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ON LIMIT PROPERTIES OF PHASES AND OF CENTRAL DISPERSIONS IN THE OSCILLATORY EQUATION $y'' = q(t)y$ WITH A PERIODIC COEFFICIENT

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1. INTRODUCTION

O. Borůvka in [2–5] and F. Neuman in [9] brought into relation the Floquet theory for the oscillatory equation (q): $y'' = q(t)y$, $q \in C^0(\mathbf{R})$, $q(t + \pi) = q(t)$ for $t \in \mathbf{R}$ $(=(-\infty, \infty))$ with the theory of phases and of central dispersions. They found thereby an expression for characteristic multipliers of (q) through an appropriate phase or through a central dispersion of (q). The present paper refers to the above results. Let α be a phase and let φ_n denote the central dispersion of (q) with the index n, $x \in \mathbf{R}$. There is proved the existence of the limits: $\lim_{t \to \pm\infty} (\alpha(t)/t)$, $\lim_{n \to \pm\infty} (\varphi_n(x)/n)$ and their values are expressed through the values of the characteristic multipliers of (q).

2. BASIC DEFINITIONS, NOTATIONS AND AUXILIARY RESULTS

We will discuss differential equations of the form

$$\text{(q)} \qquad y'' = q(t)y, \qquad q \in C^0(\mathbf{R}), \qquad q(t + \pi) = q(t) \qquad \text{for } t \in \mathbf{R},$$

being oscillatory on $\mathbf{R}$, that is, any nontrivial solution of (q) has an infinite number of zeros on the right and on the left from any point $t_0 \in \mathbf{R}$. The trivial solution of (q) will be excluded from our considerations.

Say that a function $\alpha : \mathbf{R} \to \mathbf{R}$, $\alpha \in C^0(\mathbf{R})$, is a (first) phase of (q) if there exist independent solutions u, v of (q):

$$\operatorname{tg} \alpha(t) = u(t)/v(t) \qquad \text{on } \mathbf{R} - \{t \in \mathbf{R}, v(t) = 0\}. \tag{1}$$

If $\mathfrak{E}$ denotes the set of phases of the equation $y'' = -y$, then for any $\varepsilon \in \mathfrak{E}$ there is $\varepsilon'(t) \neq 0$, $\varepsilon(t + \pi) = \varepsilon(t) + \pi \,.\, \operatorname{sign} \varepsilon'$ and hence

$$t \,.\, \operatorname{sign} \varepsilon' + \varepsilon(0) - \pi < \varepsilon(t) < t \,.\, \operatorname{sign} \varepsilon' + \varepsilon(0) + \pi, \qquad t \in \mathbf{R}. \tag{2}$$

It holds for any phase α of (q): $\alpha \in C^3(\mathbf{R})$, $\alpha'(t) \neq 0$ for $t \in \mathbf{R}$, $\lim_{t \to \nu\infty} \alpha(t) = \nu \,.\, \operatorname{sign} \alpha' \,.\, \infty$ $(\nu = \pm 1)$. If α fits the relation (1), where u, v are independent solutions of (q) and $w := uv' - u'v$ is their Wronskian, then

$$\alpha'(t) = -w/(u^2(t) + v^2(t)), \qquad t \in \mathbf{R}. \tag{3}$$

Let α be a phase of (q), n being an integer. Let α^{-1} denote the inverse function to the function α (on $\mathbf{R}$). Put $\varphi_n(t) := \alpha^{-1}(\alpha(t) + n\pi \,.\, \operatorname{sign} \alpha')$, $t \in \mathbf{R}$. The values of the function φ_n are independent of the choice of the phase α of (q) and the function φ_n is called the central dispersion (of the 1st kind) of (q) with the index n. This function possesses the following properties:

$$\varphi_n \in C^3(\mathbf{R}), \qquad \varphi_n(t) > 0, \qquad \varphi_n(t + \pi) = \varphi_n(t) + \pi \qquad \text{for } t \in \mathbf{R}, \tag{4}$$

and for any $x \in \mathbf{R}$ and any positive integer s there holds

$$(-\infty, x] = \bigcup_{k=0}^{\infty} (\varphi_{-(k+1)s}(x), \varphi_{-ks}(x)], \; [x, \infty) = \bigcup_{k=0}^{\infty} [\varphi_{ks}(x), \varphi_{(k+1)s}(x)). \tag{5}$$

From the definition of the function φ_n and from (4) we get

$$t - x + \varphi_n(x) - \pi < \varphi_n(t) < t - x + \varphi_n(x) + \pi, \qquad t \in \mathbf{R}, \tag{6}$$

where $x \in \mathbf{R}$ and n is an integer.

