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Fuzzy Controller Design Based on Stability Criteria

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Abstract: The paper deals with the stability of control systems with fuzzy controllers. The principle of stability in the first approximation is applied to fuzzy controllers. To do so a mathematical formulation of a certain class of Sugeno-Takagi fuzzy controllers is introduced and sufficient differentiability conditions are given in terms of the fuzzy parameters. The Jacobian of a fuzzy controller is explicitly calculated. This is used to derive stability intervals for the fuzzy parameters with the aid of stability theory for interval matrices.

Key Words: fuzzy control, Lyapunov stability, Sugeno-Takagi fuzzy controller, interval matrices, robust stability

Mathematics Subject Classification: 93C42, 93D05, 93C15

1. Introduction

A well known and successful application of fuzzy logic is fuzzy control. Therein the ideas of Zadeh's fuzzy sets (Zadeh 1965) are used to model certain vague and imprecise knowledge like human experience. The fuzzy controller can be interpreted as a logical device which tries to imitate human decisions. But in recent years fuzzy controllers are more and more regarded as nonlinear functions $f : \mathbb{R}^p \longrightarrow \longrightarrow \mathbb{R}^m$ with a special design algorithm, namely the fuzzy method. Especially Sugeno-Takagi fuzzy-controller can be viewed as approximation and interpolation methods (Möllers & van Laak 1998).

The situation in control theory is the following. Let a continuous time, dynamical system

$$\dot{x}(t) = g(x(t), u(t)), \tag{1}$$

$$y(t) = h(x(t)) \tag{2}$$

with $g : \mathbb{R}^{n+m} \longrightarrow \mathbb{R}^n$ and $h : \mathbb{R}^n \longrightarrow \mathbb{R}^p$ be given. We omit the time variable t in the sequel.

According to Figure 1 we choose the control u as

$$u = f(y). \tag{3}$$

In this regard the problem of stability (in the sense of Lyapunov) arises, that is the question whether the solution x of the system (1)-(2) with the feedback (3) tends

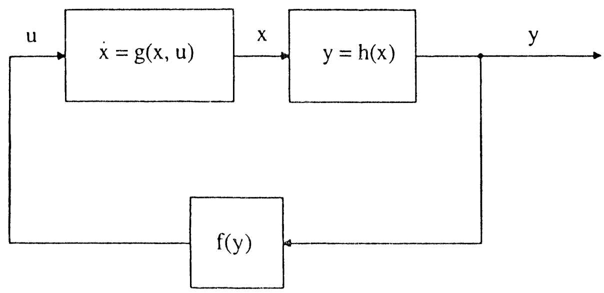

Figure 1: Nonlinear control loop

to infinity or not. For a definition of several kinds of stability see (Vidyasagar 1993). Stability theory for *linear* systems is well developed, but for *nonlinear* systems the problem is much more complicated and a general theory does not exist, indeed for special situations various stability criteria are known.

Since fuzzy controllers are nonlinear mappings we want to apply nonlinear stability theory. The analysis of stability properties of fuzzy controllers has been discussed in the literature in the last years, overviews are given by Bretthauer& Opitz (1994), Kiendl (1997), Möllers (1998). The next step beyond stability *analysis* is the controller *design in terms of stability criteria.* On this, Verbruggen & Bruijn (1997) have stated the following:

> *"The* **real design problem** *is not only to assess stability, but to describe the influence of the design parameters (the controller) and the process parameters on the stability, and to use stability criteria not only as an analysis tool but also as a design tool."*

This paper tries to make a contribution to this goal and to give explicit conditions for the parameters of the fuzzy controller in order to assure the stability of the closed loop.

The paper is organized as follows. We recall the principle of stability in the first approximation in Section 2. In Section 3 we develop a notation of fuzzy controllers suitable to derive sufficient differentiability conditions and a formula for the Jacobian of a fuzzy controller. In Section 4 the stability theory for interval matrices together with results from Section 3 are used to give stability intervals for fuzzy controller parameters. We conclude with some remarks in Section 5.

