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A MONOGENIC BAIRE MEASURE NEED NOT BE COMPLETION REGULAR *)

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A Baire measure ν on a locally compact space X is called completion regular if and only if it satisfies the following condition:

If E is any Borel set, then there exist Baire sets G and F such that $G \subset E \subset F$ and $\nu(F - G) = 0$.

A Baire measure ν is called monogenic if and only if any two Borel measures extending ν are necessarily identical.

In [1] it is proved that every completion regular Baire measure is monogenic. The converse of this proposition is false. The following is an example of a Baire measure ν which is monogenic, but not completion regular.

Example. Let X be any set with card $X = \aleph_1$. Let $x_0 \in X$ and $Y = X - \{x_0\}$. Let the topology for X be the family $\mathscr{Y} = \mathscr{Y}_1 \cup \mathscr{Y}_2$, where

$\mathscr{Y}_1$ is the family of all subsets of Y,

$\mathscr{Y}_2$ is the family of all subsets A of X such that $x_0 \in A$ and $X - A$ is finite or empty set.

We easily find out that the pair $(X, \mathscr{Y})$ is a locally compact Hausdorff topological space. If $x_0 \in U \in \mathscr{Y}$, then $U \in \mathscr{Y}_2$, hence X is compact and all sets of $\mathscr{Y}_2$ are compact G_δ. Evidently the set $\{x_0\}$ is not G_δ.

Define $\nu(F)$, for each Baire set $F \subset X$ to be 1 or 0 according as x_0 does or does not belong to F. We prove that ν is a monogenic Baire measure. Let μ be any Borel measure extending ν. If a point $x \in Y = X - \{x_0\}$, then $\{x\}$ is a Baire set and $\mu(\{x\}) = \nu(\{x\}) = 0$. Every set $E \subset X$ is Borel because $E \cup \{x_0\}$ is a compact set. Since μ is defined for every $E \subset Y$, $\mu(\{x\}) = 0$ for each $x \in Y$ and card $Y = \aleph_1$, we have (by [2], Theorem A, p. 141) $\mu(Y) = 0$. Hence $\mu(\{x_0\}) = \nu(X) - \mu(Y) = 1$. If $E \subset X$ and $x_0 \in E$, then $1 = \mu(\{x_0\}) \leqq \mu(E) \leqq \mu(X) = \nu(X) = 1$, hence $\mu(E) = 1$. If $E \subset X$ and $x_0 \notin E$, then $\mu(X - E) = 1$ and hence $\mu(E) = 0$. This means that ν is monogenic. But ν is not completion regular because $\{x_0\}$ is not Baire and if G and F are Baire sets such that $G \subset \{x_0\} \subset F$, then $G = \emptyset$ and $\nu(F - \emptyset) = \nu(F) = 1 \neq 0$.

*) This is the solution of the problem 3, [1], p. 233.

REFERENCES

[1] BERBERIAN, S. K.: Measure and Integration. New York 1965.
[2] ULAM, S.: Masstheorie in der allgemeinen Mengenlehre. Fundam. math. *16*. 1930. 140—150.

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