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Information Analysis in Variant I of Probabilistic Model of the School Achievement Test with Double Choice Response

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Abstract

The purpose of this paper is to study the uncertainty and the theoretical information contained in the school achievement test used as a method in an estimation of the student's knowledge. We consider variant I of probabilistic model of the school achievement test with a double choice response published in [1] in 1992 . The analysis in variant I begins with the assumption that when the tested person is really familiar with the topic of the question, he or she will select both the correct responses among the offered alternatives.

Key words: School-achievement test, double-choice response, probabilistic model, information theoretical analysis.

1991 Mathematics Subject Classification: 62P10, 62P15

1 Introduction

The construction of mathematical models describing the probability structure of school-achievement tests should take into account a certain difference between the real knowledge of the examinee and the result of the test evaluated by the examiner.

This paper follows the study summarized in [1], [2], [3] and [4] where the probabilistic models of the school-achievement test with a double choice response and their statistical analysis were given including certain simplifying assumptions. We consider the school-achievement test with a compulsory choice of two correct responses from $q > 3$ offered alternatives two of which are correct. A missing answer is evaluated as an incorrect one. It is also presumed that the test consists of n independent questions of the same difficulty and that the number of offered alternatives is the same.

The variant I of the probabilistic model of the school-achievement test with double choice response published in [1] proceeds from the assumption that when the person under examination is really familiar with the topic of the question he or she will select both the correct responses from the offered alternatives.

In the construction of the probabilistic model, the following notation for random events relative to every question of the test was used:

Z_i the examinee is familiar with the topic of the ith question

N_i the examinee is unfamiliar with the topic of the ith question.

According the above mentioned assumptions the random events Z_i, N_i have probabilities $P(Z_i) = 1 - \tau$, $P(N_i) = \tau$, where the parameter τ represents the proportion of the tested topic with which the examinee is unfamiliar.

In relation to the registered results of the test the following three random events are considered

S_{i0} no correct answer was given to the ith question

S_{i1} only one correct answer was given to the ith question

S_{i2} both correct answers were given to the ith question

In case of familiarity with the given topic the examinee will give the two correct responses, so that

$$P(S_{i0}|Z_i) = 0, \quad P(S_{i1}|Z_i) = 0, \quad P(S_{i2}|Z_i) = 1.$$

In case of unfamiliarity with the topic tested by the question the examinee can only use the random choice. With regard to the presence of two correct responses among $q > 3$ offered alternatives the following relations hold:

$$P(S_{i0}|N_i) = \frac{(q-2)(q-3)}{q(q-1)}, \quad P(S_{i1}|N_i) = \frac{4(q-2)}{q(q-1)}, \quad P(S_{i2}|N_i) = \frac{2}{q(q-1)}.$$

These conditional probabilities were calculated using the hypergeometric distribution which is applicable in this situation.

The unconditional probabilities of events S_{i0}, S_{i1}, S_{i2}

$$P(S_{i0}) = \tau\frac{(q-2)(q-3)}{q(q-1)}, \quad P(S_{i1}) = \tau\frac{4(q-2)}{q(q-1)}, \quad P(S_{i2}) = 1 - \tau\frac{q(q-1)-2}{q(q-1)}$$

were calculated according to the theorem of total probability.

By the Bayesian theorem, it is possible to obtain also the following conditional probabilities

$$P(N_i|S_{i0}) = 1, \quad P(N_i|S_{i1}) = 1, \quad P(N_i|S_{i2}) = \frac{2\tau}{q(q-1) - \tau(q(q-1)-2)},$$

$$P(Z_i|S_{i0}) = 0, \quad P(Z_i|S_{i1}) = 0, \quad P(Z_i|S_{i2}) = \frac{(1-\tau)q(q-1)}{q(q-1) - \tau(q(q-1)-2)}, \tag{1}$$