2. Stability in the first approximation

In order to determine the stability of a nonlinear differential equation

$$\dot{x} = g(x) \qquad \text{with} \qquad g(0) = 0 \tag{4}$$

we have a closer look at the linearized system

$$\dot{x} = A\,x \tag{5}$$

where $A := \left.\frac{\partial g(x)}{\partial x}\right|_{x=0}$ is the Jacobian of g at the equilibrium 0. A matrix A is called *stable* if and only if all its eigenvalues have negative real part, it is called *unstable* if one eigenvalue has positive real part. Observe that we consider only continuous time systems in this paper, therefore we say stable instead of Hurwitz stable (Rohn 1994). The following Theorem is known as the *Principle of Stability in the First Approximation* or as the *Linearization Principle* (Zabczyk 1992, p. 101).

Theorem 2.1. *Consider the system (4) and let g be continuously differentiable in 0. The equilibrium 0 is exponentially stable if and only if the Jacobian of g in 0*

$$A := \left.\frac{\partial g(x)}{\partial x}\right|_{x=0}$$

is stable.

If one is interested in asymptotic stability we have the following (Hahn 1967, p. 122). If A is stable, then the equilibrium 0 is asymptotically stable. If A is unstable, then the equilibrium 0 is unstable. Therefore we have to assure the stability of the linearized system (5).

3. Linearizing the fuzzy controller

We are going to develop the basis for an application of the above mentioned Linearization Principle to control loops with fuzzy controllers. We describe a special class of fuzzy controllers and suggest a certain notation of the various fuzzy controller components. Then we give sufficient differentiability conditions for this class of fuzzy controllers. Moreover the Jacobian of a fuzzy controller is explicitly calculated in terms of the fuzzy parameters, such as certain entries of the rule base and membership functions.

3.1. A standardized Sugeno-Takagi fuzzy controller

This section is concerned with the development of an formula of a multivariate Sugeno-Takagi fuzzy controller. Let denote $y = (y_1, \ldots, y_p)^T \in Y \subset \mathbb{R}^p$ the vector of inputs and $u = (u_1, \ldots, u_m)^T \in U \subset \mathbb{R}^m$ the vector of outputs of the controller. For every single input there are fuzzy sets defined on the universe of discourse. Let us label the fuzzy sets with integers. We use the same symbol μ for fuzzy sets and membership functions, i.e., we say the input y_1 is μ_{-1} or the input y_2 is μ_5 instead of something like y_1 is *negative small* or y_2 is *positive very big*. This notation will shorten the subsequent statements. The membership functions for different inputs are distinguished by there arguments, e.g. $\mu_{-1}(y_1)$ or $\mu_5(y_2)$. If there might be some misunderstandings to which component of the input the membership function belongs we write $\mu_{-1}^{(1)}(0)$ or $\mu_5^{(2)}(0)$, to indicate μ_{-1} belongs to the first input y_1 and μ_5 to the second y_2.

The Sugeno-Takagi fuzzy controller has a collection of rules of the type

$$\text{IF } y_1 \text{ IS } \mu_{-1} \text{ AND } y_2 \text{ IS } \mu_5 \text{ THEN } u_i = u_{-1,5}^{(i)}(y_1, y_2) \tag{6}$$

The AND combination in the premise can be modeled by ť-Norms (Bandemer & Gottwald 1993). In the conclusion the i-th component of the output u_i is determined by a reál valued function depending on the inputs. To indicate to which rule the *conclusion function* belongs we use the subscript "-1,5". The superscript "(i)" indicates the i-th output. This notation describes unambiguously the rule (6). If we háve specified the conclusion function, say *u_\ $_5$(yi,y$_2$)* = *y* + 2y$_{2)}$ the rule is well defined and we can omit the long notation (6). In generál, if we consider *p* inputs we use an multiindex a = *(a$_x$,...* ,a) £ Z^p of integers to describe the conclusion function and in fact the rule.

