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COMPACT ATTRACTOR FOR WEAKLY DAMPED DRIVEN KORTEWEG-DE VRIES EQUATIONS ON THE REAL LINE

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Abstract. We investigate the long-time behaviour of solutions to the Korteweg-de Vries equation with a zero order dissipation and an additional forcing term, when the space variable varies over $\mathbb{R}$, and prove that it is described by a maximal compact attractor in $H^2(\mathbb{R})$.

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1. Introduction

We investigate the long-time behaviour of solutions to

$$u_t + uu_x + u_{xxx} + \gamma u = f \text{ in } \mathbb{R} \times (0, +\infty), \tag{1.1}$$

$$u(0) = u_0 \text{ in } \mathbb{R}, \tag{1.2}$$

where γ is a positive real number and $f \in H^2(\mathbb{R})$. When $\gamma = 0$ and $f = 0$, (1.1) is the well-known Korteweg-de Vries equation which arises in many physical situations. In some cases, energy dissipation mechanisms and external excitation have to be taken into account, and yield the damping term γu and the driving force f. The presence of dissipation in (1.1) changes the long-time behaviour of the solutions: indeed, when (1.1) is considered on a bounded interval $(0, L)$ with periodic boundary conditions, J. M. Ghidaglia has shown that the long-time behaviour of the solutions to (1.1) is described by a maximal compact attractor in $H^2(0, L)$ ([Gh1], [Gh2]), which is not the case when $\gamma = 0$ and $f = 0$. Our purpose in this work is to prove that the same result holds for (1.1) when the space variable x varies over the real line and $f \in H^2(\mathbb{R})$. The main difficulty here is to obtain the compactness of the attractor in

$H^2(\mathbb{R})$ (which is not given by Sobolev embeddings since the domain is unbounded); we first use a splitting method in the spirit of [GT] and weighted Sobolev spaces ([BV], [Fe]) to get compactness in $L^2(\mathbb{R})$. A technique of J. Ball ([Ba], [Gh2]) then yields the compactness in $H^2(\mathbb{R})$.

A related result may be found in [Al], where the existence of the attractor for the weak topology of $H^2(\mathbb{R})$ is proved under an additional assumption on the decay of f as $|x| \to \infty$, namely that $f \in L^2(\mathbb{R}, (1+x^2)^{1/2}\,\mathrm{d}x)$. Such an assumption is not made here, and our result extends [Al, Thm. 3.8].

This paper is organized as follows: in the next section, we recall results concerning the well-posedness of (1.1), and state our result, namely the existence of a maximal compact attractor in $H^2(\mathbb{R})$ (Theorem 2.2). Section 3 is then devoted to the proof of Theorem 2.2.

2. Main result

We first recall the basic existence result for (1.1):

Proposition 2.1. *Let u_0, f be functions in $H^2(\mathbb{R})$. Then, the problem* (1.1)–(1.2) *has a unique solution*

$$u \in \mathcal{C}([0,+\infty), H^2(\mathbb{R})) \cap \mathcal{C}^1([0,+\infty), H^{-1}(\mathbb{R})),$$

and the mapping $S_t\colon\ u_0 \mapsto u(t)$ is continuous in $H^2(\mathbb{R})$ for each $t \geqslant 0$.

The proof of Proposition 2.1 follows the proof of the similar result by J. Bona and R. Smith in the case $\gamma = 0$, to which we refer ([BSm], see also [BSc]).

It follows from Proposition 2.1 that (S_t) is a dynamical system on $H^2(\mathbb{R})$. Our main result is then:

Theorem 2.2. *The dynamical system (S_t) defined in Proposition* 2.1 *has a maximal compact attractor in $H^2(\mathbb{R})$.*

3. Proof of Theorem 2.2

3.1. Preliminary results

We first recall the main estimates satisfied by smooth solutions to (1.1)–(1.2), which are technical consequences of the polynomial invariants of the Korteweg-de Vries equation ([MGK]).

