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Eva Tesaříková

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STATISTICAL ANALYSIS OF PROBABILISTIC MODEL OF THE SCHOOL-ACHIEVEMENT TEST WITH TRIPLE-CHOICE RESPONSE

Eva TESAŘÍKOVÁ

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Abstract

The purpose of the present paper is to estimate the parameters ν, ω and δ of the probabilistic model of school-achievement test with triple choice response described in article [5]. Numerical results of point estimation are summarized.

Key words: Statistical analysis, probalistic model, triple choice response, school-achievement test, point estimation

MS Classification: 62P10, 62P15

1 Introduction

The aim of this paper is to investigate a probabilistic model of school-achievement test with triple-choice response given in [5] where the probabilistic structure of the test was defined under certain simplifying assumptions. The test consists of n independent questions of the same difficulty, q is the number of offered alternatives, $q > 5$.

Set us M_0 denote the random variable expressing the number of questions of the test to which no correct response was given, M_1 the random variable expressing the number of questions to which only one correct response was given, M_2 the random variable expressing the number of questions to which two correct responses were given, and M_3 the random variable expressing the number of question to which all the three correct responses were given by the tested person. Then

$$P(M_0 = m_0, M_1 = m_1, M_2 = m_2, M_3 = m_3) =$$

$$= \frac{n!}{m_0!\, m_1!\, m_2!\, m_3!} \left[\nu \frac{(q-3)(q-4)(q-5)}{q(q-1)(q-2)}\right]^{m_0} \left[\left(\omega + \nu\frac{9}{q}\right) \frac{(q-3)(q-4)}{(q-1)(q-2)}\right]^{m_1} \cdot$$

$$\cdot \left[\left(\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)}\right) \frac{q-3}{q-2}\right]^{m_2} \left[1 - \delta\frac{q-3}{q-2} - \omega\frac{(q-1)(q-2)-2}{(q-1)(q-2)} - \right.$$

$$\left. - \nu\frac{q(q-1)(q-2)-6}{q(q-1)(q-2)}\right]^{m_3} \quad (1)$$

where $m_0 + m_1 + m_2 + m_3 = n$.

The distribution of the random vector $M = (M_0, M_1, M_2, M_3)$ is dependent on parameters ν, ω and δ which represent the proportion of the tested topic with which the examined person is quite unfamiliar or only "one-third-familiar" or "two-third-familiar", respectively.

The purpose of the present paper is to estimate the parameters ν, ω, δ and in this way also the parameter $\tau = \nu + 2\omega/3 + \delta/3$ representing the whole proportion of tested topic with which the examinee is unfamiliar.

2 Point estimation of the parameters

The parameters ν, ω and δ will be estimated at first by the moment method using the theoretical and empirical moments of the variables M_0, M_1, M_2, M_3 on the basis of a single application of the test. Comparing theoretical and empirical mean values and using the formulas (6), (9), (12) of [5] the following equations

$$E(M_0) = n\nu\frac{(q-3)(q-4)(q-5)}{q(q-1)(q-2)} = m_0$$

$$E(M_1) = n\left(\omega + \nu\frac{9}{q}\right)\frac{(q-3)(q-4)}{(q-1)(q-2)} = m_1$$

$$E(M_2) = n\left(\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)}\right)\left(\frac{q-3}{q-2}\right) = m_2$$

are derived.

The first equation provides the estimator of the parameter ν in the form

$$\hat{\nu} = \frac{m_0}{n} \frac{q(q-1)(q-2)}{(q-3)(q-4)(q-5)} . \quad (2)$$

After substitution to the second equation we get the expression

$$m_1 = n\omega \frac{(q-3)(q-4)}{(q-1)(q-2)} + \frac{9}{q-5} m_0$$

from where we obtain the following point estimator of the parameter ω

$$\hat{\omega} = \left(\frac{m_1}{n} - \frac{9m_0}{n(q-5)} \right) \frac{(q-1)(q-2)}{(q-3)(q-4)} . \qquad (3)$$

