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ON THE SET OF SOLUTIONS OF SOME NONCONVEX NONCLOSED HYPERBOLIC DIFFERENTIAL INCLUSIONS

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Abstract. We consider a class of nonconvex and nonclosed hyperbolic differential inclusions and we prove the arcwise connectedness of the solution set.

Keywords: hyperbolic differential inclusions, fixed point, solution set

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1. Introduction

This paper is concerned with the Darboux problem for hyperbolic differential inclusions of the form

$$\tag{1.1} \begin{gathered} u_{xy}(x,y) \in F\big(x,y,u(x,y),G(x,y,u(x,y))\big) \\ u(x,0)=\lambda(x,0), \quad u(0,y)=\lambda(0,y) \end{gathered}$$

where F is a multifunction from $Q\times\mathbb{R}^n\times\mathbb{R}^n$ to the nonempty subsets of $\mathbb{R}^n$, G is a multifunction from $Q\times\mathbb{R}^n$ to the nonempty subsets of $\mathbb{R}^n$, $Q=[0,1]\times[0,1]$ and $\lambda(x,y)=\alpha(x)+\beta(y)-\alpha(0)$ with α and β two continuous functions from $[0,1]$ to $\mathbb{R}^n$, satisfying $\alpha(0)=\beta(0)$.

When F does not depend on the last variable, (1.1) reduces to

$$\tag{1.2} \begin{gathered} u_{xy}(x,y) \in F(x,y,u(x,y)) \\ u(x,0)=\lambda(x,0), \quad u(0,y)=\lambda(0,y). \end{gathered}$$

In this case, qualitative properties and structure of the set of solutions of the Darboux problem (1.2) have been studied by many authors ([1], [2], [3], [4], [5], [9],

[10], etc.). In [4] it is shown that the solution set of (1.2) with F single valued is an R_δ-set, in [3] it is proved that the solution set is a retract of a convex subset of a Banach space and in [9] a solution of (1.2) continuous with respect to λ is constructed.

In all these results the set-valued map F is assumed to be at least closed-valued. Such an assumption is quite natural in order to obtain good properties of the solution set, but it is interesting to investigate the problem when the right-hand side of the multivalued equation may have nonclosed values.

Following the approach in [6], [7] we consider the problem (1.1), where F and G are closed-valued multifunctions Lipschitzian with respect to the second variable and F is contractive in the third variable. Obviously, the right-hand side of the differential inclusion in (1.1) is in general neither convex nor closed. We prove the arcwise connectedness of the solution set to (1.1). The main tool is a result ([6], [7]) concerning the arcwise connectedness of the fixed point set of a class of nonconvex nonclosed set-valued contractions.

The paper is organized as follows: in Section 2 we recall some preliminary results that we use in the sequel and in Section 3 we prove our main result.

2. Preliminaries

Let Z be a metric space with the distance d_Z and let 2^Z be the family of all nonempty closed subsets of Z. For $a \in Z$ and $A, B \in 2^Z$ set $d_Z(a, B) = \inf_{b\in B} d_Z(a, b)$ and $d^*_Z(A, B) = \sup_{a\in A} d_Z(a, B)$. Denote by D_Z the Hausdorff generalized metric on 2^Z defined by

$$D_Z(A, B) = \max\{d^*_Z(A, B), d^*_Z(B, A)\}, \quad A, B \in 2^Z.$$

In what follows, when the product $Z = Z_1 \times Z_2$ of metric spaces Z_i, $i = 1, 2$, is considered, it is assumed that Z is equipped with the distance $d_Z\big((z_1, z_2), (z'_1, z'_2)\big) = \sum_{i=1}^{2} d_{Z_i}(z_i, z'_i)$.

