant \frac{3\Delta-1}{2(\Delta+\delta)}n.$$

So

$$\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) = |P| - |M| = n - 2|M| \geqslant n - \frac{3\Delta-1}{\Delta+\delta}n = \frac{\delta-2\Delta+1}{\Delta+\delta}n.$$

□

Similarly we have the following result.

Theorem 9. *If a graph G has no isolated vertices and every vertex in G is even vertex, then*

$$\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) \geqslant \frac{\delta - 2\Delta + 2}{\Delta + \delta} n.$$

Theorem 10. *If G is a k-regular graph of order n, then $\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) \geqslant (1-k)n/(2k)$ if k is odd, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) \geqslant (2-k)n/(2k)$ if k is even, and these bounds are sharp.*

Proof. If k is odd, by Theorem 8, we have $\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) \geqslant (1-k)n/(2k)$. If k is even, by Theorem 9, we have $\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) \geqslant (2-k)n/(2k)$.

That the bounds are sharp may be seen by considering a complete regular bipartite graph $G = K_{k,k}$. Obviously, if k is odd, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) = -k + ((k+1)/2 - (k-1)/2) = 1 - k = (1-k)n/(2k)$; if k is even, $\gamma^{\mathrm{t}}_{\mathrm{maj}}(G) = -k + (k/2+1) - (k/2-1) = 2 - k = (2-k)n/(2k)$. □

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