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Jiří Kopřiva

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A Note on the Structure of Certain Predicates Concerning the Sublanguages of ALGOL 60

JIŘÍ KOPŘIVA

The paper only formalizes the results of Reference [1] and completes these results by some details. In [1] the struture of the particular languages $S\alpha$ of ALGOL 60 generated by the various types of the metavariables α is described. Here, the results are formulated in the form of the predicates $F_\alpha(\xi)$ expressing the property $\xi \in S\alpha$. It is shown how these predicates are formed from other predicates being more simple in a certain sense. It is established how many times one needs to apply the operations used in [1] in order that one may get a set containing the considered phrase ξ if $F_\alpha(\xi)$ is true.

1. In [2] and [3] the *primitive recursiveness* of the predicates $F_\alpha(\xi)$ is proved, where $F_\alpha(\xi)$ denotes that the sequence ξ of the *terminal symbols* (letters of a finite *terminal alphabet*) is one of the "*values*" of the *metavariable* (*auxiliary symbol*) α in the sense of [4], Section 1.1. The values of the variables are constructed sequentially by the use of the *metalinguistic formulae* (*syntactic definitions*) which describe the *syntax* of an ALGOL – *like language*. In [2] and [3] certain formal restrictive conditions are given. These conditions are not fulfilled in [4]. Of course, these conditions are only formal and they do not change the structure of the generated sublanguages. We shall give small formal restrictions here, too.

Let Γ_n be the set of all the metalinguistic variables of ALGOL 60 except the variables ⟨any sequence not containing;⟩, ⟨any sequence not containing **end** or; or **else**⟩ and ⟨code⟩. These variables are omitted from this set. Let Γ_t be the set of all the basic symbols of ALGOL 60 except the symbol **comment**, which is omitted from it. On the contrary, the variable ⟨code⟩ may be joined to this set. In the sequel, the letters α, β (also subscribted) denote the symbols from $\Gamma = \Gamma_n \cup \Gamma_t$. The other Greek letters (also subscribted) denote the phrases formed from the elements in Γ (including the *empty phrase*).

The metalinguistic formulae as

$$(1) \qquad \langle \text{empty} \rangle ::=$$

(cf. [4], 1.1) shorten the length of phrases they are applied to. In order that we may exclude this possibility we assume for the definition (1) to be eliminated. Of course, we must perform the corresponding changes in the syntax. *Then the empty phrase cannot be a value of any variable. But the sets of the values of the most important metavariables of the language* ALGOL 60 *will remain unchanged.* The detailed analysis of this matter is performed in [5], Section 2.

We write $\alpha \blacktriangleright \beta$ if there exists a metalinguistic formula of the form $\alpha ::= \varphi\beta\psi$ (i.e., the definition not containing the symbol |). Every syntactic definition of ALGOL 60 consists of such *elementary definitions.* We write $\alpha \sim \beta$ if either $\alpha = \beta$ or there are $\alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q$ such that $\alpha_i \blacktriangleright \alpha_{i+1}$ for all $1 \leqq i < p$ and $\beta_i \blacktriangleright \beta_{i+1}$ for all $1 \leqq i < q$ and $\alpha_1 = \alpha = \beta_q$ and $\alpha_p = \beta = \beta_1$. The relation $\sim$ between variables is obviously an *equivalence relation.*

Let $\Sigma_1, \ldots, \Sigma_m$ be all the *equivalence classes* of this equivalence relation. Let $\hat{\Sigma}_j$ for $1 \leqq j \leqq n$ denotes the set $\{\alpha \mid$ there are $\alpha_1, \ldots, \alpha_p$ such that $\alpha_i \blacktriangleright \alpha_{i+1}$ for all $1 \leqq i < p$ and $\alpha_1 \in \Sigma_j, \alpha_p = \alpha\}$. *It is possible to establish a simple order* $\Sigma_1, \ldots, \Sigma_m$ *of all equivalence classes such that*

$$\hat{\Sigma}_j \subseteq \bigcup_{k=1}^{j} \Sigma_k \quad \textit{for all} \quad j = 1, \ldots, m\,.$$

The detailed description of these classes is given in [5]. In the sequel, the subscripts designate the mentioned order of the classes Σ_i.

