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Correction to the paper: $C^{1,\alpha}$ local regularity for the solutions of the p-Laplacian on the Heisenberg group. The case $1 + \frac{1}{\sqrt{5}} < p \leq 2$

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Correction

to the paper "$C^{1,\alpha}$ local regularity for the solutions of the p-Laplacian on the Heisenberg group. The case $1+\frac{1}{\sqrt{5}}<p\le 2$"

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The purpose of the paper was to prove the local Hölder continuity of the gradient of local weak solutions $u \in W^{1,p}_{\mathrm{loc}}(\Omega, X)$, $1+\frac{1}{\sqrt{5}} < p \le 2$, of the p-Laplacian on the Heisenberg group

$$\operatorname{div}_H \vec{a}(Xu) = 0 \tag{1}$$

where $a^k(q) = |q|^{p-2} q_k$, $k = 1, \dots, 2n$. To this aim we introduced regularized equations

$$\operatorname{div}_H \vec{a}_\epsilon(Xu) = 0 \tag{2}$$

for small $\epsilon > 0$, where $a^k_\epsilon(q) = [(\epsilon + |q|^2)^{\frac{p-2}{2}} q_k]$, $k = 1, \dots, 2n$, and we proved that the local weak solutions $u_\epsilon \in W^{1,p}_{loc}(\Omega, X)$ of (2) are twice differentiable with respect to the horizontal fields. In particular they satisfy for $1+\frac{1}{\sqrt{5}} < p \le 2$

$$\int_{\Omega'} |T(g^4 u_\epsilon)|^p \, dx \le C R^{-4p} H_\epsilon, \tag{3}$$

$$\int_{\Omega'} |XT(g^{12} u_\epsilon)|^p \, dx \le C(R, \epsilon, H_\epsilon, p), \tag{4}$$

$$\int_{\Omega'} g^6 V_\epsilon^{p-2} |X^2 u_\epsilon|^2 \, dx \le C(\epsilon, R, H_\epsilon, p), \tag{5}$$

where g is a cut-off function with support in $B(2R)$ and $H_\epsilon = \int_{\Omega'} (V_\epsilon^p + |u_\epsilon|^p) \, dx$, $V_\epsilon^2 = \epsilon + |Xu_\epsilon|^2$ (see Sections 2, 3, 4, 7). This enables us to differentiate equation (2) and this should be, in turn, the main tool to prove uniform (with respect to ϵ) boundedness and Hölder continuity of the gradient. A limit argument for $\epsilon \to 0$ should furnish the same result for the solutions of (1).

But equation (47) of the paper obtained differentiating (2) with respect to X_i is incorrect, as not accounting for the non commutativity of the basic vector fields. Therefore, the correct equation would be

$$\int_{\Omega'} a^k_j \, X_j X_i u_\epsilon \, X_k \varphi \, dx + \int_{\Omega'} a^k_j \, T_{ij} u_\epsilon \, X_k \varphi \, dx + \int_{\Omega'} a^k \, T_{ik} \varphi \, dx = 0, \tag{6}$$

where $T_{lm} = [X_l, X_m]$.

But, specially due to the second integral on the left hand side of (6), for the moment we are able to obtain only higher integrability for the gradient, not the boundedness.

For this reason the following theorems in the paper must be considered not proved:

Theorem 5.2 and Theorem 1.1 (local boundedness of the gradient for u_ϵ and u).

Theorem 6.5 and Theorem 1.2 (local Hölder continuity of the gradient for u_ϵ and u).

Therefore the real content of the paper concerns the existence and the regularity of the second derivatives of the local weak solutions of equations (2) (Sections 2, 3, 4, 7).

References

[1] Marchi S., $C^{1,\alpha}$ *local regularity for the solutions of the p-Laplacian on the Heisenberg group. The case* $1+\frac{1}{\sqrt{5}} < p \leq 2$, Comment. Math. Univ. Carolinae **44.1** (2003), 33–56.

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