Acta Universitatis Palackianae Olomucensis. Facultas Rerum Naturalium. Mathematica-Physica-Chemica

Vlasta Peřinová

Branch points of algebraic integral equations

Acta Universitatis Palackianae Olomucensis. Facultas Rerum Naturalium. Mathematica-Physica-Chemica, Vol. 10 (1969), No. 1, 7--14

Persistent URL: http://dml.cz/dmlcz/119901

1969 — ACTA UNIVERSITATIS PALACKIANAE OLOMUCENSIS. TOM 30
FACULTAS RERUM NATURALIUM

Katedra algebry a geometrie přírodovědecké fakulty.
Vedoucí katedry: Doc. RNDr. Josef Šimek.

BRANCH POINTS OF ALGEBRAIC INTEGRAL EQUATIONS

VLASTA PEŘINOVÁ

(Received on October 30th, 1967)

In this paper we shall solve the problem of the branching of solutions, in the space of continuous functions, for the algebraic integral equation

$$\sum_{j=1}^{n} \sum_{\alpha=0}^{j} \mu^{\alpha} y^{\alpha}(s) L_j \left[y^{\alpha_1} \dots y^{\alpha_\nu}\right] = f(s), \tag{1}$$

where

$$L_j\,[\,] = \sum_{(\alpha_1 + \dots + \alpha_\nu = j - \alpha)} \int_{\mathscr{A}} \dots \int_{\mathscr{A}} L_{\alpha\alpha_1 \dots \alpha_\nu}(st_1 \dots t_\nu) \left[y^{\alpha_1}(t_1) \dots y^{\alpha_\nu}(t_\nu)\right] \mathrm{d}t_1 \dots \mathrm{d}t_\nu\,;$$

μ is a real or complex parameter, $L_{\alpha\alpha_1 \dots \alpha_\nu}(st_1 \dots t_\nu)$ and $f(s)$ are given real or complex functions of real variables $s, t_1, \dots, t_\nu$ which run the same finite one- or more-dimensional region $\mathscr{A}$. This type of integral equations was introduced by W. Schmeidler in [3].

Papers [1, 5] deal with branching of solutions of some nonlinear integral equations with nonlinear functionals. In [2] branching of solutions of the homogeneous algebraic integral equation was studied from a certain point of view; solutions of this equation in a neighbourhood of an eigenfunction $y_0(s)$ corresponding to an eigenvalue μ_0 were sought in the form of series in certain integer or rational powers of $(\mu - \mu_0)$ and convergence of these series and their number were found out in every case. There was not studied the question which powers of $(\mu - \mu_0)$ are admissible and whether examined series exhaust all small solutions in a neighbourhood of the solution $y_0(s)$.

In the following we derive the branch equation for equation (1) and for the couple $(\mu_0, y_0(s))$ which obeys (1). From this equation the form of solutions and their number in a neighbourhood of the solution $y_0(s)$ can be determined.

Let $L_{\alpha\alpha_1 \dots \alpha_\nu}(s, t_1, \dots, t_\nu)$ be continuous in all the variables in the whole definition region, $f(s)$ be continuous in $\mathscr{A}$. Let $y_0(s)$ be a solution of (1) continuous in $\mathscr{A}$ for the value $\mu_0 \neq 0$ of the parameter μ. Introducing the notation

$$\lambda = \mu - \mu_0, \quad v(s) = y(s) - y_0(s) \tag{2}$$

and assuming

$$p(s) \equiv \sum_{j=1}^{n} \sum_{\alpha=1}^{j} \alpha\, \mu_0^{\alpha} y_0^{\alpha-1}(s) L_j \left[y_0^{\alpha_1} \dots y_0^{\alpha_\nu}\right] \neq 0, \tag{3}$$

uation (1) can be written in the form

$$v(s) - \int_{\mathscr{A}} K(s, t)\, v(t) \mathrm{d}t = \frac{-1}{p(s)} P(v; \lambda), \tag{4}$$

where

$$K(s, t) = \frac{-1}{p(s)} \sum_{j=1}^{n} \sum_{\alpha=0}^{j} \mu_0^{\alpha} y_0^{\alpha}(s) \sum_{(\alpha_1 + \ldots + \alpha_\nu = j - \alpha)} \int_{\mathscr{A}} \ldots \int_{\mathscr{A}} \Big[\alpha_1 L_{\alpha\alpha_1 \ldots \alpha_\nu}(st\, t_2 \ldots t_\nu)\, y_0^{\alpha_1 - 1}(t) y_0^{\alpha_2}(t_2) \ldots$$

