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THE NONLINEAR REGULATOR PROBLEM FOR CONSTANT SIGNALS

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The general principle of synthesizing a stabilizing and regulating compensator for linear systems is considered under actuator nonlinear uncertainties. The properties of the compensator are preserved if absolute stability of all possible stationary points is ensured. A design example is given.

1. STATE OF THE ART

A *nonlinear regulator problem* has been formulated and studied via input-output approach for rather general exogeneous signals in [2]. Among the results of this paper one could mention as most important a negative one: exact regulation (i.e. zeroing the regulated output) is possible in the nonlinear case only for constant or asymptotically constant signals (see also [3]). Heuristically this can be explained by the fact that only in these cases steady-state constant solutions exist. It is probably this reason that lead other authors to formulate and solve, independently, a *nonlinear regulator problem* for the case of constant exogeneous signals [9]. Their solution consists essentially in the extension of the basic system with an ideal integrator to cope with asymptotic rejection of exogeneous constant disturbances and asymptotic tracking of constant references (specific application of the internal model principle, see [10]), stabilization of the extended system via state feedback and introduction of several uncertain nonlinear elements to describe the actuators; these functions verify a sector (Lur'e type) condition and, in order to obtain absolute stability i.e. global asymptotic stability for all nonlinear functions belonging to the considered class, a Popov-type frequency condition has been applied.

The implementation of the above approach is nevertheless concerned with the fact that not all state variables are available for feedback; also the regulated and the measured outputs do not entirely coincide. In the linear case these facts lead to the general formulation of the *measured-error-activated compensator synthesis* [4],

[5]. Here the same case of the general linear compensator will be considered under *actuator sector restricted nonlinear uncertainties*, generalizing the results of [9] and also results of [6] which dealt with nonlinear uncertainties for the linear quadratic optimal stabilization problem.

2. PRELIMINARIES. PROBLEM STATEMENT

In what follows the same notations for the linear system and for the compensator as in [4], [5] will be used. Consider the tandem composed of the linear system

$$\begin{aligned} \dot{x}_1 &= A_1 x_1 + A_3 x_2 + b_1 u(t), \quad & x_1(0) &= x_{10} \\ \dot{x}_2 &= 0, & x_2(0) &= x_{20} \\ y &= C_1 x_1 + C_2 x_2 \\ z &= d_1^{\mathrm{T}} x_1 + d_2^{\mathrm{T}} x_2 \end{aligned} \tag{2.1}$$

and the nonlinear element modeling the actuator

$$u = \varphi(\eta) \tag{2.2}$$

Here $x_1 \in \mathbb{R}^{n_1}$ is the state of the controlled system, $u \in \mathbb{R}$ is the control function, $y \in \mathbb{R}^p$ is the error (the deviation) of the measured output, $z \in \mathbb{R}$ is the error of the controlled (regulated) output and $x_2 \in \mathbb{R}^{n_2}$, $n_2 \geqq 2$, the state of the exogeneous system generating the class of the reference and of the disturbance signals. Note that in the general case [4], [5] the exogeneous signals are generated by the solutions of the linear system

$$\dot{x}_2 = A_2 x_2, \quad x_2(0) = x_{20} \tag{2.3}$$

Here, in order to obtain only constant exogeneous signals, it was assumed that $A_2 = 0$.

The nonlinear function φ: $\mathbb{R} \to \mathbb{R}$ belongs to the class

$$\mathscr{F}_\varrho = \left\{ \varphi \left| 0 < \frac{\varphi(\eta_1) - \varphi(\eta_2)}{\eta_1 - \eta_2} < \varrho, \eta_1 \neq \eta_2 ; \lim_{\eta \to \pm\infty} \varphi(\eta) = \pm\infty \right. \right\} \tag{2.4}$$

where ϱ is some positive number and, for convenience $\varrho > 1$ is assumed. Remark that this is a nonlinear system with scalar control and scalar controlled output; it is subject to exogeneous constant (step) signals.

