p-adic variant of the convergence Khintchine theorem for curves over $\mathbb{Z}_p$

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Abstract. A p-adic analogue of the convergence part of Khintchine's theorem for the linear Diophantine approximations to the points on the space curves with non-zero torsion given by normal functions is proved.

1. Introduction

In this paper we will consider Diophantine approximation of p-adic integers and generalize the convergence part of the metric theorem of Khintčhine [1]. Similar problems were first investigated for $\mathbb{Z}_p$ by K. Mahler [2].

Let $p \geq 2$ be a prime number, $\mathbb{Q}_p$ be the field of p-adic numbers with the Haar measure μ, $\mathbb{Z}_p$ be the ring of p-adic integers, $|\cdot|_p$ be the p-adic valuation. Throughout $\Psi(h) : \mathbb{R} \to \mathbb{R}^+$ is a monotonically decreasing function such that

$$\sum_{h=1}^{\infty} h^3 \Psi(h) < \infty. \tag{1.1}$$

Now we recall the definition of a normal function (by Mahler [2], see also [3]).

Definition. The function $f : \mathbb{Z}_p \to \mathbb{Z}_p$ is called a normal function if and only if $f(x) = \sum\limits_{n=0}^{\infty} b_n (x-b)^n$ where $|b|_p \leq 1$, $|b_n|_p \leq 1$ for all n and $\lim_{n\to\infty} |b_n|_p = 0$.

The class of normal functions is quite wide: given any analytic function $g(z)$ we can find integers r, s such that $p^r g(p^s z)$ is a normal function. Also if $f(x)$ is normal so are $f^{(k)}(x)$ $(k = 1, 2, \ldots)$. Besides any normal function is expanded as Taylor's series. It is not true for an arbitrary p-adic function [4, p. 223].

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Let $f_i : \mathbb{Z}_p \to \mathbb{Z}_p$ $(i = 1, 2, 3)$ be normal functions and

$$(1.2) \qquad W_3(x) = \det\left(\frac{d^j f_i(x)}{dx^j}\right)_{1\le i,j\le 3} \neq 0$$

almost everywhere in $\mathbb{Z}_p$. Let $a_i \in \mathbb{Z}$ $(i = 0, 1, 2, 3)$, $|a_i|$ be the absolute value of a_i and let $h = \max_{0\le i\le 3} |a_i| \neq 0$. Let $S_\Psi(f_1, f_2, f_3)$ be the set of $x \in \mathbb{Z}_p$ such that the inequality

$$|a_0 + a_1 f_1(x) + a_2 f_2(x) + a_3 f_3(x)|_p < \Psi(h)$$

holds for infinitely many integer vectors (a_0, a_1, a_2, a_3).

Theorem *The set $S_\Psi(f_1, f_2, f_3)$ has zero Haar measure.*

The theorem is about the linear Diophantine approximation to the points on the curves in $\mathbb{Z}_p^3$. The condition on $W_3(x)$ is equivalent to the condition that the torsion of the curve $(f_1(x), f_2(x), f_3(x))$ is non-zero almost everywhere on $\mathbb{Z}_p$.

In order to prove Theorem we use the effective version of Sprindžuk's method of essential and inessential domains. This version was eleborated in [5] where the convergence and the divergence parts of Khintchine's theorem for the curves $(x, f(x))$ on $\mathbb{Z}_p$ was proved. See also [8].

We notice that the problem under consideration belongs to the metric theory of Diophantine approximations of dependent values. It originates from Mahler's paper (1932) about the measure of S-numbers in the field $\mathbb{R}$ and $\mathbb{C}$. Then it was developed very intensively in the case of $\mathbb{R}$ by V.G. Sprindžuk, W.M. Schmidt, V.I. Bernik, M.M. Dodson and others [6]. But there are only a few results in the field $\mathbb{Q}_p$ [5 – 9]. Recently Khintchine's theorem for the case of $\mathbb{C}$ was proved in [10]. We remark also that the proofs of the aforementioned results have their own specialities depending on the fields.