Let X be a nonempty set and let $F\colon X \to 2^Z$ be a set-valued map from X to Z. The range of F is the set $F(X) = \bigcup_{x\in X} F(x)$. Let $(X, \mathscr{F})$ be a measurable space. The multifunction $F\colon X \to 2^Z$ is called measurable if $F^{-1}(\Omega) \in \mathscr{F}$ for any open set $\Omega \subset Z$, where $F^{-1}(\Omega) = \{x \in X;\ F(x) \cap \Omega \neq \emptyset\}$. Let (X, d_X) be a metric space. The multifunction F is called Hausdorff continuous if for any $x_0 \in X$ and every $\varepsilon > 0$ there exists $\delta > 0$ such that $x \in X$, $d_X(x, x_0) < \delta$ implies $D_Z(F(x), F(x_0)) < \varepsilon$.

Let $(T, \mathscr{F}, \mu)$ be a finite, positive, nonatomic measure space and let $(X, |.|_X)$ be a Banach space. We denote by $L^1(T, X)$ the Banach space of all (equivalence classes of) Bochner integrable functions $u\colon T \to X$ endowed with the norm

$$|u|_{L^1(T,X)} = \int_T |u(t)|_X \,\mathrm{d}\mu.$$

A nonempty set $K \subset L^1(T,X)$ is called decomposable if, for every $u, v \in K$ and every $A \in \mathscr{F}$, one has

$$\chi_A \cdot u + \chi_{T \setminus A} \cdot v \in K$$

where χ_B, $B \in \mathscr{F}$ indicates the characteristic function of B.

A metric space Z is called an absolute retract if, for any metric space X and any nonempty closed set $X_0 \subset X$, every continuous function $g\colon X_0 \to Z$ has a continuous extension $g\colon X \to Z$ over X. It is obvious that every continuous image of an absolute retract is an arcwise connected space.

In what follows we recall some preliminary results that are the main tools in the proof of our result.

Let $(T, \mathscr{F}, \mu)$ be a finite, positive, nonatomic measure space, S a separable Banach space and let $(X, |.|_X)$ be a real Banach space. To simplify the notation we write E in place of $L^1(T,X)$.

Lemma 2.1 ([7]). *Assume that $\varphi\colon S \times E \to 2^E$ and $\psi\colon S \times E \times E \to 2^E$ are Hausdorff continuous multifunctions with nonempty, closed, decomposable values, satisfying the following conditions:*

a) *There exists $L \in [0,1)$ such that, for every $s \in S$ and every $u, u' \in E$,*

$$D_E\big(\varphi(s,u), \varphi(s,u')\big) \leqslant L|u - u'|_E.$$

b) *There exists $M \in [0,1)$ such that $L + M < 1$ and for every $s \in S$ and every $(u,v), (u',v') \in E \times E$,*

$$D_E\big(\psi(s,u,v), \psi(s,u',v')\big) \leqslant M(|u-u'|_E + |v-v'|_E).$$

Set $\mathrm{Fix}(\Gamma(s,.)) = \{u \in E;\ u \in \Gamma(s,u)\}$, where

$$\Gamma(s,u) = \psi\big(s,u,\varphi(s,u)\big),\ \ (s,u) \in S \times E.$$

Then

1) *For every $s \in S$ the set $\mathrm{Fix}(\Gamma(s,.))$ is nonempty and arcwise connected.*
2) *For any $s_i \in S$, and any $u_i \in \mathrm{Fix}(\Gamma(s,.))$, $i = 1, \ldots, p$ there exists a continuous function $\gamma\colon S \to E$ such that $\gamma(s) \in \mathrm{Fix}(\Gamma(s,.))$ for all $s \in S$ and $\gamma(s_i) = u_i$, $i = 1, \ldots, p$.*

Lemma 2.2 ([7]). *Let $U\colon T \to 2^X$ and $V\colon T\times X \to 2^X$ be two nonempty closed-valued multifunctions satisfying the following conditions:*

a) *U is measurable and there exists $r \in L^1(T)$ such that $D_X(U(t),\{0\}) \leqslant r(t)$ for almost all $t \in T$.*

b) *The multifunction $t \to V(t,x)$ is measurable for every $x \in X$.*

c) *The multifunction $x \to V(t,x)$ is Hausdorff continuous for all $t \in T$.*

Let $v\colon T \to X$ be a measurable selection from $t \to V(t,U(t))$. Then there exists a selection $u \in L^1(T,X)$ such that $v(t) \in V(t,u(t))$, $t \in T$.