By setting expressions (2), (3) to the third equation we derive the point estimator of the parameter δ

$$\hat{\delta} = \left(\frac{m_2}{n} - \frac{4m_1}{n(q-4)} + \frac{18m_0}{n(q-4)(q-5)} \right) \frac{q-2}{q-3} . \qquad (4)$$

The same solution can be obtaind also by the maximum likelihood method on the basis of a single application of the test ussing the multinomial distribution (1) of the random vector $M = (M_0, M_1, M_2, M_3)$. Setting

$$L(\nu, \omega, \delta, m) =$$

$$= \frac{n!}{m_0!\, m_1!\, m_2!\, m_3!} \left[\nu \frac{(q-3)(q-4)(q-5)}{q(q-1)(q-2)} \right]^{m_0} \left[\left(\omega + \nu \frac{9}{q} \right) \frac{(q-3)(q-4)}{(q-1)(q-2)} \right]^{m_1} \cdot$$

$$\cdot \left[\left(\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)} \right) \frac{q-3}{q-2} \right]^{m_2} \left[1 - \delta \frac{q-3}{q-2} - \omega \frac{(q-1)(q-2)-2}{(q-1)(q-2)} - \right.$$

$$\left. - \nu \frac{q(q-1)(q-2)-6}{q(q-1)(q-2)} \right]^{m_3}$$

we will solve following system of equations

$$\frac{\partial \ln L(\nu, \omega, \delta, m)}{\partial \nu} = 0, \quad \frac{\partial \ln L(\nu, \omega, \delta, m)}{\partial \omega} = 0, \quad \frac{\partial \ln L(\nu, \omega, \delta, m)}{\partial \delta} = 0. \qquad (5)$$

Expressing the logarithm of the likelihood function

$$\ln L(\nu, \omega, \delta, m) = \ln \frac{n!}{m_0!\, m_1!\, m_2!\, m_3!} + m_0 \ln \left[\nu \frac{(q-3)(q-4)(q-5)}{q(q-1)(q-2)} \right] +$$

$$+ m_1 \ln \left[\left(\omega + \nu \frac{9}{q} \right) \frac{(q-3)(q-4)}{(q-1)(q-2)} \right] + m_2 \ln \left[\left(\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)} \right) \frac{q-3}{q-2} \right] +$$

$$+ m_3 \ln \left[1 - \delta \frac{q-3}{q-2} - \omega \frac{(q-1)(q-2)-2}{(q-1)(q-2)} - \nu \frac{q(q-1)(q-2)-6}{q(q-1)(q-2)} \right]$$

and partial derivatives (5) we get system of equations

$$\frac{m_0}{\nu} + \frac{m_1 \frac{9}{q}}{\omega + \nu\frac{9}{q}} + \frac{m_2 \frac{18}{q(q-1)}}{\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)}} - \\ - \frac{m_3 \frac{q(q-1)(q-2)-6}{q(q-1)(q-2)}}{1 - \delta\frac{q-3}{q-2} - \omega\frac{(q-1)(q-2)-2}{(q-1)(q-2)} - \nu\frac{q(q-1)(q-2)-6}{q(q-1)(q-2)}} = 0$$

$$\frac{m_1}{\omega + \nu\frac{9}{q}} + \frac{m_2 \frac{4}{q-1}}{\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)}} - \frac{m_3 \frac{(q-1)(q-2)-2}{(q-1)(q-2)}}{1 - \delta\frac{q-3}{q-2} - \omega\frac{(q-1)(q-2)-2}{(q-1)(q-2)} - \nu\frac{q(q-1)(q-2)-6}{q(q-1)(q-2)}} = 0$$

$$\frac{m_2}{\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)}} - \frac{m_3 \frac{q-3}{q-2}}{1 - \delta\frac{q-3}{q-2} - \omega\frac{(q-1)(q-2)-2}{(q-1)(q-2)} - \nu\frac{q(q-1)(q-2)-6}{q(q-1)(q-2)}} = 0 \; .$$