The concept of the Sugeno-Takagi fuzzy controller determines the degree of truth of a single rule by the degree of truth of the premise. Therefore the conclusion function must be combined with this degree of truth. That could be doně by the use of a í-Norm. We choose here the algebraic product, i.e., the conclusion function has to be multiplied with the value of truth of the premise. Due to Sugeno and Takagi the finál output of the controller is determined by a weighted average. Hence we get a formula for the Sugeno-Takagi fuzzy controller.

The following Definition summarizes up the previous notations and assumes some additional properties. Therefore we get a speciál class of Sugeno-Takagi fuzzy controllers.

Definition 3.1. A Sugeno-Takagi fuzzy controller with the following structure is called a *standardized Sugeno-Takagi fuzzy controller (SFC),* for short.

(SFC 1) For every input y_k there is an odd number of normalized fuzzy sets, labeled with N_k := {—i/fc,... , *v$_k$]* C Z. Let the support of each membership function be a reál interval.

(SFC 2) The conclusion functions are continuously differentiable.

(SFC 3) The AND composition in the premise is modeled by a continuous č-Norm.

(SFC 4) The i-th output component of the (SFC), *i* € {1,... ,ra}, is

/<(*)= ^{E} « f $^{(y}$ 1 ; ? ° $^{(y)}$ forall.er. (7)

The symbol *Yl$_a$* $^{stan(}$3s for the p-times sum ^^{l} • • •]T^^{P} =_„ with *v$_k$*, *k* = = 1,... ,p as in (SFC 1). For a chosen í-norm *t* we abbreviate the rule premise

^{at}*a(Vv- >V$_P$)* :=A*aifoi)* M a ^) * ••* *tV>a$_p$(y$_p$)* for all (l/l > • • • *'Vpt'* € *Y*

and call a^ *premise function.* For the £-norm Minimum, t_M := min, the premise function in rule (6) is *a$_a^{r M}$ (y)* = min{/x_$_1$(y$_1$),^$_5$(í/$_2$)}.

The above algebraic formula for the fuzzy controller (7) is well known (Driankov, Hellendorn & Reinfrank 1993). Here we háve just introduced some additional assumptions and a different notation. This will lead to a deeper insight in the structure of fuzzy controllers.

3.2. The Jacobian of the standardized Sugeno-Takagi fuzzy controller

We only consider the standardized Sugeno-Takagi fuzzy controller according to Definition 3.1 with the t-norm minimum in this paper. Without great effort analogous results can be shown for the t-norms algebraic product and bounded difference (Möllers 1998).

Let us collect some assumptions needed in the sequence. First we assume some properties of the membership functions. In real applications they are often fulfilled.

> Let all membership functions be
> - continuous in 0,
> - left- and right-hand differentiable in 0,
> - differentiable in a neighborhood of 0 except in 0.
>
> The zero membership functions have the value 1 in the point 0, i.e., for all $k \in \{1, \ldots, p\}$ let
> $$\mu_0^{(k)}(0) = 1.$$
> Every other membership function vanishes in 0, i.e., for all $k \in \{1, \ldots, p\}$ and all $\alpha_k \in N_k \setminus \{0\}$ let
> $$\mu_{\alpha_k}(0) = 0.$$

(P1)

Next we assume that there is always a pair of symmetric membership functions in a neighborhood of 0.

> For all $k \in \{1, \ldots, p\}$ and all $\alpha_k \in N_k$ exists an $\varepsilon > 0$, such that
> $$\mu_{\alpha_k}(y_k) = \mu_{-\alpha_k}(-y_k) \qquad \text{for all} \quad y_k \in B_\varepsilon(0).$$

(P2)

Finally we suppose a in a certain sense symmetric rule base. Denote the set of all fuzzy sets which right-hand derivative $D^+\mu$ doesn't vanish in 0 with

$$N_k^{\neq 0} := \{\alpha_k \in N_k |\ D^+\mu_{\alpha_k}(0) \neq 0\}.$$

> For all $i \in \{1, \ldots, m\}$, $k \in \{1, \ldots, p\}$ and all $\alpha_k \in N_k^{\neq 0}$ let
> $$u^{(i)}_{0\cdots 0\alpha_k 0\cdots 0}(0) = -u^{(i)}_{0\cdots 0-\alpha_k 0\cdots 0}(0).$$

(P3)

The properties of the (SFC) are sufficient to prove the Linearization Formula (8) in the next Theorem. Indeed a lot of fuzzy controllers developed for real applications are (SFC) with the properties (P1)–(P3).

