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SPECIAL AUTOMORPHISMS OF NETS WITH BAER SUBNETS

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I. A *net* is an ordered triple $(\mathscr{P}, \mathscr{L}, (\mathscr{L}_\iota)_{\iota \in I})$ where $\mathscr{P}$ is a set having at least two elements (called *points*), $\mathscr{L}$ is a set of some subsets of $\mathscr{P}$ (called *lines*) and $(\mathscr{L}_\iota)_{\iota \in I}$ is a system of mutually disjoint subsets of $\mathscr{L}$, the union of which is $\mathscr{L}$, I is a set of indices; $\# I \geqq 3$ and the following conditions are satisfied.

(i) $\forall P \in \mathscr{P}, \forall \iota \in I, \exists! a \in \mathscr{L}_\iota, P \in a,$

(ii) $\forall \alpha, \beta \in I; \alpha \neq \beta, \forall a \in \mathscr{L}_\alpha, \forall b \in \mathscr{L}_\beta, \# (a \cap b) = 1.$

From (i) it follows:

$$\forall \iota \in I, \forall a, b \in \mathscr{L}_\iota; \qquad a \neq b, a \cap b = \emptyset.$$

The set $\mathscr{L}_\iota$ is called the ι^{th} *pencil*, its lines the ι-lines. Lines of the same pencil (distinct pencils) are called *parallel* (*non-parallel*). Points A, B are termed *joinable* if there is a line p such that $A \in p$, $B \in p$; if moreover $A \neq B$, then this line is called a *join* of A, B and is written as AB. A point P, for which $P \in a$, $P \in b$ with $a, b \in \mathscr{L}$; $a \neq b$ is called the *point of intersection* and is written as $a \sqcap b$. As customary we say P is "on" p or p "passes through" P if $P \in p$. A line from the α^{th} pencil passing through the point P is written as $\alpha(P)$. The cardinality of the set I is called the *degree of a net*.

Let $\mathscr{N}, \mathscr{N}'$ be nets. By a *homomorphism* of $\mathscr{N}$ into $\mathscr{N}'$ we shall mean a mapping $\varkappa : \mathscr{P} \to \mathscr{P}'$ for which

$$\forall \iota \in I, \forall a \in \mathscr{L}_\iota, \exists a' \in \mathscr{L}'_\iota, a^\varkappa := \{X^\varkappa \mid X \in a\} \subseteqq a'.$$

If $\mathscr{P}^\varkappa = \mathscr{P}'$ we speak of an *epimorphism*, if $\varkappa$ is bijective and if $\varkappa^{-1}$ is also an epimorphism, then we speak of an *isomorphism*. If the given nets satisfy $\mathscr{P} \subseteqq \mathscr{P}'$ and the mapping id_P is a homomorphism $\mathscr{N}$ into $\mathscr{N}'$ then we say that $\mathscr{N}$ is a *subnet* of $\mathscr{N}'$.

The subnet $\mathscr{N} = (\mathscr{P}, \mathscr{L}, (\mathscr{L}_\iota)_{\iota \in I})$ of a net $\mathscr{N}' = (\mathscr{P}', \mathscr{L}', (\mathscr{L}'_\iota)_{\iota \in I})$ is called a *Baer subnet* if there exists for every point $X \in \mathscr{P}' \setminus \mathscr{P}$ at least one line $l \in \mathscr{L}'$ passing through X such that $l \cap \mathscr{P} \neq \emptyset$.

Note. Let $\mathscr{N}$ be a Baer subnet of a net $\mathscr{N}'$, $X \in \mathscr{P}' \setminus \mathscr{P}$. It is easy to werify the fact that X is exactly on one line $l \in \mathscr{L}'$, for which $l \cap \mathscr{P} \neq \emptyset$. Namely, if there exist two such lines $l_1, l_2 \in \mathscr{L}$; $l_1 \neq l_2$, then $X = l_1 \sqcap l_2 \in \mathscr{P}$, is a contradiction to $X \in \mathscr{P}' \setminus \mathscr{P}$. Let $A \in \mathscr{P}' \setminus \mathscr{P}$. The unique line passing through A containing the line from the Baer subnet is written as $A\mathscr{P}$.