After multiplication of the second equation by $(q-3)(q-2)$ and the third equation by $[(q-1)(q-2)-2]/[(q-1)(q-2)]$ we can substract these equations to obtain the equivalent expressions

$$\frac{m_1 \frac{q-3}{q-2}}{\omega + \nu\frac{9}{q}} + \frac{m_2 \frac{4(q-3)}{(q-1)(q-2)}}{\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)}} = \frac{m_2 \frac{(q-1)(q-2)-2}{(q-1)(q-2)}}{\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)}}$$

which using the relation $(q-1)(q-2)-2 = q(q-3)$ leads to the equality

$$\frac{m_1}{\omega + \nu\frac{9}{q}} = \frac{m_2 \frac{q-4}{q-1}}{\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)}} \; ,$$

resp.

$$m_1\left(\delta + \frac{4\omega}{q-1} + \frac{18\nu}{q(q-1)}\right) = m_2\frac{q-4}{q-1}\left(\omega + \nu\frac{9}{q}\right) \; . \tag{6}$$

Setting (2), (3), (4) to (6) we get identity

$$\frac{m_1 m_2}{n} - \frac{4m_1^2}{n(q-4)} + \frac{18m_0m_1}{n(q-4)(q-5)} + \frac{4m_1^2}{n(q-4)} - \frac{36m_0m_1}{n(q-4)(q-5)} + \\ + \frac{18m_0m_1}{n(q-4)(q-5)} = \frac{m_1m_2}{n} - \frac{9m_0m_2}{n(q-5)} + \frac{9m_0m_2}{n(q-5)} \; .$$

So, we can see that the estimators (2),(3),(4) are solutions of the system (5) and that they are the maximum likelihood estimators of the parameters ν, ω and δ.

However, the derived relations (2), (3), (4) are convenient solutions only under the conditions that

$$0 \le \hat{\nu} \le 1, \quad 0 \le \hat{\omega} \le 1, \quad 0 \le \hat{\delta} \le 1, \quad 0 \le \hat{\nu} + \hat{\omega} + \hat{\delta} \le 1, \tag{7}$$

If the expressions (2), (3), (4) do not fulfil the conditions (7), they can't be used. In this case we only have to use the numerical solution giving values of the parameters ν, ω and δ satisfying the conditions (7) and giving the maximum value of likelihood function at the same time.

The point estimator τ of the parameter τ, representing the whole proportion of the tested topic with which the examinee is unfamiliar, is than obtained by the relation $\hat{\tau} = \hat{\delta}/3 + 2\hat{\omega}/3 + \hat{\nu}$.

3 Numerical results

The values of the point estimator of τ for different q, n and m_0, m_1, m_2, m_3 are summarized in tables on pp. 145–155. They were calculated only with precision 0.02.

Point estimator of parameter τ

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	0	0	0.00	0.00	0.00	0.00	0.00
		1	0.04	0.04	0.04	0.04	0.04
		2	0.09	0.08	0.08	0.08	0.08
		3	0.13	0.13	0.12	0.12	0.11
		4	0.18	0.17	0.16	0.16	0.15
		5	0.22	0.21	0.20	0.19	0.19
		6	0.27	0.25	0.24	0.23	0.22
		7	0.31	0.29	0.28	0.27	0.27
		8	0.33	0.33	0.32	0.31	0.30
		9	0.33	0.33	0.33	0.33	0.33
		10	0.33	0.33	0.33	0.33	0.33

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	1	0	0.10	0.09	0.08	0.07	0.07
		1	0.19	0.15	0.14	0.13	0.13
		2	0.22	0.19	0.18	0.17	0.17
		3	0.27	0.24	0.22	0.21	0.20
		4	0.31	0.28	0.26	0.25	0.24
		5	0.36	0.32	0.30	0.29	0.28
		6	0.40	0.36	0.34	0.33	0.32
		7	0.44	0.40	0.38	0.37	0.36
		8	0.44	0.42	0.40	0.40	0.39
		9	0.44	0.42	0.40	0.40	0.39