From the above mentioned results, the multinomial probability distribution (2) of the random vektor $M = (M_0, M_1, M_2)$ was derived in the form

$$P(M_0 = m_0, M_1 = m_1, M_2 = m_2) = \tag{2}$$

$$= \frac{n!}{m_0!m_1!m_2!} \left(\tau \frac{(q-2)(q-3)}{q(q-1)}\right)^{m_0} \left(\tau \frac{4(q-2)}{q(q-1)}\right)^{m_1} \left(1 - \tau \frac{q(q-1)-2}{q(q-1)}\right)^{m_2}$$

with $m_0 + m_1 + m_2 = n$, where M_0, M_1, M_2 is a random variable expressing the number of questions of the test to which no correct response was given, only one correct response was given, both the correct responses were given by the examinee, respectively, and the binomial distribution of the random variable $X = M_0 + M_1$ in the form

$$P(X = x) = \binom{n}{x} \left(\tau \frac{q(q-1)-2}{q(q-1)}\right)^{x} \left(1 - \tau \frac{q(q-1)-2}{q(q-1)}\right)^{n-x} \tag{3}$$

If the random variable Y represents the number of questions of the test with which the examinee was really unfamiliar, the unconditional distribution of this variable is a binomial distribution again dependent on the parameter τ

$$P(Y = y) = \binom{n}{y} \tau^y (1-\tau)^{n-y} \tag{4}$$

By the Bayesian theorem, the aposteriori conditional probability distribution of the variable Y in dependence on the number of question of the test to witch both the correct response were not given, i.e. on the values of the random variable X, was derived. We have

$$P(Y = y \,|\, X = x) = \frac{P(Y = y)\, P(X = x \,|\, Y = y)}{P(X = x)} = \tag{5}$$

$$= \frac{\binom{n}{y} \tau^y (1-\tau)^{n-y} \frac{y!}{x!(y-x)!} \left(\frac{q(q-1)-2}{q(q-1)}\right)^x \left(\frac{2}{q(q-1)}\right)^{y-x}}{\frac{n!}{(n-x)!x!} \left(\tau \frac{q(q-1)-2}{q(q-1)}\right)^x \left(1 - \tau \frac{q(q-1)-2}{q(q-1)}\right)^{n-x}} =$$

$$= \binom{n-x}{y-x} \left(\frac{2\tau}{q(q-1) - \tau(q(q-1)-2)}\right)^{y-x} \left(\frac{(1-\tau)q(q-1)}{q(q-1) - \tau(q(q-1)-2)}\right)^{n-y} =$$

$$= \binom{n-x}{r} \left(\frac{2\tau}{q(q-1) - \tau(q(q-1)-2)}\right)^{r} \left(\frac{(1-\tau)q(q-1)}{q(q-1) - \tau(q(q-1)-2)}\right)^{n-x-r}$$

with $y = x, x+1, \dots, n$ or with $r = 0, 1, \dots, n-x$, respectively, where r is a value of random variable R representing the number of the questions of the test with which the examinee was really unfamiliar and to which at the same time the both correct responses were given only by the random choice.

2 Information analysis in the case of a single question

Now, let us consider the situation N_i when the tested person is really unfamiliar with the topic of the ith question of the double choice school-achievement test. If the number of the present alternatives is q and if two of them are correct, the uncertainty of the global examinee's answer A_i to this question is expected to be

$$H(A_i|N_i) = -\sum_{j=1}^{q(q-1)/2} \frac{2}{q(q-1)} \log_2 \frac{2}{q(q-1)} = \log_2 \frac{q(q-1)}{2}.$$

This uncertainty corresponds to the uniform probability distribution in the set of the different couples of the offered alternatives.

In the theoretical information analysis the random variable taken into account must be considered carefully. If the answer to the ith question is taken only as a variable X_i alternating betveen two possible values 1 or 0 corresponding to the two different levels

— correct answer (both the correct alternatives were given)

— incorrect answer (both the correct responses were not given)

respectively, the uncertainty of the answer and at the same time the entropy $H(X_i|N_i)$ is expressed as

$$H(X_i|N_i) = -\left(\frac{2}{q(q-1)} \log_2 \frac{2}{q(q-1)} + \frac{q(q-1)-2}{q(q-1)} \log_2 \frac{q(q-1)-2}{q(q-1)}\right). \quad (6)$$

Let us consider now the random variable Y_i, alternating again between two values 1 or 0, corresponding to the situations

— the examinee is familiar with the topic of ith question

— the examinee is unfamiliar with the topic of ith question

respectively. The entropy of the random variable Y_i is given by the expression

$$H(Y_i) = -(\tau \log_2 \tau + (1-\tau) \log_2 (1-\tau)) \quad (7)$$

where the parameter τ expresses the proportion of the topic of the global test with which the examinee is unfamiliar.