We are able to express the Jacobian of the (SFC) in terms of certain fuzzy parameters. For a proof of the following Theorem see (Möllers 1998, Th. 4.10, pp. 47).

Theorem 3.2. (Linearization Formula) *Let $f : \mathbb{R}^p \longrightarrow \mathbb{R}^m$ be a (SFC). Assume properties (P1)-(P3). Then f is continuously differentiable in 0 and for $j \in \{1, \dots, p\}$ the j-th column of the Jacobian in 0 is*

$$F\, e_j = \left.\frac{\partial u_{0\dots0}(y)}{\partial y_j}\right|_{y=0} + U_j \left(D^+ \mu(y_j)\big|_{y_j=0}\right) \tag{8}$$

where

$$\begin{aligned}
F &:= \left.\frac{\partial f(y)}{\partial y}\right|_{y=0}, \\
e_j &: \quad j - th \text{ unity vector}, \\
u_{0\dots0}(y) &:= \begin{pmatrix} u^{(1)}_{0\dots0}(y) \\ \vdots \\ u^{(m)}_{0\dots0}(y) \end{pmatrix}, \\
U_j &:= \begin{pmatrix} u^{(1)}_{0\dots0-\nu_j0\dots0}(0) & \cdots & u^{(1)}_{0\dots0\nu_j0\dots0}(0) \\ \vdots & \ddots & \vdots \\ u^{(m)}_{0\dots0-\nu_j0\dots0}(0) & \cdots & u^{(m)}_{0\dots0\nu_j0\dots0}(0) \end{pmatrix}, \\
D^+ &: \quad \text{right hand derivative of the subsequent function}, \\
\mu(y_j) &:= \begin{pmatrix} \mu_{-\nu_j}(y_j) \\ \vdots \\ \mu_{\nu_j}(y_j) \end{pmatrix}.
\end{aligned}$$

For an application of the Linearization Formula see Example 4.4.

4. Robust fuzzy controller design with interval matrices

Due to the Linearization Formula (8) we can use interval matrices to design robust fuzzy controllers. First we recall the definition of interval matrices and give an easy to use stability criterion. Then we state Lemma 4.3 which allows to transfer the stability theory of interval matrices to controller design. The application of this is shown in Example 4.4 which ends up with certain stability intervals for the fuzzy parameters.

We define interval matrices according to Rohn (1994).

Definition 4.1. Let the matrices $L, U \in \mathbb{R}^{n \times n}$ with $l_{i,j} \leq u_{i,j}$ for all $i, j \in$ $\in \{1, \dots, n\}$ be given. The set of matrices

$$[L, U] := \left\{X \in \mathbb{R}^{n \times n} \;\middle|\; l_{i,j} \leq x_{i,j} \leq u_{i,j} \;\text{ for all } i, j \in \{1, \dots, n\}\right\}$$

is called **interval matrix**. We define the **center matrix** $X_0 := \frac{1}{2}(U + L)$ and the **radius matrix** $\Delta X := \frac{1}{2}(U - L)$. An interval matrix is called **stable** if all its elements are stable matrices.

It is obvious that an interval matrix can either be given by lower and upper bounds L and U or its center X_0 and radius ΔX. In the latter case we write $(X_0, \Delta X)$ to denote the interval matrix. We denote the symmetric part of a real matrix by $S(X) := \frac{1}{2}(X + X^T)$.