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	2	0	0.22	0.20	0.17	0.13	0.12
		1	0.32	0.25	0.21	0.21	0.20
		2	0.37	0.28	0.27	0.26	0.25
		3	0.37	0.35	0.32	0.30	0.29
		4	0.44	0.39	0.36	0.34	0.33
		5	0.49	0.43	0.40	0.38	0.37
		6	0.53	0.47	0.44	0.42	0.41
		7	0.56	0.50	0.47	0.46	0.44
		8	0.56	0.50	0.47	0.46	0.45

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	3	0	0.32	0.30	0.26	0.21	0.20
		1	0.42	0.41	0.28	0.28	0.27
		2	0.52	0.36	0.35	0.35	0.34
		3	0.54	0.43	0.42	0.40	0.38
		4	0.55	0.50	0.46	0.44	0.42
		5	0.59	0.54	0.50	0.47	0.46
		6	0.67	0.58	0.54	0.51	0.50
		7	0.67	0.58	0.54	0.52	0.50

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	4	0	0.42	0.40	0.36	0.30	0.28
		1	0.52	0.46	0.38	0.36	0.34
		2	0.64	0.49	0.45	0.43	0.41
		3	0.70	0.57	0.49	0.48	0.47
		4	0.73	0.60	0.56	0.53	0.51
		5	0.76	0.64	0.60	0.57	0.55
		6	0.80	0.67	0.63	0.58	0.56

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	5	0	0.52	0.50	0.47	0.38	0.35
		1	0.64	0.60	0.45	0.42	0.41
		2	0.74	0.65	0.49	0.47	0.46
		3	0.84	0.67	0.56	0.54	0.53
		4	0.91	0.72	0.63	0.61	0.60
		5	1.00	0.82	0.67	0.64	0.62

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	6	0	0.64	0.60	0.52	0.45	0.41
		1	0.74	0.66	0.60	0.56	0.48
		2	0.84	0.72	0.58	0.55	0.54
		3	0.94	0.76	0.65	0.63	0.61
		4	1.00	0.79	0.67	0.65	0.64

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	7	0	0.74	0.72	0.68	0.54	0.48
		1	0.84	0.82	0.70	0.57	0.55
		2	0.94	0.90	0.75	0.63	0.61
		3	1.00	0.97	0.78	0.67	0.65

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	8	0	0.84	0.82	0.78	0.60	0.55
		1	0.94	0.92	0.84	0.65	0.61
		2	1.00	0.98	0.86	0.68	0.66

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	9	0	0.94	0.90	0.86	0.72	0.62
		1	1.00	1.00	0.92	0.78	0.67

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
0	10	0	1.00	1.00	0.96	0.79	0.67

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
1	0	0	0.10	0.10	0.10	0.10	0.10
		1	0.21	0.18	0.17	0.16	0.15
		2	0.25	0.23	0.21	0.20	0.19
		3	0.29	0.27	0.25	0.23	0.22
		4	0.34	0.31	0.29	0.28	0.27
		5	0.38	0.35	0.33	0.31	0.30
		6	0.43	0.39	0.37	0.35	0.34
		7	0.47	0.43	0.41	0.39	0.38
		8	0.48	0.44	0.43	0.42	0.41
		9	0.51	0.45	0.44	0.43	0.42

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
1	1	0	0.22	0.21	0.20	0.20	0.20
		1	0.32	0.30	0.29	0.28	0.26
		2	0.41	0.36	0.33	0.31	0.30
		3	0.45	0.40	0.37	0.35	0.34
		4	0.50	0.44	0.41	0.39	0.38
		5	0.54	0.48	0.45	0.43	0.41
		6	0.59	0.53	0.49	0.47	0.45
		7	0.64	0.55	0.52	0.51	0.49
		8	0.73	0.57	0.54	0.53	0.51