Assume for a while that $\varphi(\eta) = \eta$; obviously $\varphi \in \mathscr{F}_\varrho$ since $\varrho > 1$; this is the case of a linearized control system for which the *Linear Structurally Stable Regulator Problem* (LSSRP) [4], [5] is formulated:

Find a measured-error-activated linear compensator

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c y, \quad x_c(0) = x_{c0} \\ \eta &= f_c^{\mathrm{T}} x_c + g_c^{\mathrm{T}} y \end{aligned} \tag{2.5}$$

in order that the resulting closed loop linear system

$$\begin{aligned}\dot{x}_1 &= (A_1 + b_1 g_c^T C_1) x_1 + (A_3 + b_1 g_c^T C_2) x_{20} + b_1 f_c^T x_c \\ \dot{x}_c &= B_c C_1 x_1 + B_c C_2 x_{20} + A_c x_c\end{aligned} \tag{2.6}$$

should be asymptotically stable in the autonomous case (in the absence of the exogeneous signals i.e. for $x_{20} = 0$), $\lim_{t\to\infty} z(t) = 0$ for any $x_{20} \neq 0$; also these properties should hold for any A_3 and for small parameter uncertainties of the pair (A_1, b_1).

In most practical situations a compensator which is synthesized to solve LSSRP has to cope also with actuator nonlinear uncertainties. This means that the compensator (2.5) should be viewed as working together with the system (2.1)–(2.2). Therefore the following nonlinear closed loop system occurs

$$\begin{aligned}\dot{x}_R &= A_R x_R + b_R\, \varphi(\eta) + E_R x_{20}, \quad x_R(0) = x_{R0} \\ \eta &= f_R^T x_R + g_R^T x_{20} \\ z &= d_R^T x_R + d_2^T x_{20}\end{aligned} \tag{2.7}$$

where

$$x_R = \begin{pmatrix} x_1 \\ x_c \end{pmatrix} \in \mathbb{R}^{n_R}, \quad n_R = n_1 + n_c; \quad A_R = \begin{pmatrix} A_1 & 0 \\ B_c C_1 & A_c \end{pmatrix}, \quad b_R = \begin{pmatrix} b_1 \\ 0 \end{pmatrix},$$

$$L_R = \begin{pmatrix} A_3 \\ B_c C_2 \end{pmatrix}, \quad f_R^T = (g_c^T C_1 \;\; f_c^T), \quad g_R^T = g_c^T C_2, \quad d_R^T = (d_1^T \;\; 0) \tag{2.8}$$

The requirement that the properties obtained by solving LSSRP should remain true in the nonlinear case is dictated by common sense but, due to the fact that system (2.7) is nonlinear and the characteristic of the nonlinearity is not well known except that it exists in a sector, the asymptotic stability requirement should be replaced by *absolute stability* (i.e. asymptotic stability for all nonlinear functions of the considered class) *of all possible stationary solutions* of (2.7). The stationary solutions are defined from

$$A_R x_R + b_R \varphi(f_R^T x_R + g_R^T x_{20}) + E_R x_{20} = 0 \tag{2.9}$$

and it can be seen that for each x_{20} there corresponds, provided some conditions are fulfilled, a certain stationary point. We are now in position to state

Nonlinear Structurally Stable Regulator Problem (NSSRP). Given the system (2.1)–(2.2) find a compensator (2.5) such that the closed loop system (2.7) has the following properties:

(AS) (*Absolute stability.*) For any $\varphi \in \mathscr{F}_\varrho$ and any $x_{20} \in \mathbb{R}^{n_2}$ a unique stationary point exists (i.e. (2.9) has a unique solution) and this stationary point is globally asymptotically stable; moreover this property holds uniformly with respect to all functions from $\mathscr{F}_\varrho$.

(R) (*Regulator property.*) Together with (AS) (for specified x_{20}) one has $\lim_{t\to\infty} z(t) = 0$.

(STS) (*Structural stability.*) The above properties should remain valid for any A_3 and also for small parametric uncertainties of the pair (A_1, b_1).

3. COMPENSATOR STRUCTURE DESIGN

Consider the system (2.1)–(2.2) under the following basic assumptions:

(i) the pair (A_1, b_1) is stabilizable and the pair (C_1, A_1) is detectable;

(ii) the matrix $\begin{pmatrix} A_1 & b_1 \\ d_1^T & 0 \end{pmatrix}$ is nonsingular, i.e. $\lambda = 0$ is not a transmission zero of the triple (A_1, b_1, d_1^T); there exists $q \in \mathbb{R}^p$, $q \neq 0$ such that $(d_1^T \; d_2^T) = q^T(C_1 \; C_2)$, i.e. controlled output z is readable from the measured output $y (z = q^T y)$.