Lemma 3.1. *Let u be a smooth solution to* (1.1)–(1.2). *It holds:*

$$\frac{\mathrm{d}}{\mathrm{d}t}|u|_{L^2}^2 + 2\gamma|u|_{L^2}^2 = 2\int fu\,\mathrm{d}x, \tag{3.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\varphi_1(u) + 2\gamma\varphi_1(u) = \psi_1(u), \tag{3.2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\varphi_2(u) + 2\gamma\varphi_2(u) = \psi_2(u), \tag{3.3}$$

where

$$\begin{aligned}
\varphi_1(z) &= \int\left(z_x^2 - \frac{z^3}{3}\right)\mathrm{d}x, \quad z\in H^1(\mathbb{R}),\\
\psi_1(z) &= \int\left(\frac{\gamma}{3}z^3 - 2f_{xx}z - fz^2\right)\mathrm{d}x, \quad z\in H^1(\mathbb{R}),\\
\varphi_2(z) &= \int\left(\frac{9}{5}z_{xx}^2 - 3zz_x^2 + \frac{z^4}{4}\right)\mathrm{d}x, \quad z\in H^2(\mathbb{R}),\\
\psi_2(z) &= \int\left(3\gamma zz_x^2 - \frac{\gamma}{2}z^4\right)\mathrm{d}x\\
&\quad + \int\left(\frac{18}{5}f_{xx}z_{xx} + 3f_{xx}z^2 - 3f\ z_x^2 + fz^3\right)\mathrm{d}x, \quad z\in H^2(\mathbb{R}).
\end{aligned}$$

P r o o f o f L e m m a 3.1. We first take the scalar product in $L^2(\mathbb{R})$ of (1.1) with u, and obtain (3.1). We next take the scalar product in $L^2(\mathbb{R})$ of (1.1) with $(u^2/2 + u_{xx})$, and find (3.2). Finally, multiplying (1.1) in $L^2(\mathbb{R})$ by

$$\frac{18}{5}u_{xxxx} + 6uu_{xx} + 3u_x^2 + u^3$$

yields (3.3). □

We next investigate further the relationship between φ_1, ψ_1, φ_2, ψ_2 and the norms of $L^2(\mathbb{R})$, $H^1(\mathbb{R})$ and $H^2(\mathbb{R})$.

Lemma 3.2. *There exist positive constants c_1, c_2 depending only on γ such that*

$$\left|\varphi_1(z) - |z_x|^2_{L^2}\right| \leqslant \frac{1}{2}\, |z_x|^2_{L^2} + c_1|z|^{10/3}_{L^2}, \tag{3.4}$$

$$\psi_1(z) \leqslant \frac{\gamma}{2}|z_x|^2_{L^2} + c_1\left(|z|^2_{L^2} + |z|^4_{L^2} + |f|^2_{H^1}\right), \tag{3.5}$$

$$\left|\varphi_2(z) - \frac{9}{5}|z_{xx}|^2_{L^2}\right| \leqslant c_2|z|^4_{H^1}, \tag{3.6}$$

$$\psi_2(z) \leqslant \frac{9\gamma}{5}|z_{xx}|^2_{L^2} + c_2\left(|z|^2_{H^1} + |z|^6_{H^1} + |f|^2_{H^2}\right). \tag{3.7}$$

P r o o f o f L e m m a 3.2. In the following, we denote by C any constant depending only on γ. It first follows from Gagliardo-Nirenberg inequality that

$$|z|^3_{L^3} \leqslant C|z_x|^{1/2}_{L^2}|z|^{5/2}_{L^2} \leqslant \frac{1}{2}|z_x|^2_{L^2} + C|z|^{10/3}_{L^2},$$

hence (3.4). Next, since $H^1(\mathbb{R})$ is continuously embedded in $L^\infty(\mathbb{R})$, we have:

$$\psi_1(z) \leqslant C|z_x|^{1/2}_{L^2}|z|^{5/2}_{L^2} + C\ |f_{xx}|_{L^2}|z|_{L^2} + |f|_{L^\infty}|z|^2_{L^2}.$$