The above definitions and properties are stated in [1, 5] or they can be deduced very simply.

Let $x \in \mathbf{R}$ and y_1, y_2 be solutions of (q) satisfying the initial conditions $y_1(x) = y_2'(x) = 0$, $y_1'(x) = y_2(x) = 1$. Following the Floquet theory ([5, 7, 8]), we associate with (q) a quadratic algebraic equation

$$\lambda^2 - A\lambda + 1 = 0, \qquad (A := y_1'(x + \pi) + y_2(x + \pi)),$$

whose roots (generally complex) ϱ, ϱ^{-1} $(\varrho \neq 0)$ are called the characteristic multipliers of (q). It holds thereby:

Lemma 1 ([2–5]).

The equation (q) *has real characteristic multipliers exactly if there exists an* $x \in \mathbf{R}$ *and a positive integer n such that*

$$\varphi_n(x) = x + \pi. \tag{7}$$

If (7) holds for an $x \in \mathbf{R}$ and $\varphi_m(x_1) = x_1 + \pi$ holds for an x_1, then it can be proved that necessarily $n = m$. Say that (q) has the cathegory $(1, n)$ (see [5]), where n is a positive integer, if the relation of (7) for an $x \in \mathbf{R}$ is satisfied. In such a case $(-1)^n(\varphi_n'(x))^{1/2}$ and $(-1)^n(\varphi_n'(x))^{-1/2}$ are characteristic multipliers of (q).

Lemma 2. ([3, 5, 9])

Let $0 < a < 1$. *Then* $e^{\pm a\pi i}$ *are characteristic multipliers of* (q) *exactly if there exists a phase* α *of* (q) *and an integer* n *such that*

$$\alpha(t + \pi) = \alpha(t) + (2n + a)\,\pi, \qquad t \in \mathbf{R}. \tag{8}$$

Say that (q) has the cathegory $(2, n)$ (see [5]) with n being an integer exactly if ther exists a number $a \in (0, 1)$ and a phase α of (q) satisfying (8).

3. PRINCIPLE RESULTS

Lemma 3.

Let n *be a positive integer and* $(1, n)$ *be the cathegory of* (q). *Then*

$$\lim_{t \to \pm\infty} \alpha(t)/t = n \,.\, \operatorname{sign} \alpha'$$

for any phase α *of* (q).

Proof. Let $(1, n)$ be the cathegory of (q). By Lemma 1 this is fulfilled exactly if (7) holds for an $x \in \mathbf{R}$, where φ_n is the central dispersion of (q) with the index n. Let α be a phase of (q) and sign $\alpha' = 1$. Let $\{a_k\}$, $\{b_k\}$ be arbitrary sequences of numbers, $\lim_{k\to\infty} a_k = \infty$, $\lim_{k\to\infty} b_k = -\infty$. If we prove $\lim_{k\to\infty} \alpha(a_k)/a_k = \lim_{k\to\infty} \alpha(b_k)/b_k = n$, then necessarily $\lim_{k\to\pm\infty} \alpha(t)/t = n$. We can without any loss of generality assume for any positive integer k that $b_k < x < a_k$. Now we get from (4) and (7) that for $j = 0, 1, 2, \ldots$ $\varphi_{jn}(x) = x + j\pi$ and thus it follows from (6) that

$$t + \varphi_{jn}(x) - \pi - x = t + (j - 1)\,\pi < \varphi_{jn}(t) < t + \varphi_{jn}(x) + \pi - x = \\ = t + (j + 1)\,\pi.$$