Consider $\varphi(\eta) = \eta$; the above basic assumptions are necessary and sufficient [4], [5] for the existence of a compensator (2.5) which solves the LSSRP. Its structure is given by

$$A_c = \begin{pmatrix} A_w & a_a \\ 0 & 0 \end{pmatrix}, \quad B_c = \begin{pmatrix} B_w \\ q^T \end{pmatrix}, \quad f_c^T = (f_w^T \; f_a), \quad g_c^T \tag{3.1}$$

where $(A_w, a_a, B_w, f_w^T, f_a, g_c^T)$ defines a stabilizing compensator for the extended system

$$\begin{aligned} \dot{x}_e &= A_e x_e + b_e \eta(t) \\ y_e &= C_e x_e \end{aligned} \tag{3.2}$$

where

$$A_e = \begin{pmatrix} A_1 & 0 \\ d_1^T & 0 \end{pmatrix}, \quad b_e = \begin{pmatrix} b_1 \\ 0 \end{pmatrix}, \quad C_e = \begin{pmatrix} C_1 & 0 \\ 0 & 1 \end{pmatrix}, \quad x_e = \begin{pmatrix} x_1 \\ x_a \end{pmatrix}, \quad y_e = \begin{pmatrix} y \\ x_a \end{pmatrix} \tag{3.3}$$

and the compensator is given by

$$\begin{aligned} \dot{w} &= A_w w + B_w y + a_a x_a \\ \eta &= f_w^T w + g_c^T y + f_a x_a \end{aligned} \tag{3.4}$$

Worth to mention that due to (i) and (ii) such a stabilizing compensator can always be constructed but the structure resulting from the separation theorem – state feedback + observer – is not compulsory. On the contrary the integrator occurs in a necessary way and must be always present in the structure of the compensator; moreover, for a structurally stable design with respect to A_1, b_1, A_3 this integrator must be contained in the compensator even if the initial system contains an integrator (i.e. has a zero eigenvalue). From (3.1) and (3.4) the following compensator equations can be written

$$\begin{aligned} \dot{w} &= A_w w + a_a x_a + B_w y \\ \dot{x}_a &= q^T y \\ \eta &= f_w^T w + f_a x_a + g_c^T y \end{aligned} \tag{3.5}$$

The compensator (3.5) with the subcompensator (3.4) designed in order to stabilize the extended system (3.2) will be implemented in the nonlinear system (2.1)–(2.2). In what follows it will be shown that, with some additional, absolute stability-type assumptions, this compensator solves also the NSSRP — the problem just formulated.

4. MAIN RESULT

In order to show that the compensator solving LSSRP solves also NSSRP the following should be proved: a) the closed loop system has a unique stationary point for any exogeneous signal x_{20} and any $\varphi \in \mathcal{F}_\varrho$; b) the stationary regulated output i.e. the regulated output corresponding to the stationary point is zero; c) each stationary point is globally asymptotically stable for all $\varphi \in \mathcal{F}_\varrho$. This last property means that, even if the stationary point is defined for some nonlinear function from $\mathcal{F}_\varrho$, its stability is ensured for all $\varphi \in \mathcal{F}_\varrho$ uniformly. The main result is the following.

Theorem. Consider the system (2.1)–(2.2) under the basic assumptions (i)–(iii) with a compensator (2.4) designed in order to solve LSSRP for the linearized system (2.1)–(2.2). Assume, additionally, that the compensator is such that: (iv) there exists $\alpha \in \mathbb{R}$ such that

$$1/\varrho - \operatorname{Re}(1 + \mathrm{j}\omega\alpha)\, H_{\mathrm{R}}(\mathrm{j}\omega) > 0 \tag{4.1}$$

for all $\omega \in \mathbb{R}$ such that $\det(\mathrm{j}\omega I - A_{\mathrm{R}}) \neq 0$; here $H_{\mathrm{R}}(s) = f_{\mathrm{R}}^{\mathrm{T}}(sI - A_{\mathrm{R}})^{-1}\, b_{\mathrm{R}}$, f_{R}, A_{R}, b_{R} being those of (2.8). Then: a) the nonlinear closed loop system (2.7) has a unique stationary point for any x_{20} and $\varphi \in \mathcal{F}_\varrho$ and the corresponding stationary controlled output is zero; b) each stationary point is absolutely stable in the class $\mathcal{F}_\varrho$ where $\varrho > 1$ is the one in the frequency domain inequality (4.1) and also $\lim_{t \to \infty} z(t) = 0$. Moreover these properties are true for any A_3 and for small variations of (A_1, b_1).