2. Lemmas

According to the assumption of Theorem we have $|W_3(x)|_p \neq 0$ almost everywhere in $\mathbb{Z}_p$. Now we remove a set of arbitrary small measure θ from $\mathbb{Z}_p$ in such a way that the inequality

$$(2.1) \qquad |W_3(x)|_p \geq C_1$$

takes place in the complementary part $\mathbb{Z}_p(\theta)$ of $\mathbb{Z}_p$, where $0 < C_1 = C_1(\theta) < 1/2$. We can represent the set $\mathbb{Z}_p(\theta)$ as a countable sum of discs K_j having the Haar measure $\mu K_j \leq C_1/2$. The following investigation can be applied to any K_j, therefore we will write K_0 instead K_j. Without loss of generality we can assume that the radius of K_0 is equal r_0 and $r_0 < C_1/p^3$.

Lemma 1. *Let $g_i(x)$ $(i = 1, 2, \ldots, n)$ be normal functions. Suppose $G(x) = g_1(x) + r_2 g_2(x) + \ldots + r_n g_n(x)$ where $(r_2, \ldots, r_n) \in \mathbb{Q}^{n-1}$ and*

$$V_n(x) = \det\left(\frac{d^j g_i(x)}{dx^j}\right)_{1\le i,j\le n}.$$

Let $0 < \delta < 1$ *and let* $|V_n(x)|_p \geq \delta/\Delta > 0$, *when* $x \in K_0$. *Then* $\max\limits_{1\leq i\leq n}\left|\frac{d^iG(x)}{dx^i}\right|_p \geq \delta/\Delta$ *at every* $x \in K_0$ *where* $\Delta = \max\limits_{1\leq i\leq n}\max\limits_{x\in K_0}|\Delta_i(x)|_p$ *and* $\Delta_i(x)$ *is a cofactor of* $\frac{d^ig_1(x)}{dx^i}$ *in* $V_n(x)$.

The proof is similar to the proof of Lemma 4 in [11].

Suppose that the set $\mathcal{F}$ contains all non-zero linear forms $F(x) = a_0+a_1f_1(x)+a_2f_2(x)+a_3f_3(x)$ where $(a_0, a_1, a_2, a_3) \in \mathbb{Z}^4$ and $h_F = \max\limits_{0\leq i\leq 3}|a_i|$. Clearly that every function $F \in \mathcal{F}$ is normal. It follows from (2) that the functions $1, f_1(x), f_2(x), f_3(x)$ are linearly independent over $\mathbb{Q}$. Lemma 1 and (3) imply that

$$\min_{x\in K_0}\max(|F'(x)|_p, |F''(x)|_p, |F'''(x)|_p) \geq C_1. \tag{2.2}$$

The followong lemma is the important part of the proof of Theorem.

Lemma 2. *Suppose* $F \in \mathcal{F}$, $0 < \alpha, \beta_1, \beta_2, \beta_3 \leq 1$ *be real numbers. Let* $\sigma_{\alpha,\beta}(F)$ *be the set of points* $x \in K_0$ *satisfying*

$$|F(x)|_p < \alpha,\ \beta_1 \leq |F'(x)|_p,\ \beta_2 \leq |F''(x)|_p,\ \beta_3 \leq |F'''(x)|_p.$$

Then $\sigma_{\alpha,\beta}(F)$ *is covered by at most three discs of radius*

$$r = \min(\alpha\beta_1^{-1}, (\alpha\beta_2^{-1})^{1/2}, (\alpha\beta_3^{-1})^{1/3}).$$

Proof. The proof is similar to the proof of Lemma 1 in [5] but it needs some additional investigation. Suppose that $\sigma_{\alpha,\beta}(F)$ contains at least two points. As $\mathbb{Z}_p$ is compact, there exist the points $x_1, x_2 \in \sigma_{\alpha,\beta}(F)$ such that $|x_1 - x_2|_p \geq |x-y|_p$ for any $x, y \in \sigma_{\alpha,\beta}(F)$. It follows from (4) that there exist three cases be considered.