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
1	2	0	0.32	0.31	0.30	0.30	0.30
		1	0.42	0.41	0.40	0.39	0.38
		2	0.52	0.49	0.45	0.43	0.41
		3	0.61	0.54	0.49	0.47	0.45
		4	0.66	0.58	0.53	0.51	0.49
		5	0.70	0.62	0.57	0.55	0.53
		6	0.76	0.67	0.60	0.58	0.56
		7	0.84	0.73	0.62	0.60	0.57

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
1	3	0	0.42	0.41	0.40	0.40	0.40
		1	0.52	0.51	0.50	0.49	0.46
		2	0.64	0.62	0.58	0.54	0.50
		3	0.74	0.68	0.62	0.57	0.54
		4	0.82	0.74	0.68	0.61	0.58
		5	0.86	0.80	0.70	0.65	0.62
		6	1.00	0.86	0.72	0.66	0.63

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
1	4	0	0.52	0.51	0.50	0.50	0.50
		1	0.64	0.63	0.62	0.59	0.54
		2	0.74	0.72	0.67	0.63	0.59
		3	0.84	0.81	0.72	0.67	0.63
		4	0.94	0.87	0.76	0.70	0.67
		5	1.00	0.90	0.78	0.72	0.69

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
1	5	0	0.64	0.63	0.62	0.60	0.60
		1	0.74	0.73	0.71	0.68	0.63
		2	0.84	0.82	0.79	0.71	0.67
		3	0.94	0.92	0.81	0.75	0.72
		4	1.00	0.96	0.85	0.78	0.74

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
1	6	0	0.74	0.72	0.71	0.70	0.70
		1	0.84	0.83	0.82	0.79	0.71
		2	0.94	0.92	0.90	0.80	0.75
		3	1.00	0.98	0.93	0.87	0.79

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
1	7	0	0.84	0.83	0.82	0.80	0.80
		1	0.94	0.92	0.91	0.87	0.81
		2	1.00	1.00	0.98	0.89	0.82

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
1	8	0	0.94	0.92	0.91	0.91	0.89
		1	1.00	1.00	1.00	0.98	0.90

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
1	9	0	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
2	0	0	0.22	0.20	0.20	0.20	0.20
		1	0.32	0.30	0.29	0.28	0.26
		2	0.41	0.36	0.33	0.31	0.30
		3	0.45	0.40	0.37	0.35	0.34
		4	0.50	0.44	0.41	0.39	0.38
		5	0.54	0.48	0.45	0.43	0.41
		6	0.59	0.53	0.49	0.47	0.45
		7	0.60	0.55	0.52	0.50	0.49
		8	0.73	0.57	0.53	0.51	0.50

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
2	1	0	0.32	0.30	0.30	0.30	0.30
		1	0.42	0.41	0.40	0.39	0.37
		2	0.52	0.49	0.45	0.43	0.41
		3	0.61	0.53	0.49	0.47	0.45
		4	0.66	0.58	0.53	0.51	0.49
		5	0.70	0.62	0.57	0.55	0.53
		6	0.75	0.65	0.60	0.58	0.57
		7	0.84	0.73	0.62	0.60	0.59

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
2	2	0	0.42	0.41	0.40	0.40	0.40
		1	0.52	0.51	0.50	0.50	0.49
		2	0.64	0.62	0.58	0.55	0.53
		3	0.74	0.67	0.62	0.59	0.58
		4	0.82	0.71	0.66	0.63	0.60
		5	0.86	0.75	0.70	0.67	0.64
		6	1.00	0.80	0.75	0.73	0.72

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
2	3	0	0.52	0.51	0.50	0.50	0.50
		1	0.64	0.63	0.62	0.61	0.60
		2	0.74	0.72	0.71	0.67	0.65
		3	0.84	0.81	0.75	0.71	0.69
		4	0.94	0.85	0.79	0.75	0.73
		5	1.00	0.92	0.83	0.77	0.75