Hence

$$t + (j - 1)\,\pi < \varphi_{jn}(t) < t + (j + 1)\,\pi, \qquad t \in \mathbf{R},\ j = 0, 1, 2, \ldots \tag{9}$$

Writing $\varphi_{-jn}(t)$ for t in (9) gives

$$\varphi_{-jn}(t) + (j - 1)\,\pi < t < \varphi_{-jn}(t) + (j + 1)\,\pi,$$

because $\varphi_{jn}(\varphi_{-jn}(t)) \equiv t$. This implies

$$t - (j + 1)\,\pi < \varphi_{-jn}(t) < t - (j - 1)\,\pi, \qquad t \in \mathbf{R}, j = 0, 1, 2, \ldots \tag{10}$$

From (9) and (10) we obtain

$$t + (j - 1)\,\pi < \varphi_{jn}(t) < t + (j + 1)\,\pi, \qquad t \in \mathbf{R}, j = 0, \pm 1, \pm 2, \ldots \tag{11}$$

From (5) and from the properties of the sequences $\{a_k\}$, $\{b_k\}$ it is clear that the members of these sequences a_k, b_k are expressable in the form: $a_k = \varphi_{ni_k}(x_k)$, $b_k = \varphi_{-nj_k}(y_k)$ where $x_k \in [x, \varphi_n(x))$ $(= [x, x + \pi))$, $y_k \in (\varphi_{-n}(x), x]$ $(= (x - \pi, x])$ and $\{i_k\}$, $\{j_k\}$ are the sequences of positive integers, $\lim_{k\to\infty} i_k = \lim_{k\to\infty} j_k = \infty$.

Because of

$$\alpha(a_k) = \alpha\varphi_{ni_k}(x_k) = \alpha(x_k) + ni_k\pi,$$
$$\alpha(b_k) = \alpha\varphi_{-nj_k}(y_k) = \alpha(y_k) - nj_k\pi,$$
$$\alpha(x) \leqq \alpha(x_k) < \alpha(x + \pi), \qquad \alpha(x - \pi) < \alpha(y_k) \leqq \alpha(x),$$

we find that

$$\begin{aligned} &\alpha(x) + ni_k\pi \leqq \alpha(a_k) < \alpha(x + \pi) + ni_k\pi, \\ &\alpha(x - \pi) - nj_k\pi < \alpha(b_k) \leqq \alpha(x) - nj_k\pi. \end{aligned} \tag{12}$$

From (11) we now find the following inequalities

$$x + (i_k - 1)\pi \leqq x_k + (i_k - 1)\pi < \varphi_{ni_k}(x_k) < x_k + (i_k + 1)\pi < x + (i_k + 2)\pi,$$
$$x - (j_k + 2)\pi < y_k - (j_k + 1)\pi < \varphi_{-nj_k}(y_k) < y_k - (j_k - 1)\pi \leqq x - (j_k - 1)\pi.$$

Hence, for sufficiently large k

$$\frac{\alpha(x) + ni_k\pi}{x + (i_k + 2)\pi} < \frac{\alpha(a_k)}{a_k} < \frac{\alpha(x + \pi) + ni_k\pi}{x + (i_k - 1)\pi},$$
$$\frac{\alpha(x) - nj_k\pi}{x - (j_k + 2)\pi} < \frac{\alpha(b_k)}{b_k} < \frac{\alpha(x - \pi) - nj_k\pi}{x - (j_k - 1)\pi},$$

and, according to the Theorem on the limit of three sequences, we get $\lim_{k\to\infty} \alpha(a_k)/a_k =$ $= \lim_{k\to\infty} \alpha(b_k)/b_k = n$.

Let us assume that $\operatorname{sign} \alpha' = -1$ and let us put $\beta(t) := -\alpha(t)$, $t \in \mathbf{R}$. Then $\operatorname{sign} \beta' =$ $= 1$ and since β is a phase of (q), it follows from the first part of the proof of Lemma 3 that $\lim_{k\to\pm\infty} \alpha(t)/t = -\lim_{k\to\pm\infty} \beta(t)/t = -n$ and Lemma 3 is thus proved.

Lemma 4.

