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ACTA UNIVERSITATIS PALACKIANAE OLOMUCENSIS FACULTAS RERUM NATURALIUM

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ON A CLASS OF FUNCTIONAL BOUNDARY VALUE PROBLEMS FOR THIRD-ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS WITH PARAMETER

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Abstract. In this paper we study the existence of solutions of a one parameter value problem $y'''(t)-Q[y,y',y''](t).y'(t)=F[y,y',y'',\mu](t)$, $\alpha(y)=0$, $y'(t_1)=y'(t_2)=y'(t_3)=0$ in which $Q:X^3\longrightarrow X$, $F:X^3\times I\longrightarrow X$ are continuous operators, $\alpha:X\longrightarrow R$ is a continuous increasing functional, $\alpha(0)=0$, where $X=C^0(\langle t_1,t_3\rangle)$, $I=\langle a,b\rangle$, $-\infty<t_1<t_2<t_3<\infty$, $-\infty<a<b<\infty$.

Key words: Third-order functional differential equation depending on a paramerer, functional boundary value problem, Schauder linearization technique, Schauder fixed point theorem.

MS Classification: 34K10, 34B15.

1. Introduction

Let $-\infty<t_1<t_3<t_3<\infty$, $-\infty<a<b<\infty$, $J=\langle t_1,t_3\rangle$, $I=\langle a,b\rangle$ and let X be the Banach space of C^0-functions on J with the norm $\|y\|=\max\{|y(t)|;t\in J\}$. Consider the functional differential equation

(1) $y'''(t) - Q[y,y',y''](t).y'(t) = F[y,y',y'',\mu](t)$

in which $Q: X^3 \longrightarrow X$, $F: X^3 \times I \longrightarrow X$ are continuous operators, $Q[y,z,w](t)>0$ on X^3 for all $t \in J$, depending on the parameter μ.

Let $\alpha: X \longrightarrow \mathbf{R}$ be a continuous increasing (i.e. $\alpha(x)<\alpha(y)$ for all $x,y \in X$, $x(t)<y(t)$ on J) functional, $\alpha(0)=0$. The purpose of this paper is to obtain by the Schauder linearization technique sufficient conditions imposed on Q, F such that equation (1) admits, for a suitable value of the paramerer μ, a solution y satisfying the boundary conditions

(2) $\alpha(y)=0,\ y'(t_1)=y'(t_2)=y'(t_3)=0.$

A special case of (1) is the differential equation

$$y'''-q(t,y,y',y'').y'=f(t,y,y',y'',\mu)$$

in which $q \in C^0(J \times \mathbf{R}^3)$, $f \in C^0(J \times \mathbf{R}^3 \times I)$ and $q(t,y,z,w)>0$ for all $(t,y,z,w) \in J \times \mathbf{R}^3$.

Many sufficient conditions are known for the existence and uniqueness of solutions of boundary value problems for the third-order differential equations under various types of boundary conditions using different techniques (see for example [1]-[31], [34]).

The boundary value problem

$$y'''=h(t,y,y',y'')f(t,y,y',y'',y''',\mu),$$

$y(t_1)=y(t_2)=y(t_3)=0$ where μ is a parameter, was investigated in [25] using a suitable version of the Banach fixed point theorem.

2. NOTATION, LEMMAS

Let $\varphi \in C^2(J)$ and let u_φ, v_φ be the solution of the differential equation

$$y''=Q[\varphi,\varphi',\varphi''](t).y,$$

$u_\varphi(t_1)=0$, $u'_\varphi(t_1)=1$, $v_\varphi(t_1)=1$, $v'_\varphi(t_1)=0$. For $(t,s) \in J \times J$ define $r(t,s;\varphi)$ by

$$r(t,s;\varphi)=u_\varphi(t)v_\varphi(s)-u_\varphi(s)v_\varphi(t)\ \ (=-r(s,t;\varphi)).$$

Then $r(t,s;\varphi)>0$ for $t_1 \le s<t \le t_3$ and $r(t,s;\varphi)<0$ for $t_1 \le t<s \le t_3$ (see [32]).