$$\ldots y_0^{\alpha_\nu}(t_\nu) + \sum_{k=2}^{\nu} \alpha_k L_{\alpha\alpha_1 \ldots \alpha_\nu}(st_k\, t_2 \ldots t_{k-1}\, t\, t_{k+1} \ldots t_\nu)\, y_0^{\alpha_k - 1}(t)\, y_0^{\alpha_1}(t_k) \prod_{\substack{i=2 \\ i \neq k}}^{\nu} y_0^{\alpha_i}(t_i)\Big] \mathrm{d}t_2 \ldots \mathrm{d}t_\nu, \tag{5}$$

$$P(v; \lambda) = \sum_{j=1}^{n} \sum_{\alpha=0}^{j}{}' (\lambda + \mu_0)^{\alpha} (v(s) + y_0(s))^{\alpha} L_j \left[(v + y_0)^{\alpha_1} \ldots (v + y_0)^{\alpha_\nu}\right];$$

the sum $\Sigma\Sigma'$ is taken under the condition that expressions with $\lambda^\circ v^\circ$ and $\lambda^\circ v$ are missing.

Let 1 be p-multiple eigenvalue of the kernel $K(s, t)$, $\varphi_i(s)$ $(i = \overline{1, p})$ be corresponding eigenfunctions and $\psi_i(s)$ $(i = \overline{1, p})$ associated eigenfunctions continuous in $\mathscr{A}$. Introducing the kernel

$$C(s, t) = K(s, t) - \sum_{i=1}^{p} \overline{\psi}_i(s)\, \overline{\varphi}_i(t) \tag{6}$$

instead of $K(s, t)$, we can write (4) in the form

$$v(s) - \int_{\mathscr{A}} C(s, t)\, v(t)\, \mathrm{d}t = \sum_{i=1}^{p} C_i \overline{\psi}_i(s) - \frac{1}{p(s)} P(v; \lambda), \tag{7}$$

where

$$C_i = \int_{\mathscr{A}} \overline{\varphi}_i(t)\, v(t)\, \mathrm{d}t. \tag{8}$$

As 1 is not an eigenvalue of the kernel $C(s, t)$, there exists the continuous resolving kernel $E(s, t; \mu_0)$ of $C(s, t)$. Designating the operator $(1 + \int_{\mathscr{A}} E(s, t; \mu_0) \ldots \mathrm{d}t)$ as $(I + E)$ the solution of equation (7) can be written

$$v(s) = (I + E)\left[\sum_{i=1}^{p} C_i \overline{\psi}_i(s) - \frac{1}{p(s)} P(v; \lambda)\right]. \tag{9}$$

Let us seek the solution of (9) in the form of the series

$$v(s) = \sum_{l_1 + \ldots + l_p + l = 1}^{\infty} C_1^{l_1} \ldots C_p^{l_p} \lambda^l v_{l_1 \ldots l_p l}(s) . \tag{10}$$

Substituting (10) into (9) and equating coefficients of $C_1^{l_1} \ldots C_p^{l_p} \lambda^l$ we obtain the following system of equations for the determination of the functions $v_{l_1 \ldots l_p l}(s)$

$$v_{0 \ldots \underset{m}{010} \ldots 0}(s) = (I + E)\, \overline{\varphi}_m(s),\ m = \overline{1, p},$$

$$v_{0 \ldots 01}(s) = -\frac{1}{\mu_0}(I + E)\, y_0(s),$$

$$v_{0 \ldots \underset{m}{020} \ldots 0}(s) = (I+E)\Bigg(-\frac{1}{p(s)} \sum_{j=1}^{n} \sum_{\alpha=0}^{j} \mu_0^\alpha \Big\{\alpha y_0^{\alpha-1}(s) v_{0 \ldots \underset{m}{010} \ldots 0}(s) R[v_{0 \ldots \underset{m}{010} \ldots 0}] +$$

$$+ \binom{\alpha}{2} y_0^{\alpha-2}(s)\, v^2_{0 \ldots \underset{m}{010} \ldots 0}(s)\, Q\,[y_0] + y_0^\alpha(s)\, \big(T\,[v_{0 \ldots \underset{m}{010} \ldots 0},\ v_{0 \ldots \underset{m}{010} \ldots 0}] +$$

$$+ S\,\big[v^2_{0 \ldots \underset{m}{010} \ldots 0}\big]\big)\Big\}\Bigg),\ m = \overline{1, p},$$