From the relations (1), mentioned in the introduction of this text, we have the following expressions of the conditional entropy

$$
\begin{aligned}
&H\left(Y_i|(S_{i0}\cup S_{i1})\right) = H\left(Y_i|(X_i=0)\right) = 0,\\
&H\left(Y_i|S_{i2}\right) = H\left(Y_i|(X_i=1)\right) = \qquad\qquad (8)\\
&\quad = -\left(\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}\log_2\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}+\right.\\
&\quad\left. + \frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\log_2\frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\right)
\end{aligned}
$$

which means that in case of examinee's incorrect answer his or her unfamiliarity with the matter of ith question can be taken as certain, but the case of correct examinee's answer does not give us an opportunity for such a similar unambiguous prediction. Here is included the possibility to give the correct answer by the random choice, too.

Thus, the examiner's effort to deduce from the response of the examinee his or her real familiarity or unfamiliarity with topic of the ith question can be expressed by the mean entropy

$$
\begin{aligned}
H(Y_i|X_i) = &\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)\cdot\\
&\cdot\left(\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}\log_2\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}+\right.\\
&\left. + \frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\log_2\frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\right)
\end{aligned}
$$

which can be adjusted into the form

$$
\begin{aligned}
H(Y_i|X_i) = &-\left((1-\tau)\log_2(1-\tau)+\tau\log_2\tau\right)+ \qquad\qquad (9)\\
&+\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)\log_2\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)+\\
&+\tau\frac{q(q-1)-2}{q(q-1)}\log_2\tau\frac{q(q-1)-2}{q(q-1)}-\\
&-\tau\left(\frac{2}{q(q-1)}\log_2\frac{2}{q(q-1)}+\frac{q(q-1)-2}{q(q-1)}\log_2\frac{q(q-1)-2}{q(q-1)}\right).
\end{aligned}
$$

The difference between the entropies $H(Y_i)$ and $H(Y_i|X_i)$ can be interpreted as an amount of information about the variable Y_i contained in the known value of the variable X_i. From (7) and (9) we have

$$I(Y_i, X_i) = H(Y_i) - H(Y_i|X_i) = \tag{10}$$

$$= -\left\{\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)\log_2\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right) + \right.$$

$$\left. + \tau\frac{q(q-1)-2}{q(q-1)}\log_2 \tau\frac{q(q-1)-2}{q(q-1)}\right\} +$$

$$+\left\{\tau\left(\frac{2}{q(q-1)}\log_2\frac{2}{q(q-1)} + \frac{q(q-1)-2}{q(q-1)}\log_2\frac{q(q-1)-2}{q(q-1)}\right)\right\}.$$

The relationship (6) gives us the evaluation of the uncertainty of the response X_i of the examinee, who is unfamaliar with the topic of the ith question. This uncertainty equals to the conditional entropy $H(X_i|Y_i = 0)$, on the other hand the conditional entropy $H(X_i|Y_i = 1)$ equals zero, because the examinee, who is familiar with the topic of the question, gives a full correct answer with probability one. So the mean entropy $H(X_i|Y_i)$ can be expressed in the form

$$H(X_i|Y_i) = -\tau\left(\frac{2}{q(q-1)}\log_2\frac{2}{q(q-1)} + \frac{q(q-1)-2}{q(q-1)}\log_2\frac{q(q-1)-2}{q(q-1)}\right) \tag{11}$$

because we can with probability τ expect the examinee to be unfamiliar with the topic of the question. This expression equals to the last part of the formula (10). The first part of formula (10) contains the expression of entropy $H(X_i)$ in the form

$$H(X_i) = -\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)\log_2\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right) +$$

$$+\tau\frac{q(q-1)-2}{q(q-1)}\log_2 \tau\frac{q(q-1)-2}{q(q-1)}. \tag{12}$$

So, in formula (10) the general relationship of the information theory

$$I(X_i, Y_i) = H(Y_i) - H(Y_i|X_i) = H(X_i) - H(X_i|Y_i)$$

is fullfiled.