Lemma 1. *Assume* $\varphi\in C^2(J)$, $h\in C^0(J\times I)$, $h(t,.)$ *is* ***increasing*** *on* I *for each fixed* $t\in J$ ***and***

(3) $$h(t,a).h(t,b)\le 0 \quad \text{for all } t\in J.$$

Then there is a unique $\mu_0\in I$ *such that the differential equation*

(4) $$y''=Q[\varphi,\varphi',\varphi''](t).y+h(t,\mu)$$

with $\mu=\mu_0$ *admits a solution* y *satisfying*

(5) $$y(t_1)=y(t_2)=y(t_3)=0.$$

Moreover this solution y *is unique.*

Proof. Setting

$$y(t,\mu)=\frac{r(t_2,t;\varphi)}{r(t_2,t_1;\varphi)}\int_{t_1}^{t_2} r(t_1,s;\varphi)h(s,\mu)ds+\int_{t_2}^{t} r(t,s;\varphi)h(s,\mu)ds$$

for $(t,\mu)\in J\times I$, y is the unique solution of (4) satisfying the boundary conditions $y(t_1,\mu)=0=y(t_2,\mu)$ and since

$$y(t_3,\mu)=\frac{r(t_2,t_3;\varphi)}{r(t_2,t_1;\varphi)}\int_{t_1}^{t_2} r(t_1,s;\varphi)h(s,\mu)ds+\int_{t_2}^{t_3} r(t_3,s;\varphi)h(s,\mu)ds,$$

$y(t_3,.)$ is an increasing function on I, $y(t_3,a).y(t_3,b)\le 0$ (by (3)) and hence $y(t_3,\mu)=0$ only for a unique $\mu=\mu_0\in I$. Consequently, equation (4) admits a solution y satisfying (5) if and only if $\mu=\mu_0$. This solution is necessary unique.

Lemma 2. *Assume the assumptions of Lemma 1 are fulfilled. Then there is a unique* $\mu_0\in I$ *such that the differential equation*

(6) $$y'''=Q[\varphi,\varphi',\varphi''](t).y'+h(t,\mu)$$

with $\mu=\mu_0$ *admits a solution* y *satisfying (2). Moreover this solution* y *is unique.*

Proof. By Lemma 1 there is a unique $\mu_0\in I$ such that equation (4) with $\mu=\mu_0$ admits a (and then unique) solution z satisfying $z(t_1)=z(t_2)=z(t_3)=0$ consequently, equation (6) admits a solution y satisfying $y'(t_1)=y'(t_2)=y'(t_3)=0$ exactly if $\mu=\mu_0$ and then $y(t)=\int_t^t z(t)ds+c$ are all such solutions, where c is an arbitrary constant.

Since $\alpha(\int_{t_0}^{t} z(s)ds+c_1)<0$, $\alpha(\int_{t_0}^{t} z(s)ds+c_2)>0$ for $c_1<-\|\int_{t_0}^{t} z(s)ds\|$, $c_2>\|\int_{t_1}^{t} z(s)ds\|$ and $p(c):=\alpha(\int_{t_1}^{t} z(s)ds+c)$ is a continuous increasing function on R, $p(c_1)<0$, $p(c_2)>0$, the equation $p(c)=0$ has a unique solution c_0. Therefore $y=\int_{t_0}^{t} z(s)ds+c_0$ is a (and then unique) solution of (6) with $\mu=\mu_0$ satisfying (2).

Remark 1. If $\alpha(y)=0$ for some $y\in X$, then $y(\xi)=0$ for a $\xi\in J$. In the opposite case either $y(t)>0$ or $y(t)<0$ for all $t\in J$ and since $\alpha(0)=0$ and α is increasing we have either $\alpha(y)>0$ or $\alpha(y)<0$, respectively.