Let Q be the square $I \times I$, where $I = [0,1]$. We denote by C the Banach space of all continuous functions $u\colon Q \to \mathbb{R}^n$ endowed with the norm $|u|_C = \sup\limits_{(x,y)\in Q} |u(x,y)|$. Given a continuous strictly positive function $a\colon Q \to \mathbb{R}$, we denote by L^1 the Banach space of all (equivalence classes of) Lebesgue measurable functions $\sigma\colon Q \to \mathbb{R}^n$, endowed with the norm

$$|\sigma|_1 = \iint_Q a(x,y)|\sigma(x,y)|\,\mathrm{d}x\,\mathrm{d}y. \tag{2.1}$$

By Λ we mean the linear subspace of C consisting of all $\lambda \in C$ such that there exist continuous functions $\alpha\colon I \to \mathbb{R}^n$, $\beta\colon I \to \mathbb{R}^n$, with $\alpha(0) = \beta(0)$, satisfying $\lambda(x,y) = \alpha(x) + \beta(y) - \alpha(0)$ for every $(x,y) \in Q$. Observe that Λ, equipped with the norm of C, is a separable Banach space.

In order to study problem (1.1) we introduce the following

Hypothesis 2.3. *Let $F\colon Q\times\mathbb{R}^n\times\mathbb{R}^n \to 2^{\mathbb{R}^n}$ and $G\colon Q\times\mathbb{R}^n \to 2^{\mathbb{R}^n}$ be two set-valued maps with nonempty closed values, satisfying the following assumptions:*

i) *The set-valued maps $(x,y) \to F(x,y,u,v)$ and $(x,y) \to G(x,y,u)$ are measurable for all $u,v \in \mathbb{R}^n$.*

ii) *There exist $l \in L^1(Q)$ such that, for every $u,u' \in \mathbb{R}^n$,*

$$D\big(G(x,y,u),G(x,y,u')\big) \leqslant l(x,y)|u-u'| \quad \text{a.e. } (Q).$$

iii) *There exist $m \in L^1(Q)$ and $\theta \in [0,1)$ such that, for every $u,v,u',v' \in \mathbb{R}^n$,*

$$D\big(F(x,y,u,v),F(x,y,u',v')\big) \leqslant m(x,y)|u-u'| + \theta|v-v'| \quad \text{a.e. } (Q).$$

iv) *There exist $f,g \in L^1(Q)$ such that*

$$d\big(\{0\},F(x,y,\{0\},\{0\})\big) \leqslant f(x,y),$$
$$d\big(\{0\},G(x,y,\{0\})\big) \leqslant g(x,y) \quad \text{a.e. } (Q).$$

For $(x,y)\in Q$ and $\varepsilon>0$, we put

$$Q(x,y)=[0,x]\times[0,y],\quad R(x,y)=[x,1]\times[y,1].$$

For $\sigma\in L^1(Q)$ let us consider the following Darboux problem:

$$\begin{aligned} u_{xy}(x,y)&=\sigma(x,y)\\ u(x,0)=\lambda(x,0),&\quad u(0,y)=\lambda(0,y). \end{aligned} \tag{2.2}$$

Definition 2.4. Let $\lambda\in\Lambda$. The function $u\in C$ given by

$$u(x,y)=\lambda(x,y)+\iint_{Q(x,y)}\sigma(\xi,\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta\quad (x,y)\in Q$$

is said to be a solution of (2.2).

Definition 2.5. Let Hypothesis 2.3 be satisfied and let $\lambda\in\Lambda$. A function $u\in C$ is said to be a solution of (1.1) if there exists a function $\sigma\in L^1$ such that

$$\begin{aligned} \sigma(x,y)&\in F\bigl(x,y,u(x,y),G(x,y,u(x,y))\bigr)\quad \text{a.e. } (Q)\\ u(x,y)&=\lambda(x,y)+\iint_{Q(x,y)}\sigma(\xi,\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta\quad \text{a.e. } (Q) \end{aligned}$$

where $F\bigl(x,y,u,G(x,y,u)\bigr)=\bigcup\limits_{v\in G(x,y,u)}F(x,y,u,v)$.