Let $\mathscr{N}' = (\mathscr{P}', \mathscr{L}', (\mathscr{L}'_\iota)_{\iota \in I})$ be a net and $\mathscr{N} = (\mathscr{P}, \mathscr{L}, (\mathscr{L}_\iota)_{\iota \in I})$ be its Baer subnet. The ordered tetrad (P, Q, R, S) of points of $\mathscr{P}'$ is called a *parallelogram* if there exist $\alpha, \beta \in I$; $\alpha \neq \beta$ such that $PQ, RS \in \mathscr{L}'_\alpha$ and $QR, PS \in \mathscr{L}'_\beta$. A tripl (A, B, C) is called a *triangle* if A, B, C either is not on the same line or they are not mutually distinct. The tetrad (P, Q, R, S) of points of $\mathscr{P}' \setminus \mathscr{P}$ is termed an $\mathscr{N}$*-trapezium*, if $PQ, RS \in \mathscr{L}'_\alpha$, $QR \in \mathscr{L}'_\beta$, $PS \in \mathscr{L}'_\gamma$, $\alpha, \beta, \gamma \in I$; $\beta \neq \alpha \neq \gamma$ ($\beta = \gamma$ is possible) and moreover $PS = P\mathscr{P}$, $QR = Q\mathscr{P}$. The triangle (A, B, C) is called an $\mathscr{N}$*-triangl* if $A, B, C \in \mathscr{P}' \setminus \mathscr{P}$.

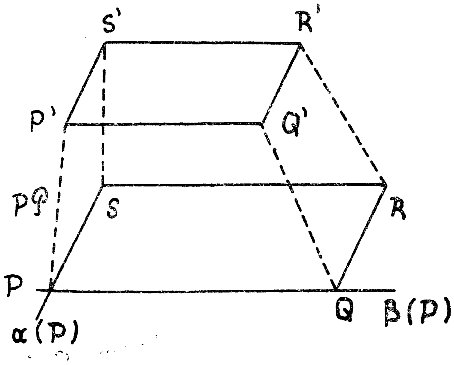

Fig.1.

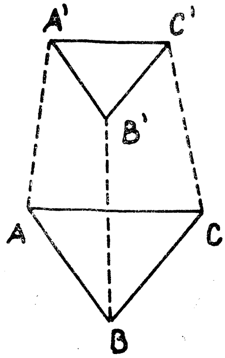

Fig.2.

Let $\mathscr{N}' = (\mathscr{P}', \mathscr{L}', (\mathscr{L}'_\iota)_{\iota \in I})$ be a net and $\mathscr{N} = (\mathscr{P}, \mathscr{L}, (\mathscr{L}_\iota)_{\iota \in I})$ be its Baer subnet, $\alpha, \beta \in I$; $\alpha \neq \beta$. Then Reisemeistr's condition of the type $(\mathscr{N}, \alpha, \beta)$ in $\mathscr{N}'$ is defined as the following implication: If (P, Q, R, S) is a paralelogram, (Q, R, R', Q') and (P, S, S', P') are $\mathscr{N}$-trapeziums such that $PS \in \mathscr{L}_\alpha$, $PQ \in \mathscr{L}_\beta$, $P' \in P\mathscr{P}$ then (P', Q', R', S') is a paralelogram. (See Fig. 1.)

Let $\mathscr{N}' = (\mathscr{P}', \mathscr{L}', (\mathscr{L}'_\iota)_{\iota \in I})$ be a net of at least fourth order and $\mathscr{N} = (\mathscr{P}, \mathscr{L}, (\mathscr{L}_\iota)_{\iota \in I})$ be its Baer subnet. Then Desargues' condition of type $(\mathscr{N})$ in $\mathscr{N}'$ is defined as the following implication: If (A, B, C), (A', B', C') are $\mathscr{N}$-triangles in $\mathscr{N}'$, (A, B, B', A') and (A, C, C', A') are $\mathscr{N}$-trapeziums, points B, C are joinable and for $B \neq C$ $BC \cap \mathscr{P} = \emptyset$, then (B, C, C', B') is an $\mathscr{N}$-trapezium. (See Fig. 2.)

Proposition 1.