Let n be an integer, $0 < a < 1$. Let $(2, n)$ be the cathegory of (q) *and $e^{\pm a\pi i}$ its characteristic multipliers. Then, for any phase α of* (q)

$$\lim_{t\to\pm\infty} \alpha(t)/t = |\,2n + a\,|\operatorname{sign} \alpha'.$$

Proof. Let n be an integer, $a \in (0, 1)$. Then, by Lemma 2, the equation (q) has the cathegory $(2, n)$ and its characteristic multipliers equal to $e^{\pm a\pi i}$ precisely if there exists a phase α of (q) satisfying (8). Let $\sigma := \operatorname{sign} \alpha'$. We prove first that the statement of Lemma 4 holds for this phase. Letting $\{a_k\}$, $\{b_k\}$ be arbitrary sequences, $\lim_{k\to\infty} a_k = \infty$, $\lim_{k\to\infty} b_k = -\infty$, then $a_k = x_k + i_k\pi$, $b_k = y_k - j_k\pi$, where $0 \leqq x_k$, $y_k < \pi$ for any positive integer k and $\{i_k\}$, $\{j_k\}$ are the sequences of the integers, $\lim_{k\to\infty} i_k = \lim_{k\to\infty} j_k = \infty$. Therefore

$$\lim_{k\to\infty} \alpha(a_k)/a_k = \lim_{k\to\infty} \alpha(x_k + i_k\pi)/(x_k + i_k\pi) =$$
$$= \lim_{k\to\infty} (\alpha(x_k) + i_k(2n + a)\pi)/(x_k + i_k\pi) = 2n + a = |\,2n + a\,|\,\sigma,$$

$$\lim_{k\to\infty} \alpha(b_k)/b_k = \lim_{k\to\infty} \alpha(y_k - j_k\pi)/(y_k - j_k\pi) =$$

$$= \lim_{k\to\infty} (\alpha(y_k) - j_k(2n + a)\pi)/(y_k - j_k\pi) = 2n + a = |2n + a|\,\sigma.$$

Let α_1 be an arbitrary phase of (q). Then $\alpha_1 = \varepsilon\alpha$ for a phase $\varepsilon \in \mathfrak{E}$, $\operatorname{sign}\alpha_1 = \sigma\,.\,\operatorname{sign}\varepsilon'$. From (2) we find that

$$\operatorname{sign}\varepsilon' + (\varepsilon(0) - \pi)\,t^{-1} < \varepsilon(t)/t < \operatorname{sign}\varepsilon' + (\varepsilon(0) + \pi)\,t^{-1} \qquad \text{for } t > 0$$

and

$$\operatorname{sign}\varepsilon' + (\varepsilon(0) + \pi)\,t^{-1} < \varepsilon(t)/t < \operatorname{sign}\varepsilon' + (\varepsilon(0) - \pi)\,t^{-1} \qquad \text{for } t < 0.$$

Hence $\lim_{t\pm\to\infty} \varepsilon(t)/t = \operatorname{sign}\varepsilon'$. Herefrom and from the first past of the proof of Lemma 4 follows

$$\lim_{t\to\pm\infty} \alpha_1(t)/t = \lim_{t\to\pm\infty} \varepsilon\alpha(t)/t = \lim_{t\to\pm\infty} \varepsilon\alpha(t)/\alpha(t)\,.\,\lim_{t\to\pm\infty} \alpha(t)/t =$$

$$= |2n + a|\,\sigma\,.\,\operatorname{sign}\alpha' = |2n + a|\operatorname{sign}\alpha_1'.$$

Theorem 1.

Let (q) *be an oscillatory equation,* $0 < a < 1$, *and* n *be an integer. Then*

(i) *the equation* (q) *has the cathegory* $(1, n)$, *where* n *is a positive integer exactly if it holds for any* (*and then for every*) *phase* α *of* (q) *that*

$$\lim_{t\to\pm\infty} \alpha(t)/t = n\,.\,\operatorname{sign}\alpha'; \tag{13}$$

(ii) *the equation* (q) *has the cathegory* $(2, n)$ *and* $e^{\pm a\pi i}$ *are its characteristic multipliers exactly if it holds for any* (*and then for every*) *phase* α *of* (q) *that*

$$\lim_{t\to\pm\infty} \alpha(t)/t = |2n + a|\operatorname{sign}\alpha'; \tag{14}$$

Proof. If the equation (q) has the cathegory $(1, n)$, then the equality (13) follows from Lemma 3.