We denote by $S(\lambda)$ the solution set of (1.1).

Lemma 2.6 ([3]). *Let $\alpha\in(0,1)$ and let $N\colon Q\to\mathbb{R}$ be a positive integrable function. Then there exists a continuous strictly positive function $a\colon Q\to\mathbb{R}$ which, for every $(x,y)\in Q$, satisfies*

$$\iint_{R(x,y)}N(\xi,\eta)a(\xi,\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta=\alpha\bigl(a(x,y)-1\bigr).$$

In what follows $N(x,y)=\max\{l(x,y),m(x,y)\}$, $(x,y)\in Q$, $\alpha\in(0,1)$ will be taken such that $2\alpha+\theta<1$ and $a\colon Q\to\mathbb{R}$ in (2.1) is the corresponding mapping found in Lemma 2.6.

3. The main result

Even if the multifunction from the right-hand side of (1.1) has, in general, nonclosed nonconvex values, the solution set $S(\lambda)$ has some meaningful properties, stated in Theorem 3.1 below.

Theorem 3.1. *Suppose $F\colon Q\times\mathbb{R}^n\times\mathbb{R}^n\to 2^{\mathbb{R}^n}$ and $G\colon Q\times\mathbb{R}^n\to 2^{\mathbb{R}^n}$ satisfy Hypothesis 2.3. Then:*

1) *For every $\lambda\in\Lambda$, the solution set $S(\lambda)$ of (1.1) is nonempty and arcwise connected in the space C.*
2) *For any $\lambda_i\in\Lambda$ and any $u_i\in S(\lambda_i)$, $i=1,\dots,p$, there exists a continuous function $s\colon\Lambda\to C$ such that $s(\lambda)\in S(\lambda)$ for any $\lambda\in\Lambda$ and $s(\lambda_i)=u_i$, $i=1,\dots,p$.*
3) *The set $S=\bigcup\limits_{\lambda\in\Lambda}S(\lambda)$ is arcwise connected in C.*

Proof. 1) For $\lambda\in\Lambda$ and $u\in L^1$, set

$$u_\lambda(x,y)=\lambda(x,y)+\iint_{Q(x,y)}u(\xi,\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta,\quad (x,y)\in Q.$$

We prove that the multifunctions $\varphi\colon\Lambda\times L^1\to 2^{L^1}$ and $\psi\colon\Lambda\times L^1\times L^1\to 2^{L^1}$ given by

$$\varphi(\lambda,u)=\bigl\{v\in L^1;\ v(x,y)\in G\bigl(x,y,u_\lambda(x,y)\bigr)\quad\text{a.e. }(Q)\bigr\},$$
$$\psi(\lambda,u,v)=\bigl\{w\in L^1;\ w(x,y)\in F\bigl(x,y,u_\lambda(x,y),v(x,y)\bigr)\quad\text{a.e. }(Q)\bigr\},$$

$\lambda\in\Lambda$, $u,v\in L^1$ satisfy the hypotheses of Lemma 2.1.

Since u_λ is measurable and G satisfies Hypothesis 2.3 i) and ii), the multifunction $(x,y)\to G\bigl(x,y,u_\lambda(x,y)\bigr)$ is measurable and nonempty closed-valued, it has a measurable selection. Therefore due to Hypothesis 2.3 iv), the set $\varphi(\lambda,u)$ is nonempty. The fact that the set $\varphi(\lambda,u)$ is closed and decomposable follows by simple computation. In the same way we obtain that $\psi(\lambda,u,v)$ is a nonempty closed decomposable set.

Set $b:=\iint_Q a(x,y)\,\mathrm{d}x\,\mathrm{d}y$.