Let us look now at the dependence of the information (10) on the increasing quantity of the offered alternatives q. It holds

$$\lim_{q\to\infty} I(X_i, Y_i) = -\left(\tau\log_2\tau + (1-\tau)\log_2(1-\tau)\right)$$

as far as the ratio $(q(q-1)-2)/(q(q-1))$ approaches to one for increasing q. In this limit case the entropy $H(Y_i|X_i)$ equals zero. It follows that both variables X_i, Y_i are equivalent in the case that $q = \infty$ and that all the information about Y_i is contained in X_i. Thus we have

$$\lim_{q\to\infty} H(X_i) = H(Y_i).$$

3 Information analysis in the case of a test of n questions

In agreement with the assuptions for variant I of probabilistic model of the school-achivement test with a double choice response we will consider now such a test of n independent questions. So, the information analysis mentioned above can be applied to an ordered n-tuple of questions according to a theoretical information rule. The rule states that the entropy as well as information from n equivalent independent sources is a n-multiple of these quantities from a single source.

If the random variable X represents the number of the questions to which the entirely correct answer was given (i.e. the examinee has chosen both the correct alternatives) and if the random variale Y represents the number of the questions with which the examinee is not really familiar we will be interested in quantity of information on variable Y contained in the knowledge of the value of X.

The entropy of Y can be calculated as the entropy of a binomial variable with probability distribution (4) in the form

$$H(Y) = -\sum_{y=0}^{n}\binom{n}{y}\tau^y(1-\tau)^{n-y}\left(\log_2\binom{n}{y}+y\log_2\tau+(n-y)\log_2(1-\tau)\right)=$$
$$= -n\left(\tau\log_2\tau+(1-\tau)\log_2(1-\tau)\right)-\sum_{y=0}^{n}\log_2\binom{n}{y}\binom{n}{y}\tau^y(1-\tau)^{n-y} \tag{13}$$

The last part of this expression follows from the derivation of the entropy of a binomial random variable mentioned in [5].

From (7) it is obvious that the uncertainty of the binomial variable Y equals to the n-multiple of the uncertainty (7) for a single question minus the expectation of the logarithm of the binomial coefficient. Second member on the right-hand side represents a quantification of the fact that the identity of the questions answered incorrectly is omitted in Y, and only the number of these questions is considered.

Conditional entropy of Y under the condition $X = x$ can be expressed as

$$H(Y|X=x)= \tag{14}$$
$$=-\sum_{y=x}^{n}\binom{n-x}{y-x}\left(\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}\right)^{y-x}\left(\frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\right)^{n-y}\cdot$$
$$\cdot\left[\log_2\binom{n-x}{y-x}+(y-x)\log_2\left(\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}\right)+\right.$$
$$\left.+(n-y)\log_2\left(\frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\right)\right]=$$
$$=-(n-x)\left(\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}\log_2\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}+\right.$$

$$+\frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\log_2\frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\Bigg)-$$

$$-\sum_{y=x}^{n}\log_2\binom{n-x}{y-x}\binom{n-x}{y-x}\left(\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}\right)^{y-x}\left(\frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\right)^{n-y}.$$

Given that the number of questions incorrectly answered by the examinee is known, the mean uncertainty about the number of the questions of the test with which the examinee is really unfamiliar can be calculated as follows

$$H(Y|X)= \tag{15}$$

$$=\sum_{x=0}^{n}H(Y|X=x)\binom{n}{x}\left(\tau\frac{(q(q-1)-2)}{q(q-1)}\right)^{x}\left(1-\tau\frac{(q(q-1)-2)}{q(q-1)}\right)^{n-x}=$$

$$=-n\left(1-\tau\frac{(q(q-1)-2)}{q(q-1)}\right)\left(\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}\log_2\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}+\right.$$

$$\left.+\frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\log_2\frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\right)-$$

$$-\sum_{y=0}^{n}\binom{n}{x}\tau^{y}(1-\tau)^{n-y}\sum_{x=0}^{y}\log_2\binom{n-x}{y-x}\binom{y}{x}\left(\frac{q(q-1)-2}{q(q-1)}\right)^{x}\left(\frac{2}{q(q-1)}\right)^{y-x}=$$

$$=-n\left(\frac{2\tau}{q(q-1)}\log_2\frac{2\tau}{q(q-1)-\tau(q(q-1)-2)}+(1-\tau)\log_2\frac{(1-\tau)q(q-1)}{q(q-1)-\tau(q(q-1)-2)}\right)-$$

$$-\sum_{y=0}^{n}\binom{n}{y}\tau^{y}(1-\tau)^{n-y}\sum_{x=0}^{y}\log_2\binom{n-x}{y-x}\binom{y}{x}\left(\frac{q(q-1)-2}{q(q-1)}\right)^{x}\left(\frac{2}{q(q-1)}\right)^{y-x}=$$