$$v_{0 \ldots \underset{m}{010} \ldots \underset{m_1}{010} \ldots 0}(s) = (I + E)\,\Bigg(-\frac{1}{p(s)} \sum_{j=1}^{n} \sum_{\alpha=0}^{j} \mu_0^\alpha \Big\{\alpha\, y_0^{\alpha-1}(s) \,.$$

$$.\, \big(v_{0 \ldots \underset{m}{010} \ldots 0}(s)\, R\,[v_{0 \ldots \underset{m_1}{010} \ldots 0}] + v_{0 \ldots \underset{m_1}{010} \ldots 0}(s)\, R\,[v_{0 \ldots \underset{m}{010} \ldots 0}]\big) +$$

$$+ 2\binom{\alpha}{2} y_0^{\alpha-2}(s)\, v_{0 \ldots \underset{m}{010} \ldots 0}(s)\, v_{0 \ldots \underset{m_1}{010} \ldots 0}(s)\, Q\,[y_0] + \tag{11}$$

$$+ y_0^\alpha(s)\, \big(2\,S\,[v_{0 \ldots \underset{m}{010} \ldots 0}\, v_{0 \ldots \underset{m_1}{010} \ldots 0}] + T\,[v_{0 \ldots \underset{m}{010} \ldots 0},\ v_{0 \ldots \underset{m_1}{010} \ldots 0}] +$$

$$+ T\,[v_{0 \ldots \underset{m_1}{010} \ldots 0},\ v_{0 \ldots \underset{m}{010} \ldots 0}]\big)\Big\}\Bigg),\ m = \overline{1, p-1},\ m_1 = \overline{m+1, p},$$

$$v_{0 \ldots \underset{m}{010} \ldots 01}(s) = (I + E)\,\Bigg(-\frac{1}{p(s)} \sum_{j=1}^{n} \sum_{\alpha=0}^{j} \Big\{\alpha\, \mu_0^\alpha y_0^{\alpha-1}(s) \,.$$

$$.\, \big(v_{0 \ldots 01}(s)\, R\,[v_{0 \ldots \underset{m}{010} \ldots 0}] + v_{0 \ldots \underset{m}{010} \ldots 0}(s)\, R\,[v_{0 \ldots 01}]\big) +$$

$$+ \alpha\, \mu_0^{\alpha-1} y_0^\alpha(s)\, R[v_{0 \ldots \underset{m}{010} \ldots 0}] + \mu_0^\alpha y_0^\alpha(s)\, T_1\,[v_{0 \ldots 01},\ v_{0 \ldots \underset{m}{010} \ldots 0}] +$$

$$+ \Big(\alpha^2 \mu_0^{\alpha-1} y_0^{\alpha-1}(s)\, v_{0 \ldots \underset{m}{010} \ldots 0}(s) + 2\binom{\alpha}{2} \mu_0^\alpha y_0^{\alpha-2}(s)\, v_{0 \ldots \underset{m}{010} \ldots 0}(s)\, v_{0 \ldots 01}(s)\Big) \,.$$

$$.\, Q\,[y_0]\Big\}\Bigg),\ m = \overline{1, p},$$

$$v_{0\cdots 02}(s) = (I+E)\Big(-\frac{1}{p(s)}\sum_{j=1}^{n}\sum_{\alpha=0}^{j}\{\mu_0^\alpha y_0^\alpha(s)\, S\,[v_{0\cdots 01}^2] +$$

$$+\Big(\Big(\binom{\alpha}{2}\mu_0^{\alpha-2} y_0^\alpha(s) + \binom{\alpha}{2}\mu_0^\alpha y_0^{\alpha-2}(s)\, v_{0\cdots 01}^2(s) +$$

$$+\alpha^2 \mu_0^{\alpha-1} y_0^{\alpha-1}(s)\, v_{0\cdots 01}(s)\Big)\, Q\,[y_0] + \alpha\,\big(\mu_0^\alpha y_0^{\alpha-1}(s)\, v_{0\cdots 01}(s) +$$

$$+\mu_0^{\alpha-1} y_0^\alpha(s)\big)\, R\,[v_{0\cdots 01}]\}\Big),$$

and in general

$$v_{l_1\cdots l_p l}(s) = h\,(s;\; y_0,\, v_{10\cdots 0},\ldots,\, v_{0\cdots 01},\ldots,\, v_{l_1-1\, l_2\cdots l_p l},$$
$$\ldots,\, v_{l_1 l_2\ldots l_{p-1} l_p-1\, l},\, v_{l_1 l_2\cdots l_p l-1}),\quad l_1+\cdots+l_p+l \geqq 2,$$