Let $\mathscr{N}' = (\mathscr{P}', \mathscr{L}', (\mathscr{L}'_\iota)_{\iota \in I})$ be a net of at least fourth order and $\mathscr{N} = (\mathscr{P}, \mathscr{L}, (\mathscr{L}_\iota)_{\iota \in I})$ be its Baer subnet. If the net $\mathscr{N}'$ satisfies Desargues' condition of the type $(\mathscr{N})$, then $\mathscr{N}'$ also satisfies Reidemeister's condition of the type $(\mathscr{N}, \xi, \eta)$ for all $\xi, \eta \in I$; $\xi \neq \eta$.

Proof. Let $P \in \mathscr{P}' \setminus \mathscr{P}$; we choose the points $P', Q, S \in \mathscr{P}' \setminus \mathscr{P}$ such that

$PS \in \mathcal{L}'_\xi$, $PQ \in \mathcal{L}'_\eta$, $P' \in P\mathcal{P}$, $\xi, \eta \in I$; $\xi \neq \eta$. Further let $R := \xi(Q) \sqcap \eta(S) \in \mathcal{P}' \setminus \mathcal{P}$ then (P, Q, R, S) is a parallelogram, $Q' := Q\mathcal{P} \sqcap \eta(P') \in \mathcal{P}' \setminus \mathcal{P}$ then (P, Q, Q', P') is $\mathcal{N}$-trapezium, $R' := R\mathcal{P} \sqcap \xi(Q') \in \mathcal{P}' \setminus \mathcal{P}$ then (Q, R, R', Q') is $\mathcal{N}$-trapezium, $S' := S\mathcal{P} \sqcap \xi(P') \in \mathcal{P}' \setminus \mathcal{P}$ then (P, S, S', P') is $\mathcal{N}$-trapezium. The points $P, Q, R, S, P', Q', R', S'$ satisfy the assumptions of Reidemeister's condition of the type $(\mathcal{N}, \xi, \eta)$. Since $P'S' \in \mathcal{L}'_\xi$, $Q'R' \in \mathcal{L}'_\eta$ and $P'Q' \in \mathcal{L}'_\eta$ it suffices to prove: $R'S' \in \mathcal{L}'_\eta$.

We choose $\alpha \in I$; $\alpha \neq \xi, \eta$, $P\mathcal{P} \notin \mathcal{L}'_\alpha$. Such an α exists, for, the net $\mathcal{N}'$ is at least fourth order. Necessarily $\alpha(P) \cap \mathcal{P} = \emptyset$ and $\alpha(P') \cap \mathcal{P} = \emptyset$. Now we consider the points $X = \alpha(P) \sqcap \eta(S) \in \mathcal{P}' \setminus \mathcal{P}$ and $X' = \alpha(P') \sqcap X\mathcal{P} \in \mathcal{P}' \setminus \mathcal{P}$. Then the points P, X, S, P', X', S' satisfy the assumptions of Desargues' condition of the type $(\mathcal{N})$ in $\mathcal{N}'$. Hence $\eta(X') = \eta(S')$. Simultaneously we consider the points $Y = \alpha(P) \sqcap \sqcap \xi(Q) \in \mathcal{P}' \setminus \mathcal{P}$ and $Y' = \alpha(P') \sqcap Y\mathcal{P} \in \mathcal{P}' \setminus \mathcal{P}$. Then the points P, Q, Y, P', Q', Y' satisfy the assumptions of Desargues' condition of the type $(\mathcal{N})$ in $\mathcal{N}'$, too. Hence $\xi(Q') = \xi(Y')$ and thus the points Q', Y', R' are on the same ξ-line. Finally, the points Y, X, R, Y', X', R' satisfy the assumptions of Desargues' condition of the type $(\mathcal{N})$ — as well. Hence $\eta(X') = \eta(R'_\eta = \eta(S')$ and thus $R'S' \in \mathcal{L}'_\eta$. (See Fig. 3.)