Next we will assume there are positive constants r_0, r_1, r_2,

(i) $$r_1(t_3-t_1)\le r_0$$

such that the operators Q, F satisfy the following assumptions :

(ii) $|F[y_0,y_1,y_2,\mu](t)|\le r_1.Q[y_0,y_1,y_2](t)$ for all $t\in J$ and $[y_0,y_1,y_2,\mu]\in D\times I$, where $D=\{[y_0,y_1,y_2];\ y_i\in X, \|y_i\|\le r_i$ for $i=0,1,2\}$;

(iii) $F[y_0,y_1,y_2,\mu_1](t)<F[y_0,y_1,y_2,\mu_2](t)$ for all $t\in J$, $[y_0,y_1,y_2]\in D$ and μ_1, $\mu_2\in I$, $\mu_1<\mu_2$;

(iv) $F[y_0,y_1,y_2,a](t).F[y_0,y_1,y_2,b](t)\le 0$ for all $t\in J$ and $[y_0,y_1,y_2]\in D$;

(v) $\min\{(A+r_1B)\tau, 2(r_1(A+r_1B))^{\frac{1}{2}}\}\le r_2$, where
$A=\sup\{\|F[y_0,y_1,y_2,\mu]\|;\ [y_0,y_1,y_2,\mu]\in D\times I\}$,
$B=\sup\{\|Q[y_0,y_1,y_2]\|;\ [y_0,y_1,y_2]\in D\}$, $\tau=\max\{t_2-t_1,t_3-t_2\}$.

Lemma 3. *Assume assumptions* (i)-(v) *are fulfilled for positive constants* r_0, r_1 *and* r_2. *If* $\varphi\in C^2(J)$, $\|\varphi^{(i)}\|\le r_i$ *(i=0,1,2), then there is a unique* $\mu_0\in I$ *such that the differential equation*

(7) $$y'''-Q[\varphi,\varphi',\varphi''](t).y'=F[\varphi,\varphi',\varphi'',\mu](t)$$

with $\mu=\mu_0$ *admits a (and then unique) solution y satisfying* (2) *and moreover*

(8) $$\|y^{(i)}\|\le r_i \quad \textit{for } i=0,1,2.$$

Proof. Setting $h(t,\mu)=F[\varphi,\varphi',\varphi'',\mu](t)$ for $(t,\mu)\in J\times I$, then $h\in C^0(J\times I)$, $h(t,.)$ is increasing on I for every fixed $t\in J$ (by (iii)), $h(t,a).h(t,b)\le 0$ on J (by (iv)) consequenty, by **Lemma 2** there is a unique $\mu_0\in I$ such that equation (7) with $\mu=\mu_0$ admits a (and then unique) solution y satisfying (2).

Let $|y'(t)|\le|y'(\xi)|>r_1$ for all $t\in J$ and a $\xi\in(t_1,t_3)$. If $y'(\xi)>r_1$ $(y'(\xi)<-r_1)$ then by (ii) we have $y'''(\xi)>0$ $(y'''(\xi)<0)$ which contradicts that y' has a local maximum (minimum) at the point $t=\xi$. Thus $\|y'\|\le r_1$. Since $y(\eta)=0$ for a $\eta\in J$ (see Remark 1) and

$$y(t)=\int_\eta^t y'(s)\mathrm{d}s$$

we have $\|y\|\le(t_3-t_1)\|y'\|\le r_0$.

Let $r_2\ge(A+r_1B)\tau$. Since $y'(t_1)=y'(t_2)=y'(t_3)=0$ there are $\xi_1\in(t_1,t_2)$, $\xi_2\in(t_2,t_3)$ such that $y''(\xi_1)=0=y''(\xi_2)$ and form the equalities

$$y''(t)=\int_{\xi_i}^t\Big\{F[\varphi,\varphi',\varphi'',\mu_0](s)+Q[\varphi,\varphi',\varphi''](s).y'(s)\Big\}\mathrm{d}s,\quad t\in J, i=1,2,$$

we obtain

$$|y''(t)|\le(A+r_1B)(t_2-t_1)\quad\text{for } t\in\langle t_1,t_2\rangle$$

and

$$|y''(t)|\le(A+r_1B)(t_3-t_2)\quad\text{for } t\in\langle t_2,t_3\rangle,$$

consequently, $\|y''\|\le r_2$.