where the notation

$$Q\,[y_0] = L_j\left[y_0^{\alpha_1}\cdots y_0^{\alpha_\nu}\right],$$

$$R\,[u] = L_j\left[\sum_{k=1}^{\nu}\alpha_k y_0^{\alpha_k-1} u \prod_{\substack{i=1\\ i\neq k}}^{\nu} y_0^{\alpha_i}\right],$$

$$T\,[u,z] = L_j\left[\sum_{k=1}^{\nu-1}\sum_{l=k+1}^{\nu}\alpha_k\alpha_l y_0^{\alpha_k-1} u\, y_0^{\alpha_l-1} z \prod_{\substack{i=1\\ i\neq k,l}}^{\nu} y_0^{\alpha_i}\right], \tag{12}$$

$$T_1\,[u,z] = L_j\left[\sum_{k=1}^{\nu}\sum_{\substack{l=1\\ l\neq k}}^{\nu}\alpha_k\alpha_l y_0^{\alpha_k-1} u\, y_0^{\alpha_l-1} z \prod_{\substack{i=1\\ i\neq k,l}}^{\nu} y_0^{\alpha_i}\right],$$

$$S\,[u\, z] = L_j\left[\sum_{k=1}^{\nu}\binom{\alpha_k}{2} y_0^{\alpha_k-2} u\, z \prod_{\substack{i=1\\ i\neq k}}^{\nu} y_0^{\alpha_i}\right],$$

was introduced.

Now we shall prove the convergence of the series (10) for $|C_1|,\ldots,|C_p|$ and $|\lambda|$ sufficiently small. Let us choose such numbers A_i $(i=\overline{1,p})$, B, $B_{j\alpha}$, Y and w that

$$\|\varphi_i(s)\| = A_i,$$
$$\|I+E\| = B,$$
$$\sum_{(\alpha_1+\ldots+\alpha_\nu=j-\alpha)}\int_{\mathscr{A}}\cdots\int_{\mathscr{A}}\left\|\frac{1}{p(s)}L_{\alpha\alpha_1\ldots\alpha_\nu}(st_1\ldots t_\nu)\right\| \mathrm{d}t_1\ldots \mathrm{d}t_\nu \le B_{j\alpha}, \tag{13}$$
$$\|y_0(s)\| = Y,$$
$$\|v(s)\| = w$$

holds, where $\| u(s) \| = \max_{s \in \mathscr{A}} | u(s) |$. Using these relations we obtain from (9) the equation for w

$$F[w, |\lambda|, |C_1|, \ldots, |C_p|] \equiv w - B\Big[\sum_{i=1}^{p} A_i |C_i| + \tag{14}$$

$$+ \sum_{j=1}^{n} \sum_{\alpha=0}^{j} B_{j\alpha} \left((|\lambda| + |\mu_0|)^\alpha (w + Y)^j - |\mu_0|^\alpha Y^j - |\mu_0|^\alpha j\, Y^{j-1} w\right)\Big] = 0.$$

Seek the solution of (14) in the form

$$w = \sum_{l_1 + \ldots + l_p + l = 1}^{\infty} |C_1|^{l_1} \ldots |C_p|^{l_p} |\lambda|^l w_{l_1 \ldots l_p l} . \tag{15}$$

For the determination of $w_{l_1 \ldots l_p l}$ we obtain the system of equations

$$w_{0 \cdots \underset{m}{010} \cdots 0} = B A_m, \qquad m = \overline{1, p},$$

$$w_{0 \cdots 01} = B \sum_{j=1}^{n} \sum_{\alpha=0}^{j} \alpha |\mu_0|^{\alpha - 1} Y^j B_{j\alpha}, \tag{16}$$

$$w_{0 \cdots \underset{m}{020} \cdots 0} = B \sum_{j=1}^{n} \sum_{\alpha=0}^{j} |\mu_0|^\alpha \binom{j}{2} Y^{j-2} w^2_{0 \ldots \underset{m}{010} \ldots 0} B_{j\alpha}, m = \overline{1, p}$$

and generally

$$w_{l_1 \cdots l_p l} = H(Y, w_{10 \cdots 0}, \ldots, w_{0 \cdots 01}, \ldots, w_{l_1 - 1 l_2 \ldots l_p l},$$
$$\ldots, w_{l_1 \ldots l_{p-1} l_p - 1 l}, w_{l_1 \ldots l_p l - 1}), \; l_1 + \ldots + l_p + l \geqq 2.$$