I. There exists a point $x_{1F} \in \sigma_{\alpha,\beta}(F)$ such that

$$\min_{x\in K_0}\max(|F'(x)|_p, |F''(x)|_p, |F'''(x)|_p) = |F'(x_{1F})|_p \geq C_1. \tag{2.3}$$

Let $x \in \sigma_{\alpha,\beta}(F)$. We consider Taylor's series for $F(x)$ in the disc $K(x_{1F}, r_0) = K_0$ with the centre at x_{1F} and of radius $r_0 < C_1/p^3$

$$F(x) - F(x_{1F}) = (x - x_{1F})(F'(x_{1F}) + \sum_{n=2}^{\infty}(n!)^{-1}F^{(n)}(x_{1F})(x - x_{1F})^{n-1}). \tag{2.4}$$

As $F(x)$ is normal and Taylor's series is unique, we obtain $|(n!)^{-1}F^{(n)}(x_{1F})|_p \leq 1$ for $n \geq 1$. Since $r_0 < C_1/p^3$, it follows that

$$|F'(x_{1F})|_p \geq |(n!)^{-1}F^{(n)}(x_{1F})(x - x_{1F})^{n-1}|_p \text{ for } n \geq 2. \tag{2.5}$$

According to (5), (7) and properties of the non-archimedean valuations, the p-adic valuation of the right-hand side of (6) equals $|F'(x_{1F})|_p|x - x_{1F}|_p$. Hence, $|F(x) - F(x_{1F})|_p = |F'(x_{1F})|_p|x - x_{1F}|_p > |F''(x_{1F})|_p|x - x_{1F}|_p^2$ and $|F(x) - F(x_{1F})|_p > |F'''(x_{1F})|_p|x - x_{1F}|_p^3$. So the assumptions of Lemma yield

$$|x - x_{1F}|_p \leq \min(\alpha\beta_1^{-1}, (\alpha\beta_2^{-1})^{1/2}, (\alpha\beta_3^{-1})^{1/3}) = r.$$

Thus the set $\sigma_{\alpha,\beta}(F)$ is covered by the disc $K(x_{1F}, r)$.

II. The set $\sigma_{\alpha,\beta}(F)$ has no points satisfying (5) but there exists a point $x_F \in \sigma_{\alpha,\beta}(F)$ such that

$$(2.6) \qquad \min_{x\in K_0} \max(|F'(x)|_p, |F''(x)|_p, |F'''(x)|_p) = |F''(x_F)|_p \geq C_1.$$

Let $x \in \sigma_{\alpha,\beta}(F)$. We consider Taylor's series for $F(x)$ in the disc $K(x_F, r_0) = K_0$, i.e. we have (6) where x_{1F} is replaced with x_F. Since $r_0 < C_1/p^3$, it follows that

$$(2.7) \qquad |F''(x_F)(x - x_F)/2|_p > |(n!)^{-1}F^{(n)}(x_F)(x - x_{1F})^{n-1}|_p \quad \text{for } n \geq 3.$$

Hence,

$$(2.8) \qquad |F(x) - F(x_F)|_p = |x - x_F|_p|F'(x_F) + F''(x_F)(x - x_F)/2|_p.$$

Suppose $|F'(x_F)|_p > |F''(x_F)(x - x_F)/2|_p$. It follows from the assumptions of Lemma and (10) that

$$\alpha \geq |F(x) - F(x_F)|_p = |F'(x_F)|_p|x - x_F|_p > |F''(x_F)|_p|x - x_F|_p^2 \geq \beta_2|x - x_F|_p^2.$$

Similarly we get $\alpha \geq |F(x) - F(x_F)|_p \geq \beta_3|x - x_F|_p^3$. Therefore $|x - x_F|_p \leq r$.