For any set Φ of phrases we put $\Phi^* = \bigcup_{k=1}^{\infty} \Phi^k$, where $\Phi^1 = \Phi$ and $\Phi^{k+1} = \Phi\Phi^k$ for all $k \geqq 1$ (cf. [1], p. 79). Let Δ_α be the set of all φ's with the property $\alpha ::= \varphi$. There are two types of variables $\alpha \in \Gamma_n$. α is of the type (i) if

$$\alpha \in \Sigma_i \supset [\Sigma_i = \{\alpha\} \,\&\, [\varphi \in \Delta_\alpha \supset \varphi \in (\bigcup_{j=1}^{i-1} \Sigma_j \cup \Gamma_t)^*]]\,.$$

α is of the type (ii) if

$$[\alpha \in \Sigma_i \,\&\, \Sigma_i = \{\alpha_1, \ldots, \alpha_n\}] \supset [n = 1 \supset (\exists\varphi)(\exists\psi)(\exists\omega)[\varphi \in \Delta_\alpha \,\&\, \varphi = \psi\alpha\omega]]\,.$$

The type (ii) includes the types (ii), (iii), and (iv) from [1].

Let S_α denote the set of all the values of the variable α, i.e., the set of all the phrases from Γ_t^* such that they can be obtained from α by repeated application of suitable syntactic definitions.

Let α be of the type (i) and let $\Delta_\alpha = \{\varphi_1, \ldots, \varphi_r\}$, where

$$\varphi_j = \beta_{j,1} \ldots \beta_{j,s_j} \quad \text{for all} \quad 1 \leqq j \leqq r \quad \text{and} \quad \beta_{p,q} \in \Gamma\,.$$

Then we have obviously

$$S\alpha = \bigcup_{j=1}^{r} S\beta_{j,1} \ldots S\beta_{j,s_j}\,,$$

where $S\beta = \{\beta\}$ for $\beta \in \Gamma_t$. *)

*) For any sets Φ and Ψ of phrases $\Phi\Psi$ denotes the *concatenation* of them., i.e., $\Phi\Psi = \{\varphi\psi \mid \varphi \in \Phi, \psi \in \Psi\}$.

Let α be of the type (ii). Then we get the representation as follows. Let us put $\Delta_\alpha = \Delta'_\alpha \cup \Delta''_\alpha$, where

$$\varphi \in \Delta'_\alpha \equiv (\exists\beta)(\exists\psi)(\exists\omega)[\alpha \sim \beta \,\&\, \varphi = \psi\beta\omega]$$

and $\Delta''_\alpha = \Delta_\alpha - \Delta'_\alpha$. We shall introduce some *auxiliary operations.* Let the considered variable α belong to the class $\Sigma_i = \{\alpha_1, \ldots, \alpha_n\}$ and let $\Phi, \Phi_1, \ldots, \Phi_n$ be any sets of phrases. Then

$$[\![\Phi; \Phi_1, \ldots, \Phi_n]\!] = \{\varphi_0\psi_{i_1}\varphi_1\psi_{i_2} \ldots \psi_{i_r}\varphi_r \mid \varphi_0\alpha_{i_1}\varphi_1\alpha_{i_2} \ldots \alpha_{i_r}\varphi_r \in \Phi \text{ and } \psi_{i_j} \in \Phi_{i_j} \text{ for all } 1 \leqq j \leqq r\},$$

$$[\![\ldots]\!]^1 = [\![\ldots]\!],\ [\![\ldots]\!]^{k+1} = [\![[\![\ldots]\!]^k;\ \Phi_1, \ldots, \Phi_n]\!],\ [\![\ldots]\!]^\infty = \bigcup_{k=1}^{\infty} [\![\ldots]\!]^k$$

and further

$$[\![\Phi; \Phi_1, \ldots, \Phi_n]\!]_0 = \{\varphi_0\psi_{i_1}\varphi_1\psi_{i_2} \ldots \psi_{i_r}\varphi_r \mid \varphi_0\alpha_{i_1}\varphi_1\alpha_{i_2} \ldots \alpha_{i_r}\varphi_r \in \Phi,$$

(2) $$\text{where } \varphi_i \text{ does not contain any } \alpha_j \text{ for } 1 \leqq j \leqq n,\ 0 \leqq i \leqq r,$$

$$\text{and } \psi_{i_j} \in \Phi_{i_j} \text{ for all } 1 \leqq j \leqq r\}.$$

If $\Phi_j = \varnothing$ for some j, $1 \leqq j \leqq n$, then there is no phrase obtained by the operation (2) *from a phrase $\varphi \in \Phi$ containing α_j.*

For any variable α of the type (ii) we obtain $S\alpha$ by replacing each symbol β in each phrase from the set

$$[\![[\![\Delta'_\alpha; \Delta'_{\alpha_1}, \ldots, \Delta'_{\alpha_n}]\!]^\infty;\ \Delta''_{\alpha_1}, \ldots, \Delta''_{\alpha_n}]\!]_0 \cup \Delta''_\alpha$$

by the set $S\beta$ and by the subsequent concatenation of $S\beta$'s.