Then (3.5) follows from Young inequality. Since $H^1(\mathbb{R})$ is continuously embedded in $L^\infty(\mathbb{R}) \cap L^4(\mathbb{R})$, (3.6) follows from

$$|zz_x^2|_{L^1} \leqslant |z|_{L^\infty}|z_x|^2_{L^2} \leqslant C|z|^3_{H^1}.$$

Finally, since $H^1(\mathbb{R})$ is continuously embedded in $L^\infty(\mathbb{R}) \cap L^2(\mathbb{R})$,

$$\begin{aligned}\psi_2(z) \leqslant C\big(&|z|_{L^\infty}|z_x|^2_{L^2} + |z|^4_{L^4} + |f_{xx}|_{L^2}|z_{xx}|_{L^2} + |f_{xx}|_{L^2}|z|_{L^\infty}|z|_{L^2}\\ &+ |f|_{L^\infty}|z_x|^2_{L^2} + |f|_{L^\infty}|z|^3_{L^3}\big),\end{aligned}$$

hence (3.7), thanks to Young inequality. □

We end this subsection with some continuity properties of φ_2 and ψ_2 we will use in the sequel.

Lemma 3.3. *Define $\varphi_{02}\colon\ H^1(\mathbb{R}) \to \mathbb{R}$ by:*

$$\varphi_{02}(z) = \int\left(\frac{z^4}{4} - 3zz_x^2\right)\,\mathrm{d}x.$$

If (z_n) is a sequence in $H^2(\mathbb{R})$ that converges to z for the weak topology of $H^2(\mathbb{R})$ and for the strong topology of $H^1(\mathbb{R})$, it holds

$$\lim_{n\to+\infty}\varphi_{02}(z_n) = \varphi_{02}(z), \quad \lim_{n\to+\infty}\psi_2(z_n) = \psi_2(z).$$

3.2. Bounded absorbing set in H^2

Proposition 3.4. *There exists a bounded subset $\mathcal{B}_2$ of $H^2(\mathbb{R})$ satisfying*

$$S_t\mathcal{B}_2 \subset \mathcal{B}_2, \ \ t \geqslant 0,$$

and such that, for each bounded subset B of $H^2(\mathbb{R})$, there exists $t_2(B) \geqslant 0$ such that

$$S_t B \subset \mathcal{B}_2, \ \ t \geqslant t_2(B).$$

P r o o f o f P r o p o s i t i o n 3.4. We denote by C any constant depending only on γ. We consider $u_0 \in H^2(\mathbb{R})$, and put $u(t) = S_t u_0$, $t \geqslant 0$. It first follows from (3.1), Young inequality and Gronwall lemma that

$$|u(t)|_{L^2}^2 \leqslant \mathrm{e}^{-\gamma t}|u_0|_{L^2}^2 + C|f|_{L^2}^2, \ \ t \geqslant 0. \tag{3.8}$$

We next infer from (3.2), (3.4) and (3.5) that

$$\frac{\mathrm{d}}{\mathrm{d}t}\varphi_1(u) + \gamma\varphi_1(u) \leqslant C\left(|u|_{L^2}^2 + |u|_{L^2}^4 + |f|_{H^1}^2\right).$$

Gronwall lemma, (3.4) and (3.8) then yield

$$|u_x(t)|_{L^2}^2 \leqslant C(1+t)\mathrm{e}^{-\gamma t}\left(1 + |u_0|_{H^1}^4\right) + C\left(|f|_{H^1}^2 + |f|_{H^1}^4\right).$$

We combine (3.8) and the above inequality, and find:

$$|u(t)|_{H^1}^2 \leqslant C(1+t)\mathrm{e}^{-\gamma t}\left(1 + |u_0|_{H^1}^4\right) + C\left(|f|_{H^1}^2 + |f|_{H^1}^4\right), \ \ t \geqslant 0. \tag{3.9}$$

Finally, (3.3), (3.6) and (3.7) give:

$$\frac{\mathrm{d}}{\mathrm{d}t}\varphi_2(u) + \gamma\varphi_2(u) \leqslant C\left(|u|_{H^1}^2 + |u|_{H^1}^6 + |f|_{H^2}^2\right).$$

It then follows from Gronwall lemma, (3.6) and (3.9) that

$$|u_{xx}(t)|_{L^2}^2 \leqslant C(1+t^3)\mathrm{e}^{-\gamma t}\left(1 + |u_0|_{H^2}^{12}\right) + C\left(|f|_{H^2}^2 + |f|_{H^2}^{12}\right),$$

hence, thanks to (3.9),

$$|u(t)|_{H^2}^2 \leqslant C(1+t^3)\mathrm{e}^{-\gamma t}\left(1 + |u_0|_{H^2}^{12}\right) + C\left(|f|_{H^2}^2 + |f|_{H^2}^{12}\right), \ \ t \geqslant 0. \tag{3.10}$$

Proposition 3.4 now follows from (3.10) by classical arguments. □

Note that the above computations are formal; one has first to compute them with smooth solutions and use the continuity of S_t in $H^2(\mathbb{R})$ to justify (3.10).