Remark that, A_{R}, b_{R}, f_{R} being those of (2.8), the transfer function $H_{\mathrm{R}}(s)$ reads

$$H_{\mathrm{R}}(s) = -T_{\mathrm{c}}(s)\, T(s)$$

where $T(s) = C_1(sI - A_1)^{-1}\, b_1$ is the transfer function matrix of the plant and $T_{\mathrm{c}}(s) = g_{\mathrm{c}}^{\mathrm{T}} + f_{\mathrm{c}}^{\mathrm{T}}(sI - A_{\mathrm{c}})^{-1}\, B_{\mathrm{c}}$ is the transfer function matrix of the compensator; therefore $H_{\mathrm{R}}(s)$ is the transfer function of the linear part of the system as in any absolute stability problem. If (3.1) are also taken into account then

$$T_{\mathrm{c}}(s) = g_{\mathrm{c}}^{\mathrm{T}} + f_{\mathrm{w}}^{\mathrm{T}}(sI - A_{\mathrm{w}})^{-1}\, B_{\mathrm{w}} + (1/s)\left[f_{\mathrm{a}} + f_{\mathrm{w}}^{\mathrm{T}}(sI - A_{\mathrm{w}})^{-1}\, a_{\mathrm{a}}\right] q^{\mathrm{T}} \quad \bullet$$

what shows a PI structure of the compensator. The difference between this compensator and the ideal PI controller occurs from the dynamics of the stabilizing compensator synthesized for the extended system (3.2).

5. AN EXAMPLE

Consider the system

$$\begin{aligned} \dot{\zeta}_1 &= \zeta_2 + \alpha_{31} v_0 \\ \dot{\zeta}_2 &= -\zeta_2 + \varphi(\eta) + \alpha_{32} v_0 \\ y &= r_0 - \zeta_1 , \quad z = y \end{aligned} \tag{5.1}$$

where r_0 is a constant reference, v_0 is a constant disturbance and α_{3i} $(i = 1, 2)$ are uncertain what means that the disturbance can occur in any point of the plant. The linear part has the eigenvalues $\lambda_1 = 0$, $\lambda_2 = -1$ hence it is not asymptotically stable. In the sequel a compensator solving the NSSRP will be constructed.

1° Consider the linear extended system

$$\begin{aligned} \dot{\eta}_1 &= -\eta_1 + \mu(t) \\ \dot{\eta}_2 &= -\eta_1 \\ \dot{x}_a &= \eta_2 , \qquad y_e = \begin{pmatrix} \eta_2 \\ x_a \end{pmatrix} \end{aligned} \tag{5.2}$$

For this system a stabilizing linear compensator is designed using the separation principle: first the spectrum assignment by linear state feedback is performed

$$\mu = k_1 \eta_1 + k_2 \eta_2 + k_3 x_a \tag{5.3}$$

where k_i are chosen in order to obtain the desired spectrum in $\mathscr{C}^-$; taking into account the closed loop characteristic equation the following restrictions on k_i are obtained from the Routh-Hurwitz conditions:

$$k_1 < 1 , \quad k_2 > 0 , \quad k_3 > 0 , \quad k_3 + k_2 k_1 < k_2 \tag{5.4}$$