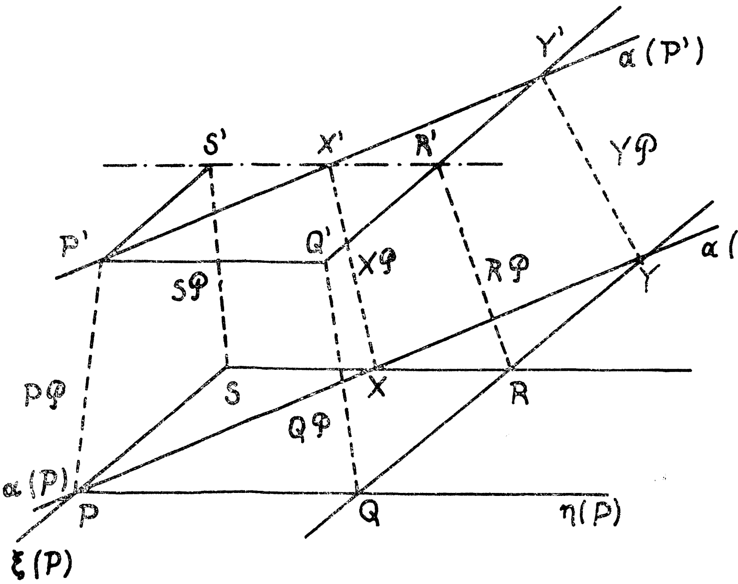

Fig. 3.

Proposition 2.

Let $\mathcal{N}' = (\mathcal{P}', \mathcal{L}', (\mathcal{L}'_\iota)_{\iota \in I})$ be a net of at least fourth order and the Desargues' condition $\mathcal{N} = (\mathcal{P}, \mathcal{L}, (\mathcal{L}_\iota)_{\iota \in I})$. Let (A, B, B', A'), (A, C, C', A') and (B, D, D', B') be $\mathcal{N}$-trapeziums in $\mathcal{N}'$ such that points C, D are joinable and for $C \neq D$, $CD \cap \mathcal{P} = \emptyset$. Then (C, D, D', C') is an $\mathcal{N}$-trapezium.

Proof. Let the points $A, B, C, D, A', B', C', D'$ satisfy the assumptions of the above proposition. If A, B, C, D are on the same line, then (C, D, D', C') is trivially an $\mathcal{N}$-trapezium.

a) Let A, B, C, D not be on the same line and (A, B, D, C) be a parallelogram. Then the points $A, B, C, D, A', B', C', D'$ satisfy the assumptions of Reidemeister's condition of the type $(\mathcal{N}, \xi, \eta)$ for suitable $\xi, \eta \in \mathrm{I}$. By proposition 1. this condition is valid in $\mathcal{N}'$; hence (A', B', D', C') is a parallelogram. Lines $A'B'$, $D'C'$ are from the same pencil and by our assumption the lines AB, DC as well. Thus (C, D, D', C') is an $\mathcal{N}$-trapezium.

b) Let (A, B, D, C) not be a parallelogram. The points C, D are joinable and for $C \neq D$, $CD \cap \mathcal{P} = \emptyset$ (by the assumptions of our proposition). Since (A, B, D, C) is not a parallelogram, there exists at least one of the points $X = AB \sqcap CD$, $Y = AC \sqcap BD$ and it is from $\mathcal{P}' \setminus \mathcal{P}$.

Let us consider the existence of the point X. As the points A, B, C, D are not on the same line, the tripls (A, C, X) and (B, D, X) are $\mathcal{N}$-triangls. Determine $X' := X\mathcal{P} \sqcap \eta(A')$ where for $\eta \in \mathrm{I}$ $A'B' \in \mathcal{L}'_\eta$ holds. Now the points A, C, X, A', C', X', as well as B, D, X, B', D', X' satisfy the assumptions of Desargues' condition of the type $(\mathcal{N})$ in the net $\mathcal{N}'$. Thus (C, X, X', C') and (D, X, X', D') are $\mathcal{N}$-trapeziums, which means that the lines CX and $C'X'$ are parallel and DX, $X'D'$ as well. Since $X \in CD$, then also $X' \in C'D'$ and hence (C, D, D', C') is $\mathcal{N}$-trapezium. (See Fig. 4.)

In analogy with the above proceeds the prove for the point Y.