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
2	4	0	0.64	0.63	0.62	0.61	0.60
		1	0.74	0.72	0.71	0.70	0.70
		2	0.84	0.83	0.82	0.79	0.75
		3	0.94	0.92	0.87	0.83	0.79
		4	1.00	0.99	0.92	0.87	0.81

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
2	5	0	0.74	0.73	0.72	0.71	0.70
		1	0.84	0.83	0.82	0.81	0.80
		2	0.94	0.93	0.92	0.90	0.85
		3	1.00	1.00	0.98	0.92	0.87

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
2	6	0	0.84	0.83	0.82	0.81	0.80
		1	0.94	0.93	0.92	0.91	0.90
		2	1.00	1.00	1.00	0.98	0.93

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
2	7	0	0.94	0.93	0.92	0.91	0.90
		1	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
2	8	0	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
3	0	0	0.32	0.31	0.30	0.30	0.30
		1	0.42	0.41	0.40	0.39	0.37
		2	0.52	0.49	0.45	0.43	0.41
		3	0.61	0.53	0.49	0.47	0.45
		4	0.66	0.58	0.53	0.51	0.49
		5	0.70	0.62	0.57	0.55	0.53
		6	0.75	0.65	0.60	0.58	0.57
		7	0.84	0.73	0.62	0.60	0.59

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
3	1	0	0.42	0.41	0.40	0.40	0.40
		1	0.52	0.51	0.50	0.50	0.49
		2	0.64	0.62	0.58	0.55	0.53
		3	0.74	0.67	0.62	0.59	0.58
		4	0.82	0.71	0.66	0.63	0.60
		5	0.86	0.75	0.70	0.67	0.64
		6	1.00	0.80	0.75	0.73	0.72

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
3	2	0	0.52	0.51	0.50	0.50	0.49
		1	0.64	0.63	0.62	0.61	0.60
		2	0.74	0.72	0.71	0.67	0.65
		3	0.84	0.81	0.75	0.71	0.69
		4	0.94	0.85	0.79	0.75	0.73
		5	1.00	0.92	0.83	0.77	0.75

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
3	3	0	0.64	0.63	0.62	0.61	0.60
		1	0.74	0.73	0.72	0.71	0.70
		2	0.84	0.83	0.82	0.79	0.77
		3	0.94	0.92	0.87	0.83	0.81
		4	1.00	0.98	0.92	0.87	0.83

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
3	4	0	0.74	0.73	0.72	0.71	0.70
		1	0.84	0.82	0.81	0.80	0.79
		2	0.94	0.93	0.92	0.91	0.88
		3	1.00	1.00	1.00	0.96	0.92

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
3	5	0	0.84	0.83	0.82	0.81	0.80
		1	0.94	0.93	0.92	0.91	0.90
		2	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
3	6	0	0.94	0.93	0.92	0.91	0.90
		1	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
3	7	0	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
4	0	0	0.42	0.41	0.40	0.40	0.40
		1	0.52	0.51	0.50	0.50	0.49
		2	0.64	0.62	0.58	0.55	0.53
		3	0.74	0.67	0.62	0.59	0.58
		4	0.82	0.71	0.66	0.63	0.60
		5	0.88	0.76	0.70	0.67	0.64
		6	1.00	0.80	0.75	0.73	0.72

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
4	1	0	0.52	0.51	0.50	0.50	0.49
		1	0.64	0.63	0.62	0.61	0.60
		2	0.74	0.72	0.71	0.67	0.65
		3	0.84	0.81	0.75	0.71	0.69
		4	0.94	0.85	0.79	0.75	0.73
		5	1.00	0.92	0.83	0.77	0.75

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
4	2	0	0.64	0.63	0.62	0.61	0.60
		1	0.74	0.73	0.72	0.71	0.70
		2	0.84	0.83	0.82	0.79	0.77
		3	0.94	0.92	0.87	0.83	0.81
		4	1.00	1.00	0.94	0.87	0.83

