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CORRIGENDUM TO "NONLINEAR DIFFERENTIAL POLYNOMIALS SHARING A SMALL FUNCTION" [ARCH. MATH. 44 (2008), 41–56]

Abhijit Banerjee and Sonali Mukherjee

This shorte note concerns the paper mentioned in the title.

The statement of **Theorem 1.1(ii)** and so that of **Corollary 1.2** given in that paper is not correct. For the uniqueness of f and g some extra conditions are required. Since the proof of that particular portion of **Theorem 1.1(ii)** depends upon **Lemma 2.11(ii)**, so the statement as well as the proof of **Lemma 2.11(ii)** should also be rectified. In the statement of **Theorem 1.1(ii)** and so in **Corollary 1.2** the extra condition "and the two expressions $\frac{b}{n+2}\, g \sum\limits_{j=0}^{n+1} \left(\frac{f}{g}\right)^j + \frac{c}{n+1} \sum\limits_{j=0}^{n} \left(\frac{f}{g}\right)^j$ and $\sum\limits_{j=0}^{n+2} \left(\frac{f}{g}\right)^j$ have no common simple zeros" should be added. Naturally in the proof of the **Lemma 2.11(ii)** some more analysis regarding the zeros of $\eta - u_k$ which are not the poles of g is required since the salient part of the proof is depending on the proof that $\eta - u_k$ has multiple zeros. The corrected statements and proofs of **Theorem 1.1(ii)** and the corresponding **Lemma 2.11(ii)** are given below.

Theorem 1.1. *Let f and g be two transcendental meromorphic functions such that $f^n(af^2+bf+c)f'$ and $g^n(ag^2+bg+c)g'$ where $a \neq 0$ and $|b|+|c| \neq 0$ share "$(\alpha,2)$". Then the following holds.*

(ii) *If $b \neq 0$, $c \neq 0$, $n > [12 - 2\Theta(\infty;f) - 2\Theta(\infty;g) - \min\{\Theta(\infty;f), \Theta(\infty;g)\}]$, the roots of the equation $az^2+bz+c=0$ are distinct, one of f and g is non entire meromorphic functions having only multiple poles and the two expressions $\frac{b}{n+2}\, g \sum\limits_{j=0}^{n+1} \left(\frac{f}{g}\right)^j + \frac{c}{n+1} \sum\limits_{j=0}^{n} \left(\frac{f}{g}\right)^j$ and $\sum\limits_{j=0}^{n+2} \left(\frac{f}{g}\right)^j$ have no common simple zeros then $f \equiv g$.*

Corollary 1.2. *Let f and g be two transcendental meromorphic functions, one of f and g is non entire meromorphic functions having only multiple poles and the two expressions $\frac{b}{n+2}\, g \sum\limits_{j=0}^{n+1} \left(\frac{f}{g}\right)^j + \frac{c}{n+1} \sum\limits_{j=0}^{n} \left(\frac{f}{g}\right)^j$ and $\sum\limits_{j=0}^{n+2} \left(\frac{f}{g}\right)^j$ have no common simple*

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zeros, such that $n > [12 - 2\Theta(\infty; f) - 2\Theta(\infty; g) - \min\{\Theta(\infty; f), \Theta(\infty; g)\}]$ *be an integer. If* $af^n(f-\beta_1)(f-\beta_2)f^{'}$ *and* $ag^n(g-\beta_1)(g-\beta_2)g^{'}$ *share "*$(\alpha, 2)$*", where* β_1 *and* β_2 *are the distinct roots of the equation* $az^2+bz+c=0$ *with* $|\beta_1| \neq |\beta_2|$*, then* $f \equiv g$*.*

Lemma 2.11. *Let* F *and* G *be given as in Lemma 2.9 and* $n(\geq 6)$ *be an integer. Suppose* $F \equiv G$*. Then the following holds.*

(ii) *If* $b \neq 0$*,* $c \neq 0$*, and the roots of the equation* $az^2+bz+c=0$ *are distinct and one of* f *and* g *is non entire meromorphic function having only multiple poles and the two expressions* $\frac{b}{n+2}\, g \sum_{j=0}^{n+1}\left(\frac{f}{g}\right)^j + \frac{c}{n+1}\sum_{j=0}^{n}\left(\frac{f}{g}\right)^j$ *and* $\sum_{j=0}^{n+2}\left(\frac{f}{g}\right)^j$ *have no common simple zero then* $f \equiv g$*.*

Proof. Case 2. Suppose $b \neq 0$ and $c \neq 0$. Then $F \equiv G$ implies

$$Af^{n+3} + Bf^{n+2} + Cf^{n+1} \equiv Ag^{n+3} + Bg^{n+2} + Cg^{n+1}, \tag{1}$$

where $A = \frac{a}{n+3}$, $B = \frac{b}{n+2}$ and $C = \frac{c}{n+1}$.
Let us assume $f \not\equiv g$.

Subcase 2.1. Suppose the roots of the equation $az^2+bz+c=0$ are distinct. Since (1) implies f, g share (∞, ∞) without loss of generality we may assume that g has some multiple poles. Putting $\eta = \frac{f}{g}$ in (1) we get

$$Ag^2(\eta^{n+3}-1) + Bg(\eta^{n+2}-1) + C(\eta^{n+1}-1) \equiv 0\,.$$

i.e.,

$$Ag^2 \equiv -Bg\ \frac{\eta^{n+2}-1}{\eta^{n+3}-1} - C\ \frac{\eta^{n+1}-1}{\eta^{n+3}-1}. \tag{2}$$

First we observe that since a meromorphic function can not have more than two Picard exceptional values, η takes at least n values among $u_k = exp(\frac{2k\pi i}{n+3})$ where $k = 1, 2, \ldots, n+2$.

Let z_0 be a pole of g with multiplicity $p(\geq 2)$, which is not a root of $\eta - u_k = 0$. Then from (2) we have

$$2p = p \quad \text{i.e.,} \quad p = 0\,,$$

which is impossible.

Hence from (2) we see that the poles of g are precisely the roots of $\eta - u_k = 0$.

Suppose z_1 is a zero of $\eta - u_k$ of multiplicity r which is a pole of g with multiplicity s (say) then from (2) we see that

$$2s = r + s$$

i.e.,

$$r = s\,.$$

Since g has no simple pole, it follows that such points are multiple zeros of $\eta - u_k$.

From (2) we know

$$Ag^2 \equiv -\frac{Bg\sum_{j=0}^{n+1}\eta^j + C\sum_{j=0}^{n}\eta^j}{\sum_{j=0}^{n+2}\eta^j}\,. \tag{3}$$

Suppose z_2 be a simple zero of $\eta - u_k$ where $k = 1, 2, \ldots, n+2$, which is a zero of multiplicity $q(\geq 2)$ of the numerator of (3). Then from (3), z_2 would be a zero of order $q-1$ of g^2. So it follows that z_2 would be a zero of $\sum_{j=0}^{n}\eta^j$. Since $\sum_{j=0}^{n}\eta^j$ and $\sum_{j=0}^{n+2}\eta^j$ may have at most one common factor, we see that $\eta - u_k$ has multiple zeros for at least $n-1$ values of $k \in \{1, 2, \ldots, n+2\}$. Hence

$$\Theta(u_k;\eta) \geq \frac{1}{2}\,,$$

for at least $n-1$ values of k, which implies a contradiction as $n \geq 6$. □

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