We put

$$\varrho_2 = \sup_{\mathcal{B}_2} |z|_{H^2}. \tag{3.11}$$

Remark 3.1. A consequence of (3.10) is that, for each $t \geqslant 0$, S_t maps bounded subsets of $H^2(\mathbb{R})$ into bounded subsets of $H^2(\mathbb{R})$.

Furthermore, we have the following result:

Lemma 3.5. *Let $t \geqslant 0$ and consider a sequence (z_n) in $H^2(\mathbb{R})$ that converges to z for the weak topology of $H^2(\mathbb{R})$ and for the strong topology of $H^1(\mathbb{R})$. Then, $(S_t z_n)$ converges to $S_t z$ for the weak topology of $H^2(\mathbb{R})$ and for the strong topology of $H^1(\mathbb{R})$.*

Proof of Lemma 3.5. We first claim that, if B is a bounded subset of $H^2(\mathbb{R})$, there exists a constant $C(B,t)$ depending on γ, f, B and t such that, for any $u_0 \in B$, $v_0 \in B$ and $t \geqslant 0$, it holds

$$|S_t u_0 - S_t v_0|_{H^1} \leqslant C(B,t) |u_0 - v_0|_{H^1}. \tag{3.12}$$

Indeed, let $t \geqslant 0$. It follows from the above remark that there exists $C(B,t)$ such that

$$|S_\tau z|_{H^2} \leqslant C(B,t), \quad \tau \in [0,t]. \tag{3.13}$$

Next, for u_0 and v_0 in B, we put $u(\tau) = S_\tau u_0$, $v(\tau) = S_\tau v_0$, and $w = u - v$. Since both u and v satisfy (1.1), we obtain

$$w_t + \left(\frac{u+v}{2} w \right)_x + w_{xxx} + \gamma w = 0.$$

Taking the scalar product of the above equation with $(w - w_{xx})$ and using (3.13) and Gronwall lemma, we obtain (3.12).

Now, let (z_n) be a sequence in $H^2(\mathbb{R})$ as in Lemma 3.5, and denote by z its limit. On the one hand, since (z_n) is bounded in $H^2(\mathbb{R})$, it follows from (3.12) that $(S_t z_n)$ converges to $S_t z$ in $H^1(\mathbb{R})$. On the other hand, it follows from Remark 3.1 that any subsequence of $(S_t z_n)$ has a subsequence that converges weakly in $H^2(\mathbb{R})$, and its limit is necessarily $S_t z$. Therefore, the whole sequence $(S_t z_n)$ also converges weakly to $S_t z$ in $H^2(\mathbb{R})$. The proof of the lemma is thus complete. □

3.3. A splitting

In the following, we denote by C any positive constant depending only on γ.

Let $u_0 \in \mathcal{B}_2$, and put $u(t) = S_t u_0$, $t \geqslant 0$. Since $f \in H^2(\mathbb{R})$, there exists a sequence $(f_\eta)_{\eta\in(0,1)}$ of compactly supported functions of $H^2(\mathbb{R})$ satisfying

$$|f - f_\eta|_{H^2} \leqslant \eta, \quad \eta \in (0,1). \tag{3.14}$$

Since f_η belongs to $H^2(\mathbb{R})$, it follows from Proposition 2.1 that the problem

$$\begin{gathered} z_t + zz_x + z_{xxx} + \gamma z = f - f_\eta \text{ in } \mathbb{R}\times(0,+\infty),\\ z(0) = u_0 \text{ in } \mathbb{R}, \end{gathered}$$

has a unique solution, which we denote by $t \mapsto S_{\eta,1}(t,u_0)$. We also put