Remark that in (5.3) only η_2 and x_a can be directly used (are "measurable"). In order to obtain an estimate for η_1 a minimal order observer is designed

$$\begin{aligned} \dot{w} &= (\theta - 1) w + \theta(\theta - 1) \eta_2 + \mu \\ \hat{\eta}_1 &= w + \theta \eta_2 \end{aligned} \tag{5.5}$$

where $\theta < 1$ in order to get a stable observer (this is a necessary condition for stabilization via state feedback + state estimation). Replacing η_1 by $\hat{\eta}_1$ in (5.3) and combining (5.3) and (5.5) a linear error-activated stabilizing compensator for system (5.2) is obtained

$$\begin{aligned} \dot{w} &= (\theta + k_1 - 1) w + (\theta(\theta - 1 + k_1) + k_2) \eta_2 + k_3 x_a \\ \mu &= k_1 w + (k_1 \theta + k_2) \eta_2 + k_3 x_a \end{aligned} \tag{5.6}$$

From (3.5) it follows that the stabilizing and regulating linear compensator is given by

$$\begin{aligned} \dot{w} &= (\theta + k_1 - 1) w + k_3 x_a + (\theta(k_1 + \theta - 1) + k_2) \eta_2 \\ \dot{x}_a &= \eta_2 \\ \eta &= k_1 w + k_3 x_a + (k_1 \theta + k_2) \eta_2 \end{aligned} \tag{5.7}$$

2^0 Taking into account that η_2 stands for the measured (and regulated) error $r_0 - \zeta_1$ and η_1 stands for the state variable ζ_2, the following closed loop system equations are obtained

$$\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 + \alpha_{31} v_0 \\
\dot{\zeta}_2 &= -\zeta_2 + \varphi(\eta) + \alpha_{32} v_0 \\
\dot{w} &= (\theta + k_1 - 1)\, w + k_3 x_a + (\theta(\theta + k_1 - 1) + k_2)(r_0 - \zeta_1) \\
\dot{x}_a &= r_0 - \zeta_1 \\
\eta &= k_1 w + k_3 x_a + (k_1 \theta + k_2)(r_0 - \zeta_1)
\end{aligned} \tag{5.8}$$

The parameters k_1, k_2, k_3 which are subject to restrictions (5.4) and $\theta < 1$ can be used for the free assignment of the spectrum for the overall system; however here a trade-off between spectrum assignment and the frequency domain inequality (4.1) must be performed. The transfer function $H_R(s)$ is

$$H_R(s) = -H_c(s)\, H(s) = -\frac{(k_1\theta + k_2)\, s^2 + (k_3 - k_2(\theta - 1))\, s - k_3(\theta - 1)}{s^2(s + 1)(s + 1 - \theta - k_1)} \tag{5.9}$$

The double pole at the origin implies that one must take $\alpha = \pm\infty$ (Popov [7]) hence (4.1) takes the form

$$\pm\omega \operatorname{Im} H_R(j\omega) < 0 \tag{5.10}$$

Some elementary manipulation shows that in order to satisfy (5.4) and (5.10) it is sufficient to assign

$$k_1 < 0\,; \quad k_2 > 0\,; \quad k_3 > 0\,; \quad -k_2 - k_1 - k_1^2 < k_3 + k_2 k_1 < k_2\,;$$

$$-\frac{k_1}{2} - \frac{1}{2}\sqrt{\left(k_1^2 - \frac{4}{k_1}(k_3 + k_2(1 + k_1))\right)} < \theta < \min\left\{1, -\frac{k_1}{2} + \right.$$

$$\left. + \frac{1}{2}\sqrt{\left(k_1^2 - \frac{4}{k_1}(k_3 + k_2(1 + k_1))\right)}\right\} \tag{5.11}$$

If θ is chosen also in order that $k_2 + \theta k_1 > 0$ a minimum phase compensator is obtained.

Due to the form of (5.10) $\varrho = \infty$ hence the NSSRP is solved for $\varphi \in \mathscr{F}_\infty$, i.e. for all monotonic nonlinear and linear functions such that $\lim_{\eta \to \pm\infty} \varphi(\eta) = \pm\infty$.

6. PROOF OF THE MAIN RESULTS

This proof will consists of several steps.