Let $r_2\ge 2(r_1(A+r_1B))^{\frac{1}{2}}$. Let $y''(t)\ne0$ on an open interval $J_1\subset J$ with an end-point ξ, $y''(\xi)=0$. Integrating the equality

$$\frac{\mathrm{d}}{\mathrm{d}t}(y''(t))^2=2Q[\varphi,\varphi',\varphi''](t).y'(t)y''(t)+2F[\varphi,\varphi',\varphi'',\mu_0](t).y''(t)$$

from ξ to t $(\in J_1)$ after evident estimates we obtain

$$(y''(t))^2\le 2r_1B\Big|\int_\xi^t|y''(s)|\mathrm{d}s\Big|+2A\Big|\int_\xi^t|y''(s)|\mathrm{d}s\Big|=$$

$$=2(A+r_1B)|y'(t)-y'(\xi)|\le 4r_1(A+r_1B)$$

hence

$$|y''(t)|\le 2(r_1(A+r_1B))^{\frac{1}{2}}\quad\text{for all } t\in J$$

and $\|y''\|\le r_2$. This completes the proof.

3. EXISTENCE THEOREM

Theorem 1. *Assume assumptions* (i)-(v) *are fulfilled for positive constants* r_0, r_1 *and* r_2. *Then there is* $\mu_0 \in I$ *such that equation* (1) *with* $\mu=\mu_0$ *admits a solution* y *satisfying* (2) *and* (8).

Proof. Let Y be the Banach space of C^2-functions on J with the norm $\|y\|_2=\|y\|+\|y'\|+\|y''\|$ and let $K=\{y; y\in Y, \ \|y^{(i)}\|\le r_i$ for $i=0,1,2\}$. K is a closed bounded convex subset of Y. Let $\varphi\in K$. By Lemma 3 there is a unique $\mu_0\in I$ such that equation (7) with $\mu=\mu_0$ admits a (and then unique) solution y satisfying (2) and (8). Setting $T(\varphi)=y$ we obtain an operator $T: K\longrightarrow K$ and to proof of Theorem 1 it is sufficient to show that T has a fixed point.

Let $\{y_n\}\subset K$ be a convergent sequence, $\lim_{n\to\infty} y_n=y$ and let $T(y_n)=z_n$, $T(y)=z$. Then there are a sequence $\{\mu_n\}\subset I$ and a $\mu_0\in I$ such that we have (see the proof of Lemma 2)

$$z_n(t)=\int_{t_1}^{t} p_n(s)ds+c_n \quad \text{for all } t\in J \text{ and } n\in N$$

and

$$z(t)=\int_{t_1}^{t} p(s)ds+c_0 \quad \text{for all } t\in J,$$

where c_0, $c_n\in\mathrm{R}$,

$$p_n(t)=\frac{r(t_2,t;y_n)}{r(t_2,t_1;y_n)}\int_{t_1}^{t_2} r(t_1,s;y_n)F[y_n,y_n',y_n'',\mu_n](s)ds+$$

$$\int_{t_2}^{t} r(t,s;y_n)F[y_n,y_n',y_n'',\mu_n](s)ds,$$

$$p(t)=\frac{r(t_2,t;y)}{r(t_2,t_1;y)}\int_{t_1}^{t_2} r(t_1,s;y)F[y,y',y'',\mu_0](s)ds+$$

$$\int_{t_2}^{t} r(t,s;y)F[y,y',y'',\mu_0](s)ds$$

and

$$\alpha\Big(\int_{t_1}^{t} p_n(s)ds+c_n\Big)=0, \qquad \alpha\Big(\int_{t_1}^{t} p(s)ds+c_0\Big)=0,$$

$$p_n(t_1)=p_n(t_2)=p_n(t_3)=0, \quad p(t_1)=p(t_2)=p(t_3)=0.$$

If $\{\mu_n\}$ is not a convergent **sequence** there are **convergent** subsequences

$$\left\{\mu_{k_n}\right\}, \quad \left\{\mu_{r_n}\right\}, \quad \lim_{n\to\infty} \mu_{k_n}=\lambda_1, \quad \lim_{n\to\infty} \mu_{r_n}=\lambda_2, \quad \lambda_1<\lambda_2,$$