$$S_{\eta,2}(t,u_0) = S_t u_0 - S_{\eta,1}(t,u_0).$$

We infer from (3.10) that

$$|S_{\eta,1}(t,u_0)|^2_{H^2} \leqslant C(1+t^3)\mathrm{e}^{-\gamma t}\left(1+|u_0|^{12}_{H^2}\right) + C\left(|f-f_\eta|^2_{H^2} + |f-f_\eta|^{12}_{H^2}\right),$$

which yields, thanks to (3.11) and (3.14),

$$|S_{\eta,1}(t,u_0)|^2_{H^2} \leqslant C\left((1+t^3)\ \mathrm{e}^{-\gamma t} + \eta^2\right), \quad t \geqslant 0. \tag{3.15}$$

A straightforward consequence of (3.11) and (3.15) is that

$$|S_t u_0|_{H^2} + |S_{\eta,1}(t,u_0)|_{H^2} + |S_{\eta,2}(t,u_0)|_{H^2} \leqslant C, \tag{3.16}$$

for any $u_0 \in \mathcal{B}_2$, $t \geqslant 0$ and $\eta \in (0,1)$.

We shall now prove that, in some sense, $S_{\eta,2}(t,u_0)$ is small as $|x| \to +\infty$. To this end, we put $\varrho(x) = |x|$ ($\varrho \in W^{1,\infty}(\mathbb{R})$) and prove the following result.

Lemma 3.6. *For any $\eta \in (0,1)$ and $t \geqslant 0$, there exists a constant $K(\eta,t)$ depending only on γ, f, η and t such that, for any u_0 in $\mathcal{B}_2$, it holds:*

$$\int |S_{\eta,2}(t,u_0)|^2\,\varrho(x)\,\mathrm{d}x \leqslant K(\eta,t). \tag{3.17}$$

Proof of Lemma 3.6. We first notice that $|\varrho'| \leqslant 1$. Next, let $u_0 \in \mathcal{B}_2$ and $\eta \in (0,1)$. We put

$$u(t) = S_t u_0, \quad v(t) = S_{\eta,1}(t,u_0), \quad w(t) = S_{\eta,2}(t,u_0).$$

(Note that both v and w depend on η.) Then, w satisfies:

$$w_t + \left(\frac{u+v}{2} w\right)_x + w_{xxx} + \gamma w = f_\eta \text{ in } \mathbb{R} \times (0, +\infty), \tag{3.18}$$

$$w(0) = 0 \text{ in } \mathbb{R}. \tag{3.19}$$

We take the scalar product in $L^2(\mathbb{R})$ of (3.18) with ϱw and find:

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int w^2 \varrho \,\mathrm{d}x \leqslant & \int \frac{u+v}{2} w^2 \varrho' \,\mathrm{d}x + \int \frac{u+v}{4}\left(w^2\right)_x \varrho \,\mathrm{d}x + \frac{1}{2}\int \left(w_x^2\right)_x \varrho \,\mathrm{d}x \\ & + \int w w_{xx} \varrho' \,\mathrm{d}x + \int f_\eta w \varrho \,\mathrm{d}x \\ \leqslant & \, C|u+v|_{L^\infty} |w|_{L^2}^2 - \int \left(\frac{u+v}{4}\right)_x w^2 \varrho \,\mathrm{d}x - \int \frac{u+v}{4} w^2 \varrho' \,\mathrm{d}x \\ & - \frac{1}{2}\int w_x^2 \varrho' \,\mathrm{d}x + C|w|_{L^2}|w_{xx}|_{L^2} + \frac{1}{2}\int (f_\eta^2 + w^2)\varrho \,\mathrm{d}x \\ \leqslant & \, C\left(1 + \left|\left(\frac{u+v}{4}\right)_x\right|_{L^\infty}\right)\int w^2 \varrho \,\mathrm{d}x + C\left(1 + |u+v|_{L^\infty}\right)|w|_{H^2}^2 \\ & + C\int f_\eta^2 \varrho \,\mathrm{d}x\end{aligned}$$

Since $H^1(\mathbb{R})$ is continuously embedded in $L^\infty(\mathbb{R})$, it follows from (3.16) and the above inequality that