A. First existence and uniqueness of the stationary point for given x_{20} and φ will be proved. The compensator solves LSSRP hence it stabilizes the system (2.1) to

(2.2) with $\varphi(\eta) = \eta$. Therefore (2.7) with $\varphi(\eta) = \eta$ is stable hence the matrix $A_{R\Sigma} = A_R + b_R f_R^T$ is a Hurwitz matrix; it is therefore nonsingular. Denoting $\eta_s = f_R^T x_{Rs} + g_R^T x_{20}$, where x_{Rs} is a solution of (2.9), (2.9) can be written as

$$A_{R\Sigma} x_{Rs} + b_R(\varphi(\eta_s) - \eta_s) + (E_R + b_R g_R^T)\, x_{20} = 0$$

and, therefore

$$x_{Rs} = -A_{R\Sigma}^{-1} b_R(\varphi(\eta_s) - \eta_s) - A_{R\Sigma}^{-1}(E_R + b_R g_R^T)\, x_{20} \tag{6.1}$$

Taking into account the definition of η_s it follows from (6.1)

$$H_{R\Sigma}(0)\, \varphi(\eta_s) = (1 + H_{R\Sigma}(0))\, \eta_s - [(1 + H_{R\Sigma}(0))\, g_R^T - f_R^T A_{R\Sigma}^{-1} E_R]\, x_{20} \tag{6.2}$$

where $H_{R\Sigma}(s) = f_R^T(sI - A_R - b_R f_R^T)^{-1}\, b_R = f_R^T(sI - A_{R\Sigma})^{-1}\, b_R$. The basic return difference formula shows that

$$H_{R\Sigma}(s) = f_R^T(sI - A_R - b_R f_R^T)^{-1}\, b_R = \frac{H_R(s)}{1 - H_R(s)}$$

where $H_R(s) = f_R^T(sI - A_R)^{-1}\, b_R$. From a well known identity [8, p. 43]:

$$1 - H_R(s) \equiv \frac{\det(sI - A_R - b_R f_R^T)}{\det(sI - A_R^T)}$$

it follows that

$$1 + H_{R\Sigma}(s) \equiv \frac{1}{1 - H_R(s)} \equiv \frac{\det(sI - A_R)}{\det(sI - A_R - b_R f_R^T)}$$

and, if $s = 0$,

$$1 + H_{R\Sigma}(0) = \frac{\det A_R}{\det A_{R\Sigma}}$$

But (2.8) and (3.1) show that $\det A_R = 0$. Therefore (6.2) becomes

$$\varphi(\eta_s) = -f_R^T A_{R\Sigma}^{-1} E_R x_{20} \tag{6.3}$$

But $\varphi \in \mathscr{F}_\varrho$ hence the mapping φ: $\mathbb{R} \to \mathbb{R}$ is invertible for any x_{20} and η_s obtained from (6.3) is unique. Replacing this η_s in (6.1) the unique stationary point x_{Rs} is obtained.

B. It will be shown that the controlled output corresponding to the stationary point is zero. Indeed, if the structure of A_R given by (2.8) and (3.1) is again taken into account, (2.9) reads

$$\begin{pmatrix} A_1 & 0 & 0 \\ B_w C_1 & A_w & a_a \\ q^T C_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{1s} \\ w_s \\ x_{as} \end{pmatrix} + \begin{pmatrix} b_1 \\ 0 \\ 0 \end{pmatrix} \varphi(f_R^T x_{Rs} + g_R^T x_{20}) + \begin{pmatrix} A_3 \\ B_w C_2 \\ d_2^T \end{pmatrix} x_{20} = 0$$

hence $q^T C_1 x_{1s} + d_2^T x_{20} = 0$. But the readability assumption (iii) gives that $q^T C_1 = d_1^T$. Therefore

$$0 = d_1^T x_{1s} + d_2^T x_{20} = d_R^T x_{Rs} + d_2^T x_{20} = z_s$$

It should be also mentioned that (6.3) holds for any matrix A_3 – a submatrix of E_R – and for small variations of A_1 and b_1 which allow the spectrum of $A_{R\Sigma}$ to remain in $\mathbb{C}^-$. Therefore the existence of the unique stationary point is robust (structurally stable) with respect to disturbance location and plant parameter uncertainties.