2. *Each of several first classes Σ_i contains only one variable α* (of the type (i)) *such that*

$$\varphi \in \Delta_\alpha \supset \varphi \in \Gamma_t.$$

(There are only two classes containing more than one element. One of them contains the variable ⟨expression⟩ and further 25 variables. The second class of them contains the variable ⟨statement⟩ and further 13 variables.) *If α is a variable belonging to some of these first classes one has obviously*

$$F_\alpha(\xi) \equiv (\exists\beta)[\beta \in \Delta_\alpha \,\&\, \xi = \beta].$$

Such variables are e.g. ⟨letter⟩, ⟨logical value⟩, ⟨relational operator⟩ etc.

Let now α be a variable of the type (i) such that there is a phrase $\varphi \in \Delta_\alpha$ containing some metavariable. *Then one has*

$$F_\alpha(\xi) \equiv (\exists r)[1 \leqq r \leqq 5 \,\&\, (p)[1 \leqq p \leqq r \supset$$
$$\supset (\exists\xi_p)(\exists\beta_p)[F_{\beta_p}(\xi_p) \,\&\, \xi = \xi_1 \ldots \xi_r \,\&\, \beta_1 \ldots \beta_r \in \Delta_\alpha]]].$$

The right side of each elementary syntactic definition of ALGOL 60 contains 5 symbols at most. The metavariables of the just described type are e.g. ⟨formal parameter⟩, ⟨procedure heading⟩, ⟨assignment statement⟩ and others.

Let now α be of the type (ii) and let it belong to the class $\Sigma_i = \{\alpha_1, \ldots, \alpha_n\}$. Let $l(\varphi)$ denote the length of the phrase φ, i.e., the number of symbols generating φ. One can establish by the detailed analysis of the syntactic definitions of ALGOL 60 that the following affirmation is true. *Let the phrase φ contain one variable from the class Σ_i at least. For the set*

$$[\![\varphi; \Delta'_{\alpha_1}, \ldots, \Delta'_{\alpha_n}]\!]^p$$

to contain all the phrases generated from φ and having the length $l(\varphi) + 1$ it is sufficient to take $p = 12$.

If the class Σ_i contains one element, i.e. $\Sigma_i = \{\alpha\}$, then the application of any syntatic definition of the form $\alpha ::= \omega$, where $\omega \in \Delta'_\alpha$, will lengthen the phrase φ immediatelly, because the syntax of ALGOL 60 does not contain the definition of the form $\alpha ::= \alpha$. If the class Σ_i contains more than one element then the most unfavourable case gives the following sequence each member of which except the last represents the left side of the elementary definition with the right side formed by the following member: ⟨actual parameter list⟩, ⟨actual parameter⟩, ⟨expression⟩, ⟨Boolean expression⟩, ⟨simple Boolean⟩, ⟨implication⟩, ⟨Boolean term⟩, ⟨Boolean factor⟩, ⟨Boolean secondary⟩, ⟨Boolean primary⟩, ⟨variable⟩, ⟨subscripted variable⟩. The definition (only) of the last member contains four symbols on the right side.

Let ξ denote the phrase whose appurtenance to the set $S\alpha$ ought to be investigated. The immediate consequence of the affirmation formulated above is following: the set

$$\Omega_1 = [\![\alpha; \Delta'_{\alpha_1}, \ldots, \Delta'_{\alpha_n}]\!]^{12[l(\xi)-1]}$$

contains all the phrases obtainable from α and having the positive length less or equal to $l(\xi)$. By the help of the set Ω_1 we shall now build the set

$$\Omega_2 = [\![\Omega_1; \Delta''_{\alpha_1}, \ldots, \Delta''_{\alpha_n}]\!]_0 \cup \Delta''_\alpha .$$