and then

$$\lim_{n\to\infty} p_{k_n}(t)=\frac{r(t_2,t;y)}{r(t_2,t_1;y)}\int_{t_1}^{t_2} r(t_1,s;y)F[y,y',y'',\lambda_1](s)ds+$$
$$\int_{t_2}^{t} r(t,s;y)F[y,y',y'',\lambda_1](s)ds,$$

$$\lim_{n\to\infty} p_{r_n}(t)=\frac{r(t_2,t;y)}{r(t_2,t_1;y)}\int_{t_1}^{t_2} r(t_1,s;y)F[y,y',y'',\lambda_2](s)ds+$$
$$\int_{t_2}^{t} r(t,s;y)F[y,y',y'',\lambda_2](s)ds$$

uniformly on J. Since $r(t_2,t_3;y)<0$, $r(t_2,t_1;y)>0$, $r(t_1,s;y)<0$ for all $s\in(t_1,t_3\rangle$, $r(t_3,s;y)>0$ for all $s\in\langle t_1,t_3)$ and (by (iii))

$$F[y,y',y'',\lambda_1](t)<F[y,y',y'',\lambda_2](t) \quad \text{on } J,$$

we have $\lim_{n\to\infty} p_{k_n}(t_3)<\lim_{n\to\infty} p_{r_n}(t_3)$ which contradicts $p_n(t_3)=0$ for all $n\in N$. Hence $\{\mu_n\}$ is convergent and let $\lim_{n\to\infty} \mu_n=\mu^*$.

Since $\left\{\int_{t_1}^{t} p_n(s)ds\right\}$ is uniformly bounded on J, α is an increasing continuous functional and $\alpha(\int_{t}^{t} p_n(s)ds+c_n)=0$ for all $n\in N$, we see $\{c_n\}$ is a bounded **sequence**. If $\{c_n\}$ is not a convergent sequence there are convergent subsequences

$$\left\{c_{k_n}\right\}, \quad \left\{c_{r_n}\right\}, \quad \lim_{n\to\infty} c_{k_n}=d_1, \quad \lim_{n\to\infty} c_{r_n}=d_2, \quad d_1<d_2,$$

and

$$\lim_{n\to\infty} z_{k_n}(t)=\int_{t_1}^{t}\bar{p}(s)ds+d_1, \quad \lim_{n\to\infty} z_{r_n}(t)=\int_{t_1}^{t}\bar{p}(s)ds+d_2$$

uniformly on J, where

$$\bar{p}(t)=\frac{r(t_2,t;y)}{r(t_2,t_1;y)}\int_{t_1}^{t} r(t_1,s;y)F[y,y',y'',\mu^*](s)ds+$$

$$\int_{t_2}^{t} r(t,s;y)F[y,y',y'',\mu^*](s)ds \quad \text{for all } t\in J.$$

Next we have

$$0=\lim_{n\to\infty} \alpha(\int_{t_1}^{t} p_{k_n}(s)ds+c_{k_n})=\alpha(\int_{t_1}^{t} \bar{p}(s)ds+d_1),$$

$$0=\lim_{n\to\infty} \alpha(\int_{t_1}^{t} p_{r_n}(s)ds+c_{r_n})=\alpha(\int_{t_1}^{t} \bar{p}(s)ds+d_2),$$

consequently, $d_1=d_2$ which contradicts $d_1<d_2$. Therefore $\{c_n\}$ is convergent, $\lim_{n\to\infty} c_n=c^*$ and then

$$(z^*(t):=)\ \lim_{n\to\infty} z_n(t)=\int_{t_1}^{t} \bar{p}(s)ds+c^*$$

uniformly on J and $\alpha(\int_{t_1}^{t} \bar{p}(s)ds+c^*)=0$. Evidently z^* is a solution of the differential equation

$$w'''-Q[y,y',y''](t).w'=F[y,y',y'',\mu^*](t)$$

and $\alpha(z^*)=0$, $z^{*\prime}(t_1)=z^{*\prime}(t_2)=z^{*\prime}(t_3)=0$.