Stability of interval matrices is still an challenging problem of current research. In recent years many different sufficient stability conditions were stated. The interested reader is referred to Mansour (1989) and Wang & Michel (1993). For further use we cite an stability criterion given by Delgado-Romero & Rojas-Estrada (1995, Th. 3.1). Let λ_{max} denote the maximum eigenvalue of an symmetric matrix.

Theorem 4.2. *Let $(X_0, \Delta X)$ be an interval matrix. If*

$$\lambda_{max}(S(X_0)) + \lambda_{max}(S(\Delta X)) < 0$$

then the interval matrix $(X_0, \Delta X)$ is stable.

In the sequel we apply stability theory for interval matrices to controller design and in the end to fuzzy controller synthesis. For the remainder of this section we introduce the following notations. For a matrix $M := (m_{i,j})_{1 \le i,j \le n}$ we denote the componentwise modulus by $|M| := (|m_{i,j}|)_{1 \le i,j \le n}$. The componentwise less than or equal to relation is denoted by $\preceq$, i.e., $M \preceq N :\Leftrightarrow m_{ij} \le n_{ij}$ for all $1 \le i,j \le n$.

Let us consider the linear differential equation

$$\dot{x} = (A + B\,F)\,x \tag{9}$$

where the state feedback matrix F is to be chosen appropriately. We intend to give upper and lower bounds for F in terms of an interval matrix which assures the stability of the system (9).

Lemma 4.3. *Let the interval matrix $(A + B\,F_0, |B|\,\Delta F)$ be stable. Then for all $F \in (F_0, \Delta F)$ the matrix $A + B\,F$ is stable.*

Proof: For $F \in (F_0, \Delta F)$ there exists a matrix δF with $|\delta F| \preceq \Delta F$ and

$$F = F_0 + \delta F.$$

Set

$$M := A + B\,F = A + B\,F_0 + B\,\delta F.$$

We have the componentwise inequality

$$B\,\delta F \preceq |B\,\delta F| \preceq |B|\,|\delta F| \preceq |B|\,\Delta F,$$

hence $M \in (A + B\,F_0, |B|\,\Delta F)$ and therefore M is stable. □

In the following Example we use Theorem 4.2, Lemma 4.3 and the Linearization Formula (8) to give stability intervals for certain parameters of the fuzzy controller.

Example 4.4. Let us consider the following system

$$\dot{x} = \left[\begin{pmatrix} 0 & 1 \\ 2 & -5 \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ 4 & -1 \end{pmatrix} F\right] x.$$

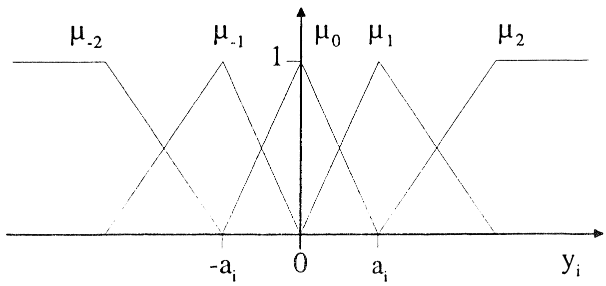

Figure 2. Membership functions for the two inputs y_1 and y_2

As center matrix for the feedback we choose

$$F_0 := \begin{pmatrix} -1.5 & 0.5 \\ -27 & 7 \end{pmatrix}$$

and as radius matrix

$$\Delta F := \begin{pmatrix} 0.1 & 0.05 \\ 0.1 & 0.1 \end{pmatrix}.$$

Then the closed loop matrix $A + B\,F_0$ is stable. With Theorem 4.2 we show that $(A + B\,X_0, |B|\,\Delta X)$ is stable. We get

$$\lambda_{max}(S(A + B\,F_0)) < -0.64$$

and

$$|B|\Delta F = \begin{pmatrix} |-1| & |1| \\ |4| & |-1| \end{pmatrix} \begin{pmatrix} 0.1 & 0.05 \\ 0.1 & 0.1 \end{pmatrix} = \begin{pmatrix} 0.2 & 0.15 \\ 0.5 & 0.3 \end{pmatrix}$$

with

$$\lambda_{max}(S(|B|\Delta F)) < 0.58$$

According to Lemma 4.3 the matrix $A + B\,F$ is stable for all

$$F \in \begin{pmatrix} [-1.6, -1.4] & [0.45, 0.55] \\ [-27.1, -26.9] & [6.9, 7.1] \end{pmatrix}.$$