Let Ω be the set of the phrases from Ω_2 having the length less or equal to $l(\xi)$. *Each symbol β in each phrase from the set Ω being replaced by the set $S\beta$ and the necessary concatenations being performed we get a finite set of phrases which must contain the considered phrase ξ if $F_\alpha(\xi)$ is true.*

We shall investigate the dependence of the predicate $F_\alpha(\xi)$ on the *more simple* predicates by the formalisation of the described process. A predicate $F_\beta(\xi)$ is called more simple than the predicate $F_\alpha(\xi)$ if β belongs to the class preceding the class containing α.

$$\zeta \in [\![\varphi; \Phi_1, \ldots, \Phi_n]\!] \equiv (\exists r)\,[0 \leqq r \leqq l(\varphi)\,\&\,(\exists\varphi_0)\,(p)\,[1 \leqq p \leqq r \supset$$
$$\subset (\exists i_p)\,(\exists\varphi_p)\,(\exists\psi_p)\,[1 \leqq i_p \leqq n\,\&\,\psi_p \in \Phi_{i_p}\,\&\,\varphi = \varphi_0\alpha_{i_1}\varphi_1\alpha_{i_2}\ldots\alpha_{i_r}\varphi_r\,\&\,\zeta =$$
$$= \varphi_0\psi_1\varphi_1\psi_2\ldots\psi_r\varphi_r]]] ,$$

$$\zeta \in \Omega_1 \equiv \zeta \in [\![\Delta'_\alpha; \Delta'_{\alpha_1}, \ldots, \Delta'_{\alpha_n}]\!]^{12[l(\xi)-1]} \equiv (\exists\zeta^{(0)})[\zeta^{(0)} \in \Delta'_\alpha \,\&\, (i)[1 \leqq i \leqq 12(l(\xi)-1) \supset$$
$$\supset (\exists\zeta^{(i)})\,[\zeta^{(i)} \in [\![\zeta^{(i-1)}; \Delta'_{\alpha_1}, \ldots, \Delta'_{\alpha_n}]\!] \,\&\, \zeta = \zeta^{(12(l(\xi)-1))}]]] \,,$$
$$\zeta \in \Omega_2 \equiv \zeta \in [\![\Omega_1; \Delta''_{\alpha_1}, \ldots, \Delta''_{\alpha_n}]\!]_0 \vee \zeta \in \Delta'_\alpha \equiv$$
$$\equiv [(\exists\omega)\,[\omega \in \Omega_1 \,\&\, \zeta \in [\![\omega; \Delta''_{\alpha_1}, \ldots, \Delta''_{\alpha_n}]\!]] \,\&\, (i)\,[1 \leqq i \leqq n \supset \neg\, \zeta = \varphi\alpha_i\psi]] \vee \zeta \in \Delta''_\alpha \,,$$
$$\zeta \in \Omega \equiv \zeta \in \Omega_2 \,\&\, l(\zeta) \leqq l(\xi) \,,$$
$$F_\alpha(\xi) \equiv (\exists\zeta)\,[\zeta \in \Omega \,\&\, (p)\,[1 \leqq p \leqq l(\zeta) \supset$$
$$\subset (\exists\beta_p)\,(\exists\xi_p)\,[F_{\beta_p}(\xi_p) \,\&\, \zeta = \beta_1 \ldots \beta_{l(\zeta)} \,\&\, \xi = \xi_1 \ldots \xi_{l(\zeta)}]]] \,.$$

It follows from the written relations that the predicates $F_\beta(\xi_p)$ are more simple than the predicate $F_\alpha(\xi)$ (in the sense mentioned above).

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VÝTAH

Poznámka k struktuře určitých predikátů v podjazycích ALGOLu 60

JIŘÍ KOPŘIVA

Pomocí symbolů a vztahů formální logiky je popsán postup sestrojení podjazyků $S\alpha$ ALGOLu 60 příslušných k metaproměnným α různých typů. Základem jsou výsledky práce [1] ze seznamu literatury, které jsou zde doplněny několika detaily. Jedná se např. o stanovení toho, kolikrát je potřeba užít vytvářející operaci, abychom dostali takovou množinu frází, která obsahuje uvažovanou frázi ξ v případě, že $\xi \in S\alpha$.

Doc. Dr. Jiří Kopřiva, Csc., Laboratoř počítacích strojů, Třída obránců míru 21, Brno.