Comparing these relations with the ones obtained from (11) and (12) with the use of (13) we get

$$\| v_{l_1 \ldots l_p l}(s) \| \leq w_{l_1 \ldots l_p l}, \; l_1 + \ldots + l_p + l \geqq 1. \tag{17}$$

From this it follows that the region of convergence of the series (15) is the one of the series (10). Let us study the implicit function $w(|\lambda|, |C_1|, \ldots, |C_p|)$ obeying equation (14). As

$$\left.\frac{\partial F[w, |\lambda|, |C_1|, \ldots, |C_p|]}{\partial w}\right|_{w = |\lambda| = |C_1| = \ldots = |C_p| = 0} = 1,$$

it is possible to determine w from (14) as unambiguous and continuous function of $|\lambda|, |C_1|, \ldots, |C_p|$. That means that the series (15) has a finite positive radius of convergence. The same is valid for the series (10). Hence, (10) converges absolutely and uniformly according to s in a neighbourhood of the point $\mu = \mu_0$ to a continuous function $v(s)$ which is the unique solution of equation (9).

Substituting (10) in (8) instead of $v(s)$ we have the following system of equations for the determination of the quantities C_i

$$C_i = \sum_{l_1+\dots+l_p+l=1}^{\infty} C_1^{l_1}\dots C_p^{l_p}\,\lambda^l\, D^{(i)}_{l_1\dots l_p l},\; i=\overline{1,p}, \tag{18}$$

where

$$D^{(i)}_{l_1\dots l_p l} = \int_{\mathscr{A}} v_{l_1\dots l_p l}\,(t)\,\overline{\varphi_i}\,(t)\,\mathrm{d}t\,.$$

Prove that it is valid

$$D^{(j)}_{\underset{i}{0\dots010\dots0}} = \int_{\mathscr{A}} \varphi_j\,(t)\,(I+E)\,\overline{\psi_i}\,\mathrm{d}t = \delta_{ji},\; i,j=\overline{1,p}\,. \tag{19}$$

From the equation for the functions $\varphi_i(s)$ $(i=\overline{1,p})$

$$\varphi_i\,(s) - \int_{\mathscr{A}} K(s,t)\,\varphi_i(t)\mathrm{d}t = 0$$

we obtain, introducing the kernel $C(s,t)$ and using the orthogonality of the eigenfunctions, the equation

$$\varphi_i\,(s) - \int_{\mathscr{A}} C(s,t)\,\varphi_i(t)\mathrm{d}t = \overline{\psi_i}(s)\,.$$

Its solution can be written in the form

$$\varphi_i\,(s) = (I+E)\,\overline{\psi_i}$$

and from this, multiplying by the function $\overline{\varphi}_j(s)$ and integrating over s, follows (19). Hence, the system (18) has the form

$$\sum_{l_1+\dots+l_p=2}^{\infty} C_1^{l_1}\dots C_p^{l_p}\,D^{(i)}_{l_1\dots l_p 0} + \sum_{l_1+\dots+l_p=0}^{\infty} C_1^{l_1}\dots C_p^{l_p}\sum_{l=1}^{\infty}\lambda^l\,D^{(i)}_{l_1\dots l_p l} = 0,\; i=\overline{1,p}. \tag{20}$$

As the system (20) does not include C_i $(i=\overline{1,p})$ in the first power, it is not unambiguously solvable. It is called the branch system. Every small solution C_i of this system gives a solution of equation (1) which is defined in a neighbourhood of the point $\mu=\mu_0$ and equals the solution $y_0(s)$ at this point.

In the case $p=0$ the system (20) does not exist and the unique solution of equation (1) in a neighbourhood of the point $\lambda=0$ is

$$y(s) = y_0(s) + \sum_{l=1}^{\infty}\lambda^l\,v_l\,(s)\,.$$

Functions $v_l\,(s)$ can be determined by the method of uncertain coefficients.