Pick $(\lambda,u),(\lambda_1,u_1)\in\Lambda\times L^1$ and choose $v\in\varphi(\lambda,u)$. For each $\varepsilon>0$ there exists $v_1\in\varphi(\lambda_1,u_1)$ such that, for every $(x,y)\in Q$, one has

$$\begin{aligned}|v(x,y)-v_1(x,y)|&\leqslant D\bigl(G(x,y,u_\lambda(x,y)),G(x,y,u_{\lambda_1}(x,y))\bigr)+\varepsilon\\&\leqslant N(x,y)[|\lambda(x,y)-\lambda_1(x,y)|\\&\qquad+\iint_{Q(x,y)}|u(\xi,\eta)-u_1(\xi,\eta)|\,\mathrm{d}\xi\,\mathrm{d}\eta]+\varepsilon.\end{aligned}$$

Hence

$$\begin{aligned}
|v - v_1|_1 &\leqslant |\lambda - \lambda_1|_C \iint_Q a(x,y)N(x,y)\,\mathrm{d}x\,\mathrm{d}y \\
&\qquad + \iint_Q a(x,y)N(x,y)\biggl(\iint_{Q(x,y)} |u(\xi,\eta) - u_1(\xi,\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta\biggr)\,\mathrm{d}x\,\mathrm{d}y + \varepsilon b \\
&\leqslant \alpha\bigl(a(1,1) - 1\bigr)|\lambda - \lambda_1|_C \\
&\qquad + \iint_Q |u(\xi,\eta) - u_1(\xi,\eta)|\biggl(\iint_{R(\xi,\eta)} a(x,y)N(x,y)\,\mathrm{d}x\,\mathrm{d}y\biggr)\,\mathrm{d}\xi\,\mathrm{d}\eta + \varepsilon b \\
&\leqslant \alpha\bigl(a(1,1) - 1\bigr)|\lambda - \lambda_1|_C \\
&\qquad + \iint_Q \alpha\bigl(a(\xi,\eta) - 1\bigr)|u(\xi,\eta) - u_1(\xi,\eta)|\,\mathrm{d}\xi\,\mathrm{d}\eta + \varepsilon b \\
&\leqslant \alpha\bigl(a(1,1) - 1\bigr)|\lambda - \lambda_1|_C + \alpha|u - u_1|_1 + \varepsilon b
\end{aligned}$$

for any $\varepsilon > 0$.

This implies

$$d_{L^1}\bigl(v, \varphi(\lambda_1, u_1)\bigr) \leqslant \alpha\bigl(a(1,1) - 1\bigr)|\lambda - \lambda_1|_C + \alpha|u - u_1|_1$$

for all $v \in \varphi(\lambda, u)$. Therefore,

$$d^*_{L^1}\bigl(\varphi(\lambda, u), \varphi(\lambda_1, u_1)\bigr) \leqslant \alpha\bigl(a(1,1) - 1\bigr)|\lambda - \lambda_1|_C + \alpha|u - u_1|_1.$$

Consequently,

$$D_{L^1}\bigl(\varphi(\lambda, u), \varphi(\lambda_1, u_1)\bigr) \leqslant \alpha\bigl(a(1,1) - 1\bigr)|\lambda - \lambda_1|_C + \alpha|u - u_1|_1,$$

which shows that φ is Hausdorff continuous and satisfies the assumptions of Lemma 2.1.

Pick $(\lambda, u, v), (\lambda_1, u_1, v_1) \in \Lambda \times L^1 \times L^1$ and choose $w \in \psi(\lambda, u, v)$. Then, as before, for each $\varepsilon > 0$ there exists $w_1 \in \psi(\lambda_1, u_1, v_1)$ such that

$$\begin{aligned}
&|w(x,y) - w_1(x,y)| \\
&\quad \leqslant D\bigl(F(x,y,u_\lambda(x,y), v(x,y)), G\bigl(x,y,u_{\lambda_1}(x,y), v_1(x,y))\bigr) + \varepsilon \\
&\quad \leqslant N(x,y)[|\lambda(x,y) - \lambda_1(x,y)| + \iint_{Q(x,y)} |u(\xi,\eta) - u_1(\xi,\eta)|\,\mathrm{d}\xi\,\mathrm{d}\eta] \\
&\qquad + \theta|v(x,y) - v_1(x,y)| + \varepsilon
\end{aligned}$$