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int w^2 \varrho \,\mathrm{d}x \leqslant C\left(1 + \int \varrho \, w^2 \,\mathrm{d}x + \int f_\eta^2 \varrho \,\mathrm{d}x\right).$$

Gronwall lemma then yields

$$\int w(t)^2 \varrho \,\mathrm{d}x \leqslant C\left(1 + \int f_\eta^2 \varrho \,\mathrm{d}x\right) \mathrm{e}^{Ct},$$

hence (3.17). □

From the study of $S_{\eta,1}$ and $S_{\eta,2}$, we infer the following result:

Proposition 3.7. *For all sequences $t_n \to +\infty$, and $U_n \in \mathcal{B}_2$, there is a subsequence of $(S_{t_n} U_n)$ that converges strongly in $H^1(\mathbb{R})$.*

Proof of Proposition 3.7. We first claim that,

for any $\varepsilon \in (0,1)$, there exists $T(\varepsilon)$ such that $S_{T(\varepsilon)}\mathcal{B}_2$ has a covering by a finite number of balls of $L^2(\mathbb{R})$ of radius ε.

Indeed, let $\varepsilon > 0$. On the one hand, it follows from (3.15) that there exist $T(\varepsilon)$ and $\eta(\varepsilon) \in (0,1)$ such that

$$\forall u_0 \in \mathcal{B}_2,\ \left|S_{\eta(\varepsilon),1}(T(\varepsilon), u_0)\right|_{H^2} \leqslant \frac{\varepsilon}{2}. \tag{3.20}$$

On the other hand, we infer from (3.16)–(3.17) that, for any u_0 in $\mathcal{B}_2$, $S_{\eta(\varepsilon),2}(T(\varepsilon), u_0)$ belongs to

$$\mathcal{K}(\varepsilon) = \left\{ z \in H^2(\mathbb{R}) \cap L^2(\mathbb{R}, \varrho\,\mathrm{d}x),\ |z|_{H^2} + \left(\int z^2 \varrho\,\mathrm{d}x\right)^{1/2} \leqslant \kappa_\varepsilon \right\},$$

where κ_ε depends only on γ, f and ε. But, $\mathcal{K}(\varepsilon)$ is a compact subset of $L^2(\mathbb{R})$ (see e.g. [BV, Lemma 2.16]), which, together with (3.20) yields the claim.

Classical arguments then ensure that, for any sequence $t_n \to +\infty$, and $U_n \in \mathcal{B}_2$, there is a subsequence of $(S_{t_n} U_n)$ that converges strongly in $L^2(\mathbb{R})$. Since $\mathcal{B}_2$ is positively invariant by S_t and bounded in $H^2(\mathbb{R})$, we may also assume that this subsequence converges weakly in $H^2(\mathbb{R})$. Proposition 3.7 then follows from an interpolation argument. □

3.4. Proof of Theorem 2.2

Theorem 2.2 follows at once from the following result.

Proposition 3.8. *Let (u_0^j) be a sequence of $\mathcal{B}_2$, and (t_j) a sequence of real numbers, $t_j \to +\infty$. Then, there exists a subsequence of $\left(S_{t_j} u_0^j\right)$ that converges strongly in $H^2(\mathbb{R})$.*

Taking Proposition 3.8 for granted, we proceed with the proof of Theorem 2.2: the obvious candidate for the maximal attractor in $H^2(\mathbb{R})$ is the ω-limit set of $\mathcal{B}_2$ in $H^2(\mathbb{R})$. Indeed, we put

$$\mathcal{A} = \bigcap_{t \geqslant 0} \mathrm{Cl}_{H^2}\left(S_t \mathcal{B}_2\right).$$

Since S_t is continuous in $H^2(\mathbb{R})$, a straightforward consequence of Proposition 3.8 is that $\mathcal{A}$ is the maximal attractor for (S_t) in $H^2(\mathbb{R})$ and is compact in $H^2(\mathbb{R})$.

Coming back to Proposition 3.8, we just mention that its proof relies on (3.3), Lemma 3.5 and Proposition 3.7 and follows the same lines as that of Theorem 4.1 in [Gh2], to which we refer.

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