C. In order to prove absolute stability of any stationary point, consider some $\varphi \in \mathscr{F}_\varrho$ where $\varrho > 1$ is the one of (4.1) and some $x_{20} \neq 0$. Let x_{Rs} be the corresponding stationary point of (2.7). Introducing the deviation from the operating point $\xi_R = x_R - x_{Rs}$ and taking into account (2.9), (6.3) and the expression of η_s it follows

$$\begin{aligned} \dot{\xi}_R &= A_R\xi_R + b_R[\varphi(f_R^T\xi_R + \eta_s) - \varphi(\eta_s)] \\ z &= d_R^T\xi_R \end{aligned} \tag{6.4}$$

Introducing the nonlinear function $\psi: \mathbb{R} \to \mathbb{R}$ defined by $\psi(\eta) = \varphi(\eta + \eta_s) - \varphi(\eta_s)$ system (6.4) becomes

$$\dot{\xi}_R = A_R\xi_R + b_R\psi(f_R^T\xi_R), \quad z = d_R^T\xi_R \tag{6.5}$$

Due to the fact that $\varphi \in \mathscr{F}_\varrho$ the function ψ verifies the sector inequalities

$$0 < \psi(\eta)/\eta < \varrho \tag{6.6}$$

Also $\psi(\eta) = 0$ iff $\eta = 0$. The only stationary solution of (6.5) is the trivial one. If absolute stability of the trivial solution of (6.5) is obtained, this property implies absolute stability of any stationary point because the system in deviations is the same for all stationary points. But the absolute stability of the zero solution of (6.5) in the class (6.6) is a *standard absolute stability problem*. Applying a quite general result [8, p. 251] the following sufficient absolute stability conditions are found:
a) there exist $\varrho > 0$ and $\alpha \in \mathbb{R}$ such that

$$1/\varrho - \operatorname{Re}(1 + j\omega\alpha)\, H_R(j\omega) \geqq 0, \quad \omega \in \mathbb{R} \setminus \{\omega \mid \det(j\omega I - A_R) = 0\};$$

b) there exists $\varrho_0 \in (0, \varrho)$ such that $A_R + \varrho_0 b_R f_R^T$ is a Hurwitz matrix;
c) the left-hand side of the frequency domain inequality is not identically zero.

But from the assumptions of the Theorem it follows that $\varrho_0 = 1$ and a) together with c) follow from the strict frequency domain inequality (4.1). Therefore absolute stability of (6.5) in the class of (6.6) follows hence absolute stability of each stationary point in the class $\mathscr{F}_\varrho$ is obtained.

Taking into account that $\lim_{t\to\infty} \xi_R(t) = 0$, $\lim_{t\to\infty} d_R^T\, \xi_R(t) = \lim_{t\to\infty} z(t) = 0$.

The robustness of the above properties is ensured by the strict frequency domain condition (4.1). Indeed, as it can be seen from the expressions of $T(s)$ and $T_c(s)$, $H_R(s)$ is independent of A_3; if "small variations" of plant parameters (A_1, b_1, C_1) are allowed such that for the modified transfer function $\tilde{H}_R(s)$ the nonstrict frequency domain inequality

$$1/\varrho - \operatorname{Re}(1 + j\omega\alpha)\, \tilde{H}_R(j\omega) \geqq 0$$

still holds then, according to the general result of [8, p. 251], absolute stability of (6.5) in the class (6.6) is still valid. Remark that usually small variations of $H_R(s)$ can be compensated in order that (4.1) holds by suitable modification of the free parameter α. Therefore robustness of statement b) of the Theorem has been obtained; this completes the proof.

7. CONCLUDING REMARKS

The result of the paper is concerned with the case of a single nonlinear element and a single regulated output while there are several measured outputs. The result can be easily extended to the case of several noninteracting nonlinear elements and of several regulated outputs. The single variable case was considered only for the simplicity of the exposition. Two facts should be mentioned.

First, the structure of the frequency domain condition (4.1)

$$1/\varrho + \operatorname{Re}(1 + j\omega\alpha)\, T_c(j\omega)\, T(j\omega) > 0$$

shows that the compensator transfer function matrix acts like a multiplier ensuring positive realness for the tandem transfer function. The idea of using multiplers for frequency domain absolute stability criteria is not new (e.g. [1]); however here the multiplier occurs in a different way, namely from a linear compensator synthesis.

From here the next fact that should be pointed out: the synthesis of the compensator should be performed – as the example shows – by taking into account the restrictions imposed by the frequency domain inequality (4.1). If the structure of the compensator is imposed (for instance, by the application of the separation principle) the parameters should be chosen according to the requirements of the frequency domain inequality.

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