for every $(x,y) \in Q$. Hence

$$\begin{aligned}
|w - w_1|_1 &\leqslant \alpha\bigl(a(1,1) - 1\bigr)|\lambda - \lambda_1|_C + \alpha|u - u_1|_1 + \theta|v - v_1|_1 + \varepsilon b \\
&\leqslant \alpha\bigl(a(1,1) - 1\bigr)|\lambda - \lambda_1|_C + (\alpha + \theta)(|u - u_1|_1 + |v - v_1|_1) + \varepsilon b \\
&= \alpha\bigl(a(1,1) - 1\bigr)|\lambda - \lambda_1|_C + (\alpha + \theta)d_{L^1 \times L^1}\bigl((u,v),(u_1,v_1)\bigr) + \varepsilon b.
\end{aligned}$$

As above, we deduce that

$$D_{L^1}\big(\psi(\lambda,u,v),\psi(\lambda_1,u_1,v_1)\big) \leqslant \alpha\big(a(1,1)-1\big)|\lambda-\lambda_1|_C \\ + (\alpha+\theta)d_{L^1\times L^1}\big((u,v),(u_1,v_1)\big),$$

namely, the multifunction ψ is Hausdorff continuous and satisfies the hypothesis of Lemma 2.1.

Define $\Gamma(\lambda,u) = \psi\big(\lambda,u,\varphi(\lambda,u)\big)$, $(\lambda,u)\in\Lambda\times L^1$. According to Lemma 2.1, the set $\mathrm{Fix}(\Gamma(s,.)) = \{u\in E;\ u\in\Gamma(s,u)\}$ is nonempty and arcwise connected in L^1. Moreover, for fixed $\lambda_i\in\Lambda$ and $u_i\in\mathrm{Fix}(\Gamma(\lambda_i,.))$, $i=1,\dots,p$, there exists a continuous function $\gamma\colon\ \Lambda\to L^1$ such that

$$\gamma(\lambda)\in\mathrm{Fix}(\Gamma(\lambda,.)),\ \ \forall\lambda\in\Lambda, \tag{3.1}$$

$$\gamma(\lambda_i)=u_i,\quad i=1,\dots,p. \tag{3.2}$$

We shall prove that

$$\mathrm{Fix}(\Gamma(\lambda,\cdot)) = \big\{u\in L^1;\ \ u(x,y)\in F\big(x,y,u_\lambda(x,y),G(x,y,u_\lambda(x,y))\big) \\ \text{a.e. } (Q)\big\}. \tag{3.3}$$

Denote by $A(\lambda)$ the right-hand side of (3.3). If $u\in\mathrm{Fix}(\Gamma(\lambda,\cdot))$ then there is $v\in\varphi(\lambda,v)$ such that $u\in\psi(\lambda,u,v)$. Therefore, $v(x,y)\in G\big(x,y,u_\lambda(x,y)\big)$ and

$$u(x,y)\in F\big(x,y,u_\lambda(x,y),v(x,y)\big)\subset F\big(x,y,u_\lambda(x,y),G(x,y,u_\lambda(x,y))\big) \\ \text{a.e. } (Q)$$

so that $\mathrm{Fix}(\Gamma(\lambda,.))\subset A(\lambda)$.

Let now $u\in A(\lambda)$. By Lemma 2.2, there exists a selection $v\in L^1$ of the multifunction $(x,y)\to G\big(x,y,u_\lambda(x,y))\big)$ satisfying

$$u(x,y)\in F\big(x,y,u_\lambda(x,y),v(x,y)\big)\quad \text{a.e. } (Q).$$

Hence, $v\in\varphi(\lambda,v)$, $u\in\psi(\lambda,u,v)$ and thus $u\in\Gamma(\lambda,u)$, which completes the proof of (3.3). □

We next note that the function $T\colon\ L^1\to C$,

$$T(u)(x,y) := \iint_{Q(x,y)} u(\xi,\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta\quad (x,y)\in Q$$

is continuous and one has

$$S(\lambda) = \lambda + T\big(\mathrm{Fix}(\Gamma(\lambda, .))\big), \quad \lambda \in \Lambda.$$

Since $\mathrm{Fix}(\Gamma(\lambda,.))$ is nonempty and arcwise connected in L^1, the set $S(\lambda)$ has the same properties in C.