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
4	3	0	0.74	0.73	0.72	0.71	0.70
		1	0.84	0.82	0.81	0.80	0.80
		2	0.94	0.93	0.92	0.91	0.88
		3	1.00	1.00	1.00	0.96	0.92

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
4	4	0	0.84	0.83	0.82	0.81	0.80
		1	0.94	0.93	0.92	0.91	0.90
		2	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
4	5	0	0.94	0.93	0.92	0.91	0.90
		1	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
4	6	0	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
5	0	0	0.52	0.51	0.50	0.50	0.50
		1	0.64	0.62	0.61	0.60	0.60
		2	0.74	0.73	0.71	0.67	0.65
		3	0.84	0.81	0.75	0.71	0.69
		4	0.94	0.85	0.79	0.75	0.73
		5	1.00	0.92	0.83	0.77	0.75

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
5	1	0	0.64	0.63	0.62	0.61	0.60
		1	0.74	0.73	0.72	0.71	0.70
		2	0.84	0.82	0.81	0.79	0.77
		3	0.94	0.92	0.87	0.83	0.81
		4	1.00	1.00	0.92	0.87	0.83

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
5	2	0	0.74	0.72	0.71	0.70	0.70
		1	0.84	0.83	0.82	0.81	0.80
		2	0.94	0.93	0.92	0.91	0.88
		3	1.00	1.00	1.00	0.96	0.92

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
5	3	0	0.84	0.83	0.82	0.81	0.80
		1	0.94	0.93	0.92	0.91	0.90
		2	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
5	4	0	0.94	0.93	0.92	0.91	0.90
		1	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
5	5	0	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
6	0	0	0.64	0.63	0.62	0.61	0.60
		1	0.74	0.73	0.72	0.71	0.70
		2	0.84	0.83	0.81	0.79	0.77
		3	0.94	0.92	0.88	0.83	0.81
		4	1.00	1.00	0.92	0.87	0.83

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
6	1	0	0.74	0.73	0.72	0.71	0.70
		1	0.84	0.83	0.82	0.81	0.80
		2	0.94	0.93	0.92	0.90	0.88
		3	1.00	1.00	1.00	0.96	0.92

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
6	2	0	0.84	0.83	0.82	0.81	0.80
		1	0.94	0.93	0.92	0.91	0.90
		2	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
6	3	0	0.94	0.93	0.92	0.91	0.90
		1	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
6	4	0	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
7	0	0	0.74	0.73	0.72	0.71	0.70
		1	0.84	0.83	0.82	0.81	0.80
		2	0.94	0.93	0.92	0.90	0.88
		3	1.00	1.00	1.00	0.96	0.92

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
7	1	0	0.84	0.83	0.82	0.81	0.80
		1	0.94	0.93	0.92	0.91	0.90
		2	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
7	2	0	0.94	0.93	0.92	0.91	0.90
		1	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
7	3	0	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
8	0	0	0.84	0.83	0.82	0.81	0.80
		1	0.94	0.93	0.92	0.91	0.90
		2	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
8	1	0	0.94	0.93	0.92	0.91	0.90
		1	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
8	2	0	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
9	0	0	0.94	0.93	0.92	0.91	0.90
		1	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
9	1	0	1.00	1.00	1.00	1.00	1.00

m_0	m_1	m_2	$q=6$	$q=7$	$q=8$	$q=9$	$q=10$
10	0	0	1.00	1.00	1.00	1.00	1.00

$n = 10$ number of questions of the test

m_0 number of questions to which no correct answer was given

m_1 number of questions to which one correct answer was given

m_2 number of questions to which two correct answers were given

m_3 number of questions to which all three correct answers were given
$m_3 = n - m_0 - m_1 - m_2$

q number of offered alternatives

τ part of real unfamiliarity

References

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Author's address: Department of Algebra and Geometry
Faculty of Science
Palacký University
Tomkova 38, Hejčín
779 00 Olomouc
Czech Republic