Now we investigate the remaining case when

$$(2.9) \qquad |F'(x_F)|_p \leq |F''(x_F)(x - x_F)/2|_p.$$

Let $x_F \neq x_2$, $|x - x_F|_p \leq |x - x_2|_p$ and

$$(2.10) \qquad |x - x_F|_p \leq |x_2 - x_F|_p.$$

Using Taylor's series, we get $F(x) - F(x_F) = \sum_{n=1}^{\infty}(n!)^{-1}F^{(n)}(x_F)(x - x_F)^n$. We form

$$(2.11) \quad \Delta_{2,F} = \frac{F(x_2) - F(x_F)}{x_2 - x_F}(x - x_F) = \sum_{n=1}^{\infty}(n!)^{-1}F^{(n)}(x_F)(x_2 - x_F)^{n-1}(x - x_F).$$

Then

$$F(x) - F(x_F) - \Delta_{2,F} = F''(x_F)(x - x_F)(x - x_2)/2+$$

$$(2.12) \qquad +\sum_{n=3}^{\infty}(n!)^{-1}F^{(n)}(x_F)(x - x_F)(x - x_2)\sum_{j=0}^{n-2}(x - x_F)^j(x_2 - x_F)^{n-2-j}.$$

It follows from the assumptions of Lemma and (12) that the p-adic valuation of the left-hand side of (14) is less than α. By (9), the p-adic valuation of the right-hand side of (14) equals $|F''(x_F)/2|_p|(x - x_F)(x - x_2)|_p$. Therefore

$$(2.13) \qquad \alpha \geq |F''(x_F)|_p|x - x_F|_p^2$$

and

$$(2.14) \qquad |x - x_F|_p \leq (\alpha\beta_2^{-1})^{1/2}.$$

If $|x - x_F|_p \geq |x - x_2|_p$ then we get (16) where x_2 is written instead x_F. If (12) does not hold but the inequality

$$(2.15) \qquad |x - x_F|_p \leq |x_1 - x_F|_p$$

is valid, then we replace x_2 by x_1 in the formulas after (11). Again we obtain (15) and (16).

The definition of the points x_1, x_2 implies that there are only two possibilities: (12) and (17). If $x_F = x_2$ we replace x_F by x_2 and x_2 by x_1 respectively in the formulas after (12). Thus the set $\sigma_{\alpha,\beta}(F)$ is covered by at most two discs from $\{K(x_F, r), K(x_2, r), K(x_1, r)\}$.

III. The set $\sigma_{\alpha,\beta}(F)$ has no points satisfying (5) or (8). Therefore

$$\min_{x\in K_0} \max(|F'(x)|_p, |F''(x)|_p, |F'''(x)|_p) = \min_{x\in K_0} |F'''(x)|_p \geq C_1. \tag{2.16}$$

Let $x \in \sigma_{\alpha,\beta}(F)$. We consider Taylor's series for $F(x)$ in the disc $K(x_1, r_0) = K_0$. As above, (18) implies

(2.17)

$$|F(x) - F(x_1)|_p = |x - x_1|_p |F'(x_1) + F''(x_1)(x - x_1)/2 + F'''(x_1)(x - x_1)^2/(3!)|_p.$$

The second multiplier of the right-hand side of (19) contains three addends. If the p-adic valuation of the j-th addend $(1 \leq j \leq 3)$ is greater than the others then similarly to the cases I, II we get $|x - x_1|_p \leq r$.

Now we consider the case when the p-adic valuations of the addends coincide, i.e.

$$|F'(x_1)|_p = |F''(x_1)(x - x_1)/2|_p = |F'''(x_1)(x - x_1)^2/(3!)|_p.$$

We can take a point x_3 such that $|x_1 - x_3|_p = |x_2 - x_3|_p$. Similarly to (13) we form the differences $\Delta_{1,2}$ and $\Delta_{2,3}$ for the points (x_1, x_2), (x_2, x_3) respectively and the second order difference $(\Delta_{2,3} - \Delta_{1,3})(x - x_2)/(x_2 - x_1)$. Then instead of (14) we consider

$$F(x) - F(x_3) - \Delta_{2,3} - (\Delta_{2,3} - \Delta_{1,3})(x - x_2)/(x_2 - x_1). \tag{2.18}$$

As above, we obtain that the p-adic valuation of the right-hand side of (20) equals

$$|F'''(x_3)(x - x_3)(x - x_2)(x - x_1)/3!|_p$$

and the p-adic valuation of the left-hand side of (19) is less than α. Therefore

$$\alpha \geq |F'''(x_3)(x - x_3)(x - x_2)(x - x_1)|_p.$$

Thus the set $\sigma_{\alpha,\beta}(F)$ is covered by $\sum_{i=1}^{3} K(x_i, r)$.