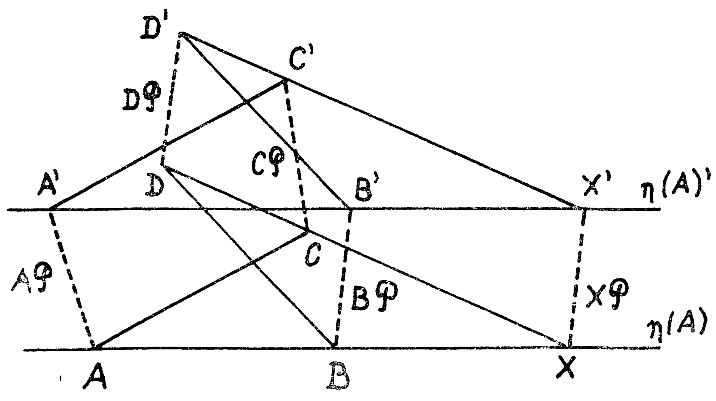

Fig. 4.

II. Let $\varkappa : \mathcal{P} \to \mathcal{P}$ be an automorphism of the net $\mathcal{N}$. Obviously it follows $\{X^\varkappa \mid X \in l\} \in \mathcal{L}$ and $\{X^{\varkappa^{-1}} \mid X \in l\} \in \mathcal{L}$ for all $l \in \mathcal{L}$. Thus $\varkappa$ induces a permutation $\bar{\varkappa}$ of $\mathcal{L}$ with $l^{\bar{\varkappa}} := \{X^\varkappa \mid X \in l\}$.

Let $\mathcal{N}' = (\mathcal{P}', \mathcal{L}', (\mathcal{L}'_\iota)_{\iota \in \mathrm{I}})$ be a net and $\mathcal{N} = (\mathcal{P}, \mathcal{L}, (\mathcal{L}_\iota)_{\iota \in \mathrm{I}})$ be its Baer subnet.

By an $\mathcal{N}$*-automorphism* of a net $\mathcal{N}'$ we mean such an automorphism for which $l^{\varkappa} = l$ for every line $l \in \mathcal{L}$ with $l \cap \mathcal{P} \neq \emptyset$. We say that the net $\mathcal{N}'$ is $\mathcal{N}$*-transitive* if there exists an $\mathcal{N}$-automorphism such that $A^{\varkappa} = A'$ for any two points $A, A' \in \in \mathcal{P}' \setminus \mathcal{P}$; $A' \in A\mathcal{P}$.

Note. It is easy to verify that a net $\mathcal{N}'$ is $\mathcal{N}$-transitive whenever there exists a line $l_0, l_0 \cap \mathcal{P} \neq \emptyset$ such that for any pair of points $A, A' \in l_0$ there exists an $\mathcal{N}$-automorphism with $A \mapsto A'$.

Theorem.

Let $\mathcal{N}' = (\mathcal{P}', \mathcal{L}', (\mathcal{L}'_\iota)_{\iota \in I})$ be a net of at least fourth order and let $\mathcal{N} = (\mathcal{P}, \mathcal{L}, (\mathcal{L}_\iota)_{\iota = I})$ be its Baer subnet. Then the net $\mathcal{N}'$ satisfies Desargues' condition of the type $(\mathcal{N})$ if and only if it is $\mathcal{N}$-transitive.

Proof. 1. Let $\mathcal{N}'$ be $\mathcal{N}$-transitive and the points A, B, C, A', B', C' satisfy the assumptions of Desargues' condition of the type $(\mathcal{N})$ in $\mathcal{N}'$. If A, B, C are not mutually different, obviously (B, C, C', B') is an $\mathcal{N}$-trapezium. Let A, B, C be mutually different, and let us consider an $\mathcal{N}$-automorphism $\varkappa$ with $A^{\varkappa} = A'$. We see that $(AB)^{\bar{\varkappa}} = A'B'$, $(AC)^{\bar{\varkappa}} = A'C'$, $(B\mathcal{P})^{\bar{\varkappa}} = B\mathcal{P}$, $(C\mathcal{P})^{\bar{\varkappa}} = C\mathcal{P}$ so that $C^{\varkappa} = (AC \sqcap C\mathcal{P})^{\varkappa} = (AC)^{\bar{\varkappa}} \sqcap (C\mathcal{P})^{\bar{\varkappa}} = A'C' \sqcap C\mathcal{P} = C'$ and also $B^{\varkappa} = (AB \sqcap B\mathcal{P})^{\varkappa} = A'B' \sqcap B\mathcal{P} = B'$. Hence $(BC)^{\bar{\varkappa}} = B'C'$ and since $\varkappa$ is an automorphism of a net $\mathcal{N}'$, then the lines BC and $B'C'$ are parallel. Therefore (B, C, C', B') is an $\mathcal{N}$-trapezium.