From Lemma 2 it follows $\mu^*=\mu_0$ and $z^*=z$. Due to the fact that

$$\lim_{n\to\infty} z_n^{(i)}(t)=z^{(i)}(t)$$

uniformly on J for $i=0,1,2$, $\lim_{n\to\infty} T(y_n)=T(y)$ and T is a continuous operator.

Let $\varphi\in K$ and $T(\varphi)=y$. From the equality

$$y'''(t)=Q[\varphi,\varphi',\varphi''](t)y'(t)+F[\varphi,\varphi',\varphi'',\mu_0](t)$$

holding on J for a $\mu_0\in I$, we get $\|y'''\|\le A+r_1B$ $(:=r_3)$ consequently, $T(K)\subset\{y;\ y\in C^3(J),\ \|y^{(i)}\|\le r_i \text{ for } i=0,1,2,3\}$ $(=:\mathcal{L})$. By the Ascoli theorem $\mathcal{L}$ is a compact subset of Y and therefore $T(K)$ is a compact subset of Y too. This proves T is a completely continuous operator and by the Schauder fixed point theorem there is a fixed point of T.

Remark 2. Using the results from the paper [33] we can prove that the boundary conditions $y'(t_1)=y'(t_2)=y'(t_3)=0$ in (2) can be repalced by $y'(t_1)-y'(t_4)=y'(t_2)=y'(t_3)-y'(t_5)=0$, where $t_1<t_4<t_2<t_5<t_3$.

Example 1.

Consider the equation

$$y'''(t)-k(t)\exp\{|y(h_0(t))y'(h_1(t))|\}y'(t)=$$
$$=m(t)\cos(s(t)y''(h_2(t)))+\mu.p(t) \tag{9}$$

in which $k,m,s,p,h_i\in C^0(\langle -1,1\rangle)$, $h_i:\langle -1,1\rangle\longrightarrow\langle -1,1\rangle$, $(i=0,1,2)$, $k_0\le k(t)\le k_1$, $p_0\le p(t)\le p_1$, $|m(t)|\le k_0p_0/(2(p_0+p_1))$ for $t\in\langle -1,1\rangle$, where k_0,k_1,p_0,p_1 are positive constants. The assumptions of Theorem 1 are fulfilled with $J=\langle -1,1\rangle$, $I=\langle -k_0/(2(p_0+p_1))$, $k_0/(2(p_0+p_1))\rangle$, $r_0=1$, $r_1=\frac{1}{2}$, $r_2=(k_0+e^{\frac{1}{2}}k_1)^{\frac{1}{2}}$. Let $t_2\in(-1,1)$ and let α be a continuous increasing functional on the Banach space $C^0(J)$ with the sup norm, $\alpha(0)=0$ (for **example** $\alpha(y)=\int_{-1}^{1}y(s)ds$ or $\alpha(y)=\sum_{k=1}^{\infty}\beta_k y(\tau_k)$, where $\beta_k\in(0,\infty)$, $\tau_k\in J$ for $k=1,2,\dots,n$). By Theorem 1 there is $\mu_0\in I$ such that equation (9) with $\mu=\mu_0$ admits a solution y satisfying

$$\alpha(y)=0,\ y'(-1)=y'(t_2)=y'(1)=0$$

and

$$|y(t)|\le 1,\ |y'(t)|\le\tfrac{1}{2},\ |y''(t)|\le(k_0+e^{\frac{1}{2}}k_1)^{\frac{1}{2}}\ \text{ for all } t\in J.$$

Next let $-1<t_4<t_2<t_5<1$. By **Remark 2** there is $\mu_1\in I$ **such that** equation (9) with $\mu=\mu_1$ admits a solution y_1 satisfying

$$\alpha(y_1)=0,\ y_1'(-1)-y_1'(t_4)=y_1'(t_2)=y_1'(1)-y_1'(t_5)=0$$

and

$$|y_1(t)|\le 1,\ |y_1'(t)|\le\tfrac{1}{2},\ |y_1''(t)|\le(k_0+e^{\frac{1}{2}}k_1)^{\frac{1}{2}}\ \text{ for all } t\in J.$$

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