3. Proof of Theorem. The case of a large first derivative.

For every $Q \in \mathbb{N}$, we define $\mathcal{F}(Q) = \{F \in \mathcal{F} : h_F \leq Q\}$. Let K be a disc in K_0 and $\gamma > 0$. Suppose that the set $\Omega(K, \gamma, Q, F)$ consists of points $x \in K$ such that

$$|F(x)|_p < \gamma Q^{-4}, \; |F'(x)|_p \geq h_F^{-1/2} \tag{3.1}$$

and $\Omega(K, \gamma, Q) = \bigcup_{F\in\mathcal{F}(Q)} \Omega(K, \gamma, Q, F)$.

Proposition 1. *There exists the constant $C_2 > 0$ such that for any disc $K \subset K_0$ and for any number γ $(0 < \gamma < 1)$ there exists the positive number $Q_0 = Q_0(K, f_1, f_2, f_3, \gamma)$ such that $\mu\Omega(K, \gamma, Q) \leq C_2\gamma\mu K$ for each $Q > Q_0$.*

Proof. We cosider the functions $F \in \mathcal{F}(Q)$ such that $\Omega(K,\gamma,Q,F) \neq \emptyset$. As $\mathbb{Z}_p$ is compact, there exists a point $\alpha_F \in \Omega(K,\gamma,Q,F)$ such that $|F'(\alpha_F)|_p = \min_{x\in\Omega(K,\gamma,Q,F)} |F'(x)|_p$. Lemma 2 implies that

$$\mu\Omega(K,\gamma,Q,F) \ll \gamma Q^{-4}|F'(\alpha_F)|_p^{-1} \tag{3.2}$$

where the Vinogradov symbol $\ll$ contains a positive constant C depending only on K_0, f_1, f_2, f_3. For every $F \in \mathcal{F}(Q)$, we define the disc

$$\bar{\Omega}(K,\gamma,Q,F) = \{x \in \mathbb{Z}_p : |x-\alpha_F|_p \leq (2pQ|F'(\alpha)|_p)^{-1}\}.$$

We notice that $\bar{\Omega}(K,\gamma,Q,F) \subset K$ for suffiiently large Q. It follows from (22) that

$$\mu\Omega(K,\gamma,Q,F) \ll \gamma Q^{-3}\mu\bar{\Omega}(K,\gamma,Q,F). \tag{3.3}$$

As in Theorem 2 of [5] using Taylor's series, (21) and (22), we get $|F(x)|_p < (2Q)^{-1}$ for any $x \in \bar{\Omega}(K,\gamma,Q,F)$. Furthermore we have $\bar{\Omega}(K,\gamma,Q,F_1)\bigcap\bar{\Omega}(K,\gamma,Q,F_2) = \emptyset$ for any $F_1, F_2 \in \mathcal{F}(Q)$ if $F_1 - F_2 \in \mathbb{Z}$. Therefore

$$\sum_{F\in\mathcal{F}(Q,a_1,a_2,a_3)} \mu\bar{\Omega}(K,\gamma,Q,F) \ll \mu K \tag{3.4}$$

where $\mathcal{F}(Q,a_1,a_2,a_3)$ is the subset of $\mathcal{F}(Q)$ such that the coefficients a_1, a_2, a_3 are fixed. It follows from (23) and (24) that

$$\sum_{F\in\mathcal{F}(Q,a_1,a_2,a_3)} \mu\Omega(K,\gamma,Q,F) \ll \gamma Q^{-3}\mu K.$$

Since the number of different classes of $F \in \mathcal{F}(Q,a_1,a_2,a_3)$ equals $(2Q+1)^3$, Proposition 1 is proved.