2. Let $\mathcal{N}'$ be a net satisfying Desargues' condition of the type $(\mathcal{N})$ and (A, A')

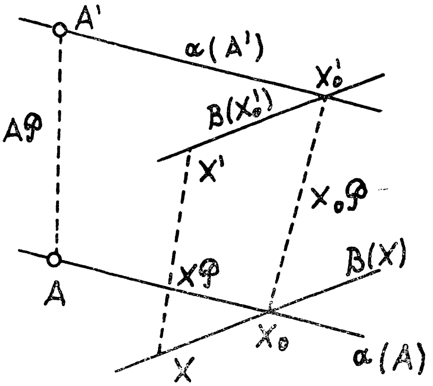

Fig.5.

be a pair of points such that $A, A' \in \mathcal{P}' \setminus \mathcal{P}$ and $A' \in A\mathcal{P}$. Now define a mapping $\varkappa_{AA'} : \mathcal{P}' \to \mathcal{P}'$ as follows:

1. $A^{\varkappa}{}_{AA'} = A'$
2. $\forall\, l \in \mathcal{L}'_\iota,\ l^{\bar{\varkappa}}{}_{AA'} \in \mathcal{L}'_\iota,\ \iota \in I$
3. $\forall\, X \in \mathcal{P},\ X \in l,\ X^{\varkappa}{}_{AA'} \in l$

4. for $X \in \mathscr{P}' \setminus \mathscr{P}$ let $X^{\varkappa}_{AA'} := X'$ is a point for which there exists an intermediating pair of (X_0, X_0'), so that (A, X_0, X_0', A') and (X_0, X, X', X_0') are $\mathscr{N}$-trapeziums.

We show that the point X' is thus determined in a unique way independently of X_0, X_0'. There exists at least such a one pair of points (X_0, X_0'), because we can choose arbitrary indices $\alpha, \beta \in \mathrm{I}$; $\alpha \neq \beta$ with $A\mathscr{P} \notin \mathscr{L}'_\alpha$, $A\mathscr{P} \notin \mathscr{L}'_\beta$. Now put $X_0 := \alpha(A) \sqcap \sqcap \beta(X)$, $X_0' := X_0\mathscr{P} \sqcap \alpha(A')$, then $X' = X\mathscr{P} \sqcap \beta(X_0')$. Furthere the independence of X' on the choice of (X_0, X_0') is guaranteed immediately by proposition 2. It is obvious that $\varkappa_{AA'}$ must be bijective (and thus it is a permutation of $\mathscr{P}'$) and $\{X^{\varkappa}_{AA'} \mid X \in l\} = l$ for every line $l \in \mathscr{L}'$, $l \cap \mathscr{P} \neq \emptyset$.

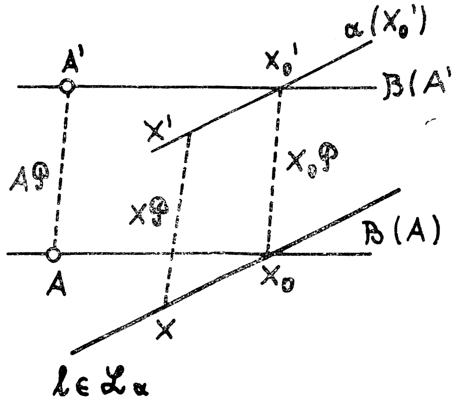

Fig.6.