If the equation (q) has the cathegory $(2, n)$ and $e^{\pm a\pi i}$ are its characteristic multipliers, then (14) follows from Lemma 4.

Let us assume that (13) holds for any phase α of (q), where n is an positive integer. Then (q) has real characteristic multipliers, because in the contrary case $\lim_{t\to\pm\infty} \alpha(t)/t$ would not be equal to an integer (by Lemma 4). Let us assume that $(1, m)$ is a cathegory of (q). Then we get by Lemma 3 that $\lim_{t\to\pm\infty} \alpha(t)/t = m\,.\,\operatorname{sign}\alpha'$, which with respect to (13) holds exactly if $m = n$.

Let (14) hold for any phase α of (q). Then (q) has complex characteristic multipliers, for in the contrary case $\lim_{t\to\pm\infty} \alpha(t)/t$ would be equal to an integer (by Lemma 3). Assume that $(2, m)$ is the cathegory of (q) and $e^{\pm b\pi i}$ where $b \in (0, 1)$ are its characteristic multipliers. Then by Lemma 4 $\lim_{t\to\pm\infty} \alpha(t)/t = |2m + b|\operatorname{sign}\alpha'$ and with respect to (14) we get $|2m + b| = |2n + a|$. The last equality is satisfied for the integer m, n and a, $b \in (0, 1)$ exactly if $m = n$ and $a = b$. This completes the proof of Theorem 1.

Corollary 1.

Let $x_0 \in \mathbf{R}$, u, v being independent solutions of (q), $w := uv' - u'v$. *Then*

(i) *the equation* (q) *has the cathegory* $(1, n)$ *exactly if*

$$\lim_{t \to \pm\infty} t^{-1} \int_{x_0}^{t} \frac{ds}{u^2(s) + v^2(s)} = n\,|w|^{-1},$$

(ii) *the equation* (q) *has the cathegory* $(2, n)$ *and* $e^{\pm a\pi i}$ $(0 < a < 1)$ *are its characteristic multipliers exactly if*

$$\lim_{t \to \pm\infty} t^{-1} \int_{x_0}^{t} \frac{ds}{u^2(s) + v^2(s)} = |w^{-1}(2n + a)|.$$

Proof. Let u, v be independent solutions of (q), $w := uv' - u'v$ be their Wronskian and α be a phase of (q) satisfying (1) and therefore also (3). Then $\operatorname{sign} \alpha' = -\operatorname{sign} w$ and after integration of (3) from x_0 to t we get

$$\alpha(t) - \alpha(x_0) = -w \int_{x_0}^{t} \frac{ds}{u^2(s) + v^2(s)}.$$

Since

$$\lim_{t \to \pm\infty} \alpha(t)/t = -w \,.\, \lim_{t \to \pm\infty} t^{-1} \int_{x_3}^{t} \frac{ds}{u^2(s) + v^2(s)}$$

we find that

$$\operatorname{sign} \alpha' . \lim_{t \to \pm\infty} \alpha(t)/t = |w| \lim_{t \to \pm\infty} t^{-1} \int_{x_0}^{t} \frac{ds}{u^2(s) + v^2(s)}$$

and Corollary 1 immediately follows from Theorem 1.

Theorem 2.

Let φ_k be the central dispersion of an oscillatory equation (q) *with the index k. Then*

(i) *the equation* (q) *has the cathegory* $(1, n)$ *exactly if for any* (*and then for every*) $x \in \mathbf{R}$

$$\lim_{k \to \pm\infty} \varphi_k(x)/k = \pi n^{-1}$$

(ii) *the equation* (q) *has the cathegory* $(2, n)$ *and* $e^{\pm a\pi i}$ $(0 < a < 1)$ *are its characteristic multipliers exactly if for any* (*and then for every*) $x \in \mathbf{R}$

$$\lim_{k \to \pm\infty} \varphi_k(x)/k = \pi\,|2n + a|^{-1}.$$

Proof. It follows from Theorem 1 that for any phase α of (q) there exist limits $\lim_{t \to \pm\infty} \alpha(t)/t$ being equal to one another and independent on the choice of the phase α of (q).