Now we consider the set of $x \in K_0$ such that the system of inequalities

$$|F(x)|_p < \Psi(h_F),\ |F'(x)|_p \geq h_F^{-1/2} \tag{3.5}$$

holds for infinitely many $F \in \mathcal{F}$. Let $t \in \mathbb{N}$ and let $\Lambda(t)$ be the set of points $x \in K_0$ such that there exists a solution $F \in \mathcal{F}(2^t)$ of (25). Since $\Psi(h)$ is monotonic, it follows from (1) that $\Psi(h) < h^{-4}$ for sufficiently large h. Hence, we have $\Lambda(t) \subset \Omega(K_0, \Psi(2^t), 2^t)$. According to Proposition 1 we get $\mu\Lambda(t) \ll 2^{4t}\Psi(2^t)$. The Borel–Cantelly lemma and (1) imply that the set under consideration has zero measure. The proof of this part of Theorem is complete.

4. Proof of Theorem. The case of a small first derivative.

Proposition 2. *For almost all $x \in K_0$ the system*

$$|F(x)|_p < h_F^{-4},\ |F'(x)|_p < h_F^{-1/2} \tag{4.1}$$

has at most finitely many solution $F \in \mathcal{F}$.

Proof. We discuss similarly to Theorem 3 in [5] and give a sketch of the proof. Let $F \in \mathcal{F}$ be such a function that there exists a point $x \in K_0$ satisfying (26). It follows from (4) and the second inequality in (26) that $\min_{x\in K_0} \max(|F''(x)|_p, |F'''(x)|_p) \geq C_1$. Therefore we introduce two sets. Let $\sigma_2(F)$ be the set of points $x \in K_0$ satisfying

(26) and the inequality $\min\limits_{x \in K_0} (|F''(x)|_p) \geq C_1$. Let $\sigma_3(F)$ be the set of points $x \in K_0$ satisfying (26) and the inequalities $|F''(x)|_p < C_1$, $\min\limits_{x \in K_0} |F'''(x)|_p \geq C_1$. At first we consider $\sigma_2(F)$. We divide $\bigcup\limits_{F \in \mathcal{F}} \sigma_2(F)$ into essential and inessential domains by the Sprindžuk method. As in Theorem 3 of [5], Lemma 2 implies that the set of $x \in K_0$ belonging to infinitely many essential domains $\sigma_2(F)$ has zero measure. As in Theorem 3 of [5], Lemma 2 and the result in [9] imply that the set of $x \in K_0$ belonging to infinitely many inessential domains $\sigma_2(F)$ has zero measure. Details see in [5].

Now we consider $\sigma_3(F)$. We divide $\sigma_3(F)$ into four subsets. Let $S_1(F)$ be the set of points $x \in \sigma_3(F)$ satisfying the inequalities

$$|F'(x)|_p < \Psi(h),\ h_F^{-1/2} \leq |F''(x)|_p < C_1.$$

Let $S_2(F)$ be the set of points $x \in \sigma_3(F)$ satisfying the inequalities

$$|F'(x)|_p < \Psi(h),\ |F''(x)|_p < h_F^{-1/2}.$$

Let $S_3(F)$ be the set of points $x \in \sigma_3(F)$ satisfying the inequalities

$$\Psi(h) \leq |F'(x)|_p < h_F^{-1/2},\ h_F^{-1/2} \leq |F''(x)|_p < C_1.$$

Let $S_4(F)$ be the set of points $x \in \sigma_3(F)$ satisfying the inequalities

$$\Psi(h) \leq |F'(x)|_p < h_F^{-1/2},\ |F''(x)|_p < h_F^{-1/2}.$$

We are interested in those $x \in K_0$ which belong to infinitely many $S_i(F)$ ($i = 1, 2, 3, 4$) for $F \in \mathcal{F}$. The measure of the set $S_1(F)$ is estimated similarly to Proposition 1. As in Theorem 3 of [5], the measures of the sets $S_2(F)$, $S_3(F)$ and $S_4(F)$ are estimated by Lemma 2, the result of [9] and with help of the Sprindžuk method. The Borel–Cantelly lemma finishes the proof.

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