For $p=1$ the branch equation has the form

$$\sum_{l=2}^{\infty} C^l D_{lo} + \sum_{l=0}^{\infty} C^l \sum_{m=1}^{\infty}\lambda^m\,D_{lm} = 0\,. \tag{21}$$

Consider some cases of equation (21) in greater detail. If all the coefficients of (21) are equal to zero ($D_{lm} = 0$), then $v(s)$ represents one-parametric set of solutions of (7). As it is possible to choose C as an arbitrary function of λ, there exists the infinite number of solutions in the form of series in arbitrary powers of λ which converge either in a neighbourhood of the point $\lambda = 0$ or only at this point. If all D_{lm} do not equal zero, there exists a finite number of small solutions of equation (7). Every such solution can be expressed in the form of a convergent series in powers of λ and exponents of these powers and the number of solutions can be determined by the method of Newton's diagram [6]. For example, if it is valid

$$D_{30} \neq 0,$$

$$D_{0j} = 0 \quad (j = \overline{1, m-1}),\ D_{0m} \neq 0,$$

$$D_{1j} = 0 \quad \left(j = \overline{1, E\left(\frac{m-k}{2}\right)}\right),$$

$$D_{2j} = 0 \quad (j = \overline{1, k-1}),\ D_{2k} \neq 0,$$

there exist three solutions of equation (7) in the form

$$v_i(s) = \sum_{l_1+l=1}^{\infty} C_i^{l_1} \lambda^l v_{l_1 l}(s), \quad i = 1, 2, 3, \tag{22}$$

where for the constants C_i

$$\begin{aligned} C_{1,2} &= \pm \sqrt{\frac{-D_{0m}}{D_{2k}}}\, \lambda^{\frac{m-k}{2}} + \sum_{i=1}^{\infty} a_i^{(1,2)} \lambda^{\frac{m-k+i}{2}}, \\ C_3 &= -\frac{D_{2k}}{D_{30}} \lambda^k + \sum_{i=1}^{\infty} b_i \lambda^{k+i} \end{aligned} \tag{23}$$

holds. Substituting (23) in (22) we seek the solutions $y_i(s)$ of equation (1) in the form of the series

$$y_i(s) = y_0(s) + \sum_{l=1}^{\infty} \lambda^{\frac{l}{2}} u_l^{(i)}(s) \qquad i = 1, 2,$$

$$y_3(s) = y_0(s) + \sum_{l=1}^{\infty} \lambda^l u_l(s);$$

the functions $u_l(s)$ can be determined using the method of uncertain coefficients again. Analogically it is possible to study all other cases in equation (21).

The system (20) for $p \geq 2$ can be studied by means of the method described in [7]. This method is based on the method of Newton's diagram and the theory of elimination.

The above method of the study of branch points of algebraic integral equations will be used in the next paper for the study of the countable infinity of the set of eigenvalues of the symmetric homogeneous algebraic integral equation. This problem was studied with the aid of another method by W. Schmeidler in [4].

REFERENCES

[1] *Lichtenstein, L.:* Vorlesungen über einige Klassen nichtlinearer Integralgleichungen und Integro-Differentialgleichungen nebst Anwendungen. Berlin, 1931.

[2] *Peřinová, V.:* Branching of Solution of Homogeneous Algebraic Integral Equation. Math. Nachr. 12, 1968, p. 171–180.

[3] *Schmeidler, W.:* Algebraische Integralgleichungen. Math. Nachr. 8, 1952, p. 31–40.

[4] *Schmeidler, W.:* Über symmetrische algebraische Integralgleichungen. Annalen der Finnischen Akademie der Wissenschaften, I. Mathematica-Physica 220. Helsinki, 1956.

[5] *Schmidt, E.:* Über die Auflösung der nichtlinearen Integralgleichungen und die Verzweigung ihrer Lösungen. Math. Ann. 65, 1908, p. 370–399.

[6] *Vajnberg, M. M.* – *Trenogin, V. A.:* Metody Ljapunova i Šmidta v teorii nelinejnych uravnenij i ich dalnejšee razvitie. Uspechi mat. nauk 17(2), 1962, p. 13–75.

[7] *Vajnberg, M. M.* – *Ajzengendler, P. G.:* Metody issledovanija v teorii razvetvlenija rešenij. Itogi nauki, Mat. analiz, 1965, p. 7–69.

SHRNUTÍ

BODY VĚTVENÍ ALGEBRAICKÝCH INTEGRÁLNÍCH ROVNIC

VLASTA PEŘINOVÁ

Práce se zabývá problémem větvení řešení obecné algebraické integrální rovnice s číselným parametrem v prostoru spojitých funkcí. Tento problém je převeden na hledání všech malých řešení (větví nulového řešení pro hodnoty parametru z okolí nuly) jisté transformované rovnice, což je ekvivalentní s určením všech malých řešení odpovídající rovnice větvení. Podrobněji jsou rozebrány některé speciální případy rovnice větvení.