Let $x \in \mathbf{R}$ and α be a phase of (q). Then $\varphi_k(t) = \alpha^{-1}(\alpha(t) + k\pi \,.\, \text{sign}\, \alpha')$. Hence

$$\lim_{k \to \pm\infty} \varphi_k(x)/k = \lim_{k \to \pm\infty} \alpha^{-1}(\alpha(x) + k\pi \,.\, \text{sign}\, \alpha')/k =$$

$$= \lim_{k \to \pm\infty} \alpha^{-1}(\alpha(x) + k\pi \,.\, \text{sign}\, \alpha')/(\alpha(x) + k\pi \,.\, \text{sign}\, \alpha') \,.\, \lim_{k \to \pm\infty} (\alpha(x) + k\pi \,.\, \text{sign}\, \alpha')/k =$$

$$= \pi \,.\, \text{sign}\, \alpha' \,.\, \lim_{t \to \pm\infty} \alpha^{-1}(t)/t = \pi \,.\, \text{sign}\, \alpha' \,.\, \lim_{t \to \pm\infty} t/\alpha(t)$$

whence we obtain

$$\lim_{k \to \pm\infty} \varphi_k(x)/k = \pi \,.\, \text{sign}\, \alpha' \,.\, \lim_{t \to \pm\infty} t/\alpha(t). \tag{15}$$

This proves that there exist proper limits $\lim_{k \to \pm\infty} \varphi_k(x)/k$ for any $x \in \mathbf{R}$ and it follows from (15) that their value is independent on the choice of x. Theorem 2 follows immediately form Theorem 1 and (15).

Remark 1.

The existence of the proper limit $\lim_{k \to \infty} \varphi_k(x)/k$ has been proved in [6].

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SOUHRN

O LIMITNÍCH VLASTNOSTECH FÁZÍ A CENTRÁLNÍCH DISPERSÍ OSCILATORICKÉ ROVNICE $y'' = q(t)\,y$ S PERIODICKÝM KOEFICIENTEM

SVATOSLAV STANĚK

V práci je vyšetřována diferenciální rovnice

$$\text{(q)} \qquad y'' = q(t)\,y, \qquad q \in C^0(\mathbf{R}), \qquad q(t + \pi) = q(t) \qquad \text{pro } t \in \mathbf{R}(=(-\infty, \infty)),$$

která je oscilatorická na **R**. Nechť α je fáze a φ_n je centrální disperse rovnice (q) s indexem n, $x \in \mathbf{R}$. Je dokázána existence limit $\lim_{t \to \pm\infty} \alpha(t)/t$, $\lim_{n \to \pm\infty} \varphi_n(x)/n$ a jejich hodnoty jsou vyjádřeny pomocí hodnot charakteristických kořenů rovnice (q).

РЕЗЮМЕ

ОБ ПРЕДЕЛЬНЫХ СВОЙСТВАХ ФАЗ И ЦЕНТРАЛЬНЫХ ДИСПЕРСИЙ КОЛЕБЛЮЩЕГО УРАВНЕНИЯ $y'' = p(t)y$ С ПЕРИОДИЧЕСКИМ КОЭФФИЦИЕНТОМ

СВАТОСЛАВ СТАНЕК

В работе изучается колеблющиеся на **R** $(= (-\infty, \infty))$ дифференциальные уравнения

$$\text{(q)} \qquad y'' = q(t)\,y,\ q \in C^0(\mathbb{R}),\ q(t + \pi) = q(t),\ t \in \mathbf{R}.$$

Пусть α фаза и φ_n центральная дисперсия уравнения (q) с индексом n, $x \in \mathbf{R}$. Доказано существование $\lim_{t \to \pm\infty} \alpha(t)/t$, $\lim_{n \to \pm\infty} \varphi_n(x)/x$ и значения этих пределов представлены с помощью значений характеристических корней уравнения (q).