2) Let $\lambda_i \in \Lambda$ and let $u_i \in S(\lambda_i)$, $i = 1, \dots, p$ be fixed. By (3.4) there exists $v_i \in \mathrm{Fix}(\Gamma(\lambda_i,.))$ such that

$$u_i = \lambda_i + T(v_i), \quad i = 1, \dots, p.$$

If $\gamma\colon \Lambda \to L^1$ is a continuous function satisfying (3.1) and (3.2) we define, for every $\lambda \in \Lambda$,

$$s(\lambda) = \lambda + T(\gamma(\lambda)).$$

Obviously, the function $s\colon \Lambda \to C$ is continuous, $s(\lambda) \in S(\lambda)$ for all $\lambda \in \Lambda$, and

$$s(\lambda_i) = \lambda_i + T(\gamma(\lambda_i)) = \lambda_i + T(v_i) = u_i, \quad i = 1, \dots, p.$$

3) Let $u_1, u_2 \in S = \bigcup_{\lambda \in \Lambda} S(\lambda)$ and choose $\lambda_i \in \Lambda$, $i = 1, 2$ such that $u_i \in S(\lambda_i)$, $i = 1, 2$. From the conclusion of 2) we deduce the existence of a continuous function $s\colon \Lambda \to C$ satisfying $s(\lambda_i) = u_i$, $i = 1, 2$ and $s(\lambda) \in S(\lambda)$, $\lambda \in \Lambda$. Let $h\colon [0,1] \to \Lambda$ be a continuous mapping such that $h(0) = \lambda_1$ and $h(1) = \lambda_2$. Then the function $s \circ h\colon [0,1] \to C$ is continuous and verifies

$$s \circ h(0) = u_1, \quad s \circ h(1) = u_2,$$
$$s \circ h(\lambda) \in S(h(\lambda)) \subset S, \quad \lambda \in \Lambda.$$

Remark 3.2. If the multifunction F does not depend on the last variable, (1.1) reduces to (1.2) and the first statement of Theorem 3.1 yields known results. More exactly, it follows from Corollary 1 in [3] that the solution set of (1.2) is arcwise connected in the space C.

References

[1] *F. S. De Blasi and J. Myjak*: On the set of solutions of a differential inclusion. Bull. Inst. Math. Acad. Sinica *14* (1986), 271–275.

[2] *F. S. De Blasi and J. Myjak*: On the structure of the set of solutions of the Darboux problem of hyperbolic equations. Proc. Edinburgh Math. Soc. *29* (1986), 7–14.

[3] *F. S. De Blasi, G. Pianigiani and V. Staicu*: On the solution sets of some nonconvex hyperbolic differential inclusions. Czechoslovak Math. J. *45* (1995), 107–116.

[4] *L. Gorniewicz and T. Pruszko*: On the set of solutions of the Darboux problem for some hyperbolic equations. Bull. Acad. Polon. Sci. Math. Astronom. Phys. *38* (1980), 279–285.

[5] *S. Marano*: Generalized solutions of partial differential inclusions depending on a parameter. Rend. Accad. Naz. Sci. Mem. Mat. *107* (1989), 281–295.

[6] *S. Marano*: Fixed points of multivalued contractions with nonclosed, nonconvex values. Atti. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. *5* (1994), 203–212.

[7] *S. Marano and V. Staicu*: On the set of solutions to a class of nonconvex nonclosed differential inclusions. Acta Math. Hungar. *76* (1997), 287–301.

[8] *S. Marano and V. Staicu*: Correction to the paper On the set of solutions to a class of nonconvex nonclosed differential inclusions. Acta Math. Hungar. *78* (1998), 267–268.

[9] *V. Staicu*: On a non-convex hyperbolic differential inclusion. Proc. Edinburgh Math. Soc. *35* (1992), 375–382.

[10] *G. Teodoru*: A characterization of the solutions of the Darboux problem for the equation $u_{xy} \in F(x, y, u)$. An. Stiint. Univ. Al. I. Cuza Iasi Mat. *33* (1987), 33–38.

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