$$=-n\Bigg\{[(1-\tau)\log_2(1-\tau)+\tau\log_2\tau]-$$

$$-\left[\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)\log_2\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)+\left(\tau\frac{q(q-1)-2}{q(q-1)}\right)\log_2\left(\tau\frac{q(q-1)-2}{q(q-1)}\right)\right]+$$

$$+\tau\left(\frac{q(q-1)-2}{q(q-1)}\log_2\frac{q(q-1)-2}{q(q-1)}+\frac{2}{q(q-1)}\log_2\frac{2}{q(q-1)}\right)\Bigg\}-$$

$$-\sum_{y=0}^{n}\binom{n}{y}\tau^{y}(1-\tau)^{n-y}\sum_{x=0}^{y}\log_2\binom{n-x}{y-x}\binom{y}{x}\left(\frac{q(q-1)-2}{q(q-1)}\right)^{x}\left(\frac{2}{q(q-1)}\right)^{y-x}.$$

The difference between the entropies $H(Y)$ and $H(Y|X)$ yields the amount of information

$$I(X,Y) = -n\left[\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)\log_2\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)+\right.$$
$$\left.+\left(\tau\frac{q(q-1)-2}{q(q-1)}\right)\log_2\left(\tau\frac{q(q-1)-2}{q(q-1)}\right)\right]+ \tag{16}$$
$$+n\tau\left[\frac{q(q-1)-2}{q(q-1)}\log_2\frac{q(q-1)-2}{q(q-1)}+\frac{2}{q(q-1)}\log_2\frac{2}{q(q-1)}\right]-$$
$$-\sum_{y=0}^{n}\binom{n}{y}\tau^y(1-\tau)^{n-y}\left[\log_2\binom{n}{y}-\sum_{x=0}^{y}\log_2\binom{n-x}{y-x}\binom{y}{x}\left(\frac{q(q-1)-2}{q(q-1)}\right)^x\left(\frac{2}{q(q-1)}\right)^{y-x}\right].$$

The first two members of (16) are n-multiples of (10).

The variable X is accompanied by the entropy $H(X)$ in the form

$$H(X) = -n\left[\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)\log_2\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)+\right.$$
$$\left.+\left(\tau\frac{q(q-1)-2}{q(q-1)}\right)\log_2\left(\tau\frac{q(q-1)-2}{q(q-1)}\right)\right]+ \tag{17}$$
$$-\sum_{x=0}^{n}\log_2\binom{n}{x}\cdot\binom{n}{x}\left(\tau\frac{q(q-1)-2}{q(q-1)}\right)^x\left(1-\tau\frac{q(q-1)-2}{q(q-1)}\right)^{n-x}$$

derived from the probability distribution (3) of variable X. From the conditional probability distribution $P(X=x|Y=y)$ is derived the conditional entropy of X

$$H(X|Y=y) = -y\left[\frac{q(q-1)-2}{q(q-1)}\log_2\frac{q(q-1)-2}{q(q-1)}+\frac{2}{q(q-1)}\log_2\frac{2}{q(q-1)}\right]-$$
$$-\sum_{x=0}^{y}\log_2\binom{y}{x}\cdot\binom{y}{x}\left(\frac{q(q-1)-2}{q(q-1)}\right)^x\left(\frac{2}{q(q-1)}\right)^{y-x}. \tag{18}$$

From (3), the mean entropy of random variable X can be expressed in the form

$$H(X|Y) = -n\tau\left[\frac{q(q-1)-2}{q(q-1)}\log_2\frac{q(q-1)-2}{q(q-1)}+\frac{2}{q(q-1)}\log_2\frac{2}{q(q-1)}\right]-$$
$$-\sum_{y=0}^{n}\binom{n}{y}\tau^y(1-\tau)^{n-y}\sum_{x=0}^{y}\log_2\binom{y}{x}\cdot\binom{y}{x}\left(\frac{q(q-1)-2}{q(q-1)}\right)^x\left(\frac{2}{q(q-1)}\right)^{y-x}. \tag{19}$$

Then the difference $H(X)-H(X|Y)$ of (17) and (19) is again the amount of information $I(X,Y)$ as can be proved after some adjustments.

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