Now we interpret the matrix F as the Jacobian of a fuzzy controller. Suppose a fuzzy controller $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ has five membership functions for each input according to Figure 2 and suppose the rule base has the following structure with constant entries, observe that the free spaces in the rule base are not included in the Linearization Formula (8) and therefore do not influence the Jacobian of the fuzzy controller in 0.

Rule base for output f_1

$y_1 \backslash y_2$	-2	-1	0	1	2
-2			$-u^{(1)}_{2,0}$		
-1			$-u^{(1)}_{1,0}$		
0	$-u^{(1)}_{0,2}$	$-u^{(1)}_{0,1}$	0	$u^{(1)}_{0,1}$	$u^{(1)}_{0,2}$
1			$u^{(1)}_{1,0}$		
2			$u^{(1)}_{2,0}$		

Rule base for output f_2

$y_1 \backslash y_2$	-2	-1	0	1	2
-2			$-u_{2,0}^{(2)}$		
-1			$-u_{1,0}^{(2)}$		
0	$-u_{0,2}^{(2)}$	$-u_{0,1}^{(2)}$	0	$u_{0,1}^{(2)}$	$u_{0,2}^{(2)}$
1			$u_{1,0}^{(2)}$		
2			$u_{2,0}^{(2)}$		

This fuzzy controller fulfills the assumptions of Theorem 3.2. With the Linearization Formula (8) we have

$$F\,e_1 = 0 + \begin{pmatrix} -u_{2,0}^{(1)} & -u_{1,0}^{(1)} & 0 & u_{1,0}^{(1)} & u_{2,0}^{(1)} \\ -u_{2,0}^{(2)} & -u_{1,0}^{(2)} & 0 & u_{1,0}^{(2)} & u_{2,0}^{(2)} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{a_1} \\ \frac{1}{a_1} \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{a_1} u_{1,0}^{(1)} \\ \frac{1}{a_1} u_{1,0}^{(2)} \end{pmatrix}$$

$$F\,e_2 = \begin{pmatrix} \frac{1}{a_2} u_{0,1}^{(1)} \\ \frac{1}{a_2} u_{0,1}^{(2)} \end{pmatrix}.$$

The condition

$$F = \begin{pmatrix} \frac{1}{a_1} u_{1,0}^{(1)} & \frac{1}{a_2} u_{0,1}^{(1)} \\ \frac{1}{a_1} u_{1,0}^{(2)} & \frac{1}{a_2} u_{0,1}^{(2)} \end{pmatrix} \in \begin{pmatrix} [-1.6, -1.4] & [0.4, 0.6] \\ [-27.1, -26.9] & [6.95, 7.05] \end{pmatrix}$$

assures the stability of the closed loop with fuzzy controller. It can be seen that we have four conditions but six free parameters. Therefore it is possible to design such a fuzzy controller.

5. Concluding Remarks

The notation of fuzzy controllers introduced in this paper leads to a deeper insight into there structure and enables us to calculate the linearization of the fuzzy controller in 0. This linearization is used both for controller analysis and design based on the Principle of Stability in the First Approximation. An application of interval matrix theory to the Linearization Formula is presented which leads to fuzzy stability intervals.

It should be mentioned that due to the Linearization Formula it is now possible to apply methods from linear control theory to fuzzy controllers. This can be regarded as a first step in dealing with stability of fuzzy controllers for design purpose. The next step, namely the analysis and design of the fuzzy controller's behavior far away from the linearization point, must be evaluated additionally, several methods for this purpose are given in Möllers (1998).

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