It remains to prove that the mapping $\varkappa_{AA'}$ is parallelity preserving for all remaining lines, i.e.

$$\forall \iota \in \mathrm{I}, \forall l \in \mathscr{L}'_\iota, \{X^{\varkappa}_{AA'} \mid X \in l\} \in \mathscr{L}'_\iota.$$

Let $l \cap \mathscr{P} = \emptyset$, $l \in \mathscr{L}'_\alpha$. We choose $\beta \in \mathrm{I}$; $\beta \neq \alpha$ such that $A\mathscr{P} \notin \mathscr{L}'_\beta$. Now put $X_0 := := \beta(A) \sqcap l$; $X_0' := \beta(A') \sqcap X_0\mathscr{P}$. Let $X \in l$. Determine $X^{\varkappa}_{AA'}$ by means of the pair of points (X_0, X_0'). Then $X^{\varkappa}_{AA'} \in \alpha(X_0')$ i.e. $\{X^{\varkappa}_{AA'} \mid X \in l, l \in \mathscr{L}'_\alpha\} \in \mathscr{L}'_\alpha$.

Construction. Let $\boldsymbol{T}$ be a field and $\boldsymbol{F}$ be a subfield of $\boldsymbol{T}$ such that the dimension of the vector space $\boldsymbol{T}$ over $\boldsymbol{F}$ is two.

Define:

$\mathscr{P}' := \{(x, y) \mid x, y \in \boldsymbol{T}\}$,
$\mathscr{L}' := \{\{(x, y) \mid x = c, y \in \boldsymbol{T}\} \mid c \in \boldsymbol{T}\} \cup \{\{(x, y) \mid y = mx + c, x \in \boldsymbol{T}\}\ m \in \boldsymbol{F}, c \in \boldsymbol{T}\}$

A system $(\mathscr{L}'_\iota)_{\iota \in \mathrm{I}}$ is defined as follows:

Let $l_1 = \{(x, y) \mid y = m_1x + c_1\}$, $l_2 = \{(x, y) \mid y = m_2x + c_2\}$ then $l_1 \parallel l_2 \Leftrightarrow m_1 = = m_2$.

Clearly, $\mathcal{N}' = (\mathcal{P}', \mathcal{L}', (\mathcal{L}'_\iota)_{\iota \in I})$ is a net. Now we define the sets $\mathcal{P}, \mathcal{L}, (\mathcal{L}_\iota)$ in the same way under the condition $x, y, c \in \mathbf{F}$. Then $(\mathcal{P}, \mathcal{L}, (\mathcal{L}_\iota)_{\iota \in I})$ is a Baer subnet of the net $\mathcal{N}'$.

Proof. Let $(a, b) \subset \mathcal{P}' \setminus \mathcal{P}$. Then either $a \in \mathbf{T} \setminus \mathbf{F}$ or $a \in \mathbf{F}$.

1. Let $a \in \mathbf{F}$, then the unique line passing through (a, b) containing a line from $\mathcal{L}$ is the line $\{(x, y) \mid x = a, \ y \in \mathbf{T}\}$. Suppousing $(a, b) \in p$, $p \cap \mathcal{P} \neq \emptyset$, $p \in \mathcal{L}'$, the line p would have the form $p = \{(x, y) \mid y = mx + c, \ m, c \in \mathbf{F}\}$.

Let $\{1, g\}$, $g \in \mathbf{T} \setminus \mathbf{F}$ be a base of $\mathbf{T}$ over $\mathbf{F}$. Then $b = b_1 + b_2 g$, $a = a_1 + a_2 g$ with $a_1, a_2, b_1, b_2 \in \mathbf{F}$. If $a \in \mathbf{F}$ then $a_2 = 0$. Then $b_1 + b_2 g = m(a_1 + a_2 g) + c$ and it follows:

$$b_1 = ma_1 + c$$
$$b_2 = ma_2$$

in a contradiction to $a_2 = 0$, $b_2 \neq 0$.

2. Let $a \in \mathbf{T} \setminus \mathbf{F}$ i.e. $a_2 \neq 0$. Then $m = b_2/a_2$, $c = b_1 - b_2 a_1/a_2$ and $m, c \in \mathbf{F}$ are thus determined uniquely and there exists only one line in $\mathcal{L}$ whose extension contains the point (a, b).

Theorem.

The net $\mathcal{N}'$ from the above construction is $\mathcal{N}$-transitive.

Proof. Let φ be an automorphism of the vector space $\mathbf{T}$ over $\mathbf{F}$ for which $x^\varphi = x$ for any $x \in \mathbf{F}$. We show that a mapping $(x, y) \mapsto (x^\varphi, y^\varphi)$ is an $\mathcal{N}$-automorphism of $\mathcal{N}'$. Let $c \in \mathbf{T}$, then $\{(x, y) \mid x = c\}^\varphi := \{(x^\varphi, y^\varphi) \mid x^\varphi = c^\varphi\} = \{(x, y) \mid x = c^\varphi\}$. Let $m \in \mathbf{F}$, $c \in \mathbf{T}$, then $\{(x, y) \mid y = mx + c\}^\varphi := \{(x^\varphi, y^\varphi) \mid y^\varphi = mx^\varphi + c^\varphi\} = \{(x, y) \mid y = mx + c^\varphi\}$.

The mapping, for which $(x, y) \mapsto (x^\varphi, y^\varphi)$, maps any line to a parallel line, i.e. the pencils are preserved, it preserves the lines for $c \in \mathbf{F}$.

Now, we show $\mathcal{N}'$ being $\mathcal{N}$-transitive: Let (a, b), (a', b') be a pair of points from $\mathcal{P}' \setminus \mathcal{P}$ on the same line l such that $l \cap \mathcal{P} \neq \emptyset$.

Let $a \in \mathbf{T} \setminus \mathbf{F}$, than $l = \{(x, y) \mid y = mx + c\}$, $m, c \in \mathbf{F}$ and $\{1, a\}$ be a base of $\mathbf{T}$ over $\mathbf{F}$ and let $z = z_1 . 1 + z_2 . a$, $z_1, z_2 \in \mathbf{F}$ for any $z \in \mathbf{T}$. Choose φ such that $z^\varphi = z_1 . 1 + z_2 . a'$. Since $b = ma + c$ and $b' = ma' + c$ and $a \to a'$ we obtain

$$b = ma + c \mapsto ma' + c = b'.$$

Let $a \in \mathbf{F}$, then $l = \{(x, y) \mid x = a\}$ and hence $a = a'$. Then $b \in \mathbf{T} \setminus \mathbf{F}$ and choosing φ such that $b^\varphi = b'$ i.e. the base $\{1, b\}$ is mapped into the base $\{1, b'\}$.

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SOUHRN

SPECIELNÍ AUTOMORFIZMY TKÁNÍ S BAEROVOU PODTKÁNÍ

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V článku se zavádí pojem tkáně $\mathcal{N}'$ s Baerovou podtkání $\mathcal{N}$, t. j. takové tkáně, že každým bodem tkáně $\mathcal{N}'$ nepatřícím do $\mathcal{N}$ prochází právě jedna přímka podtkáně $\mathcal{N}$. Zavádí se jisté modifikace Reidemeistrových a Desarguessových konfigurací právě vzhledem k Baerově podtkáni. Vyšetřují se příslušné modifikace Reidemeistrovy a Desarguessovy uzavírací podmínky a studuje se podgrupa grupy všech automorfizmů dané tkáně s Baerovou podtkání a to podgrupa automorfizmů reprodukujících tuto Baerovu podtkáň. Ukazuje se, že souvislosti mezi zmíněnými modifikacemi jsou stejné, jako pro obvyklý případ. V závěru je uveden algebraicky konstruovaný příklad $\mathcal{N}$-transitivní tkáně.

РЕЗЮМЕ

СПЕЦИАЛЬНЫЕ ТИПЫ АВТОМОРФИЗМОВ СЕТЕЙ ИМЕЮЩИХ ПОДСЕТИ БЭРА

ЯРОСЛАВА ЯАХАНОВА

Статья описивает понятие сети $\mathcal{N}'$ имеющей подсеть Бэра $\mathcal{N}$, т. е. сети в которой каждой точкой из $\mathcal{N}'$ невходящей в $\mathcal{N}$ проходит точно одна прямая принадлежащая подсети $\mathcal{N}$. Дальше изучаются специальные типы условий замыкания Рейдемейстра и Дезарга, и подгруппа группы автоморфизмов сети с подсетью Бэра — подгруппа автоморфизмов рестрикция которых на $\mathcal{N}$ является тождеством.

Дальше здесь доказывается что связности между условиями замыкания Рейдемейстра и Дезарга этих специальных типов такие как в обыкновенном случае. В конце описан алгебраически построенный пример $\mathcal{N}$-транзитивной сети.