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A Carleson inequality for *BMOA* functions with their derivatives on the unit ball

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Abstract. The main purpose of this note is to give a new characterization of the well-known Carleson measure in terms of the integral for *BMOA* functions with their derivatives on the unit ball.

Keywords: Carleson measure, *BMOA* functions, Hardy spaces

Classification: 32A10, 32A35

1. Introduction

Let B denote the unit ball in $\mathbb{C}^n (n \geq 1)$, and v the $2n$ -dimensional Lebesgue measure on B normalized so that $v(B) = 1$, while σ is the normalized surface measure on the boundary S of B .

For $z = (z_1, \dots, z_n)$ and $w = (w_1, \dots, w_n)$ in \mathbb{C}^n , we let $\langle z, w \rangle = z_1 \bar{w}_1 + \dots + z_n \bar{w}_n$ so that $|z|^2 = \langle z, z \rangle$. For $\alpha = (\alpha_1, \dots, \alpha_n)$ with each α_i a nonnegative integer, we write $|\alpha| = \alpha_1 + \dots + \alpha_n, z^\alpha = z_1^{\alpha_1} \dots z_n^{\alpha_n}, \bar{w}^\alpha = \bar{w}_1^{\alpha_1} \dots \bar{w}_n^{\alpha_n}$, and

$$\frac{\partial^{|\alpha|} f(z)}{\partial z^\alpha} = \frac{\partial^{|\alpha|} f(z)}{\partial z_1^{\alpha_1} \dots \partial z_n^{\alpha_n}},$$

where $\partial^0 f(z) / \partial z^0 = f(z)$.

For $a \in B, a \neq 0$, let φ_a denote the automorphism of B taking 0 to a defined by

$$\varphi_a(z) = \frac{a - P_a(z) - \sqrt{(1 - |z|^2)} Q_a(z)}{1 - \langle z, a \rangle},$$

where P_a is the projection of \mathbb{C}^n onto the one-dimensional subspace span of a and Q_a is $I - P_a$. If $a = 0$, let $\varphi_0(z) = z$. For $0 < r < 1$ and $a \in B$, let $E(a, r) = \{z \in B : |\varphi_a(z)| < r\}$ as a pseudohyperbolic ball on B . It is easy to see that $E(a, r) = \varphi_a(rB)$ and $v(E(a, r)) \sim (1 - |a|)^{n+1}$ (see [Ru, 2.2.7]), where the symbol “ \sim ” indicates that the quantities have ratios bounded and bounded away from zero as a varies.

The Hardy space $H^p (0 < p < \infty)$ is defined as the space of holomorphic functions f on B satisfying

$$(1.1) \quad \|f\|_p = \sup_{0 < r < 1} \left\{ \int_S |f(r\xi)|^p d\sigma(\xi) \right\}^{1/p} < \infty.$$

The space $BMOA$ consists of the functions $f \in H^1$ for which

$$\|f\|_{BMOA} = \sup \frac{1}{\sigma(Q)} \int_Q |f - f_Q| \, d\sigma < \infty,$$

where f_Q denotes the averages of f over Q and the supremum is taken over all $Q = Q_\delta(\xi) = \{\eta \in S : |1 - \langle \eta, \xi \rangle| < \delta\}$ for $\xi \in S$ and $0 < \delta \leq 2$. Here we have identified f with its boundary function.

In the work on interpolation by bounded analytic functions on the unit disc \mathcal{D} of \mathbb{C} , L. Carleson [Ca1], [Ca2] found the following well-known result:

Let μ be a positive measure on \mathcal{D} and $0 < p < \infty$. Then an estimate of the form

$$(1.2) \quad \left(\int_{\mathcal{D}} |f(z)|^p \, d\mu(z) \right)^{1/p} \leq C_p \|f\|_p$$

holds for all $f \in H^p$ if and only if there exists a constant $C' > 0$ such that

$$\mu(S(I)) \leq C'|I|$$

for all $S(I) = \{z \in \mathcal{D} : z/|z| \in I, 1 - |I| \leq |z| < 1\}$, where $|I|$ denotes the arc length of the subarc I on the unit circle and $S(I) = \mathcal{D}$ if $|I| \geq 1$. Here μ is called a Carleson measure on \mathcal{D} .

We say that a positive measure μ on B is a Carleson measure if there exists a constant $C > 0$ such that

$$\mu(B_\delta(\xi)) \leq C\delta^n$$

for all $\xi \in S$ and all $\delta(0 < \delta \leq 2)$, where $B_\delta(\xi) = \{z \in B : |1 - \langle z, \xi \rangle| < \delta\}$ is said to be a Carleson region. The definition above tells us that a Carleson measure is finite. Here and in the sequel, constants are denoted by C , they are positive and may differ from one occurrence to the other.

Hörmander [Hö] proved the higher dimensional version of Carleson’s theorem above and gave a simple proof of Carleson’s estimates. In this paper we shall give a new characterization of Carleson measures in terms of integrals for $BMOA$ functions with their derivatives on the unit ball. Our main result is the following:

Theorem 1. *Let μ be a positive Borel measure on B , $0 < p < \infty$ and α a multiindex. Then there exists a constant $C > 0$ such that*

$$(1.3) \quad \sup_{a \in B} \int_B \left| \frac{\partial^{|\alpha|} f(z)}{\partial z^\alpha} - \frac{\partial^{|\alpha|} f(a)}{\partial z^\alpha} \right|^p \frac{(1 - |a|^2)^{n + \frac{3|\alpha|p}{2}} (1 - |z|^2)^{\frac{|\alpha|p}{2}}}{|1 - \langle z, a \rangle|^{2n+2|\alpha|p}} \, d\mu(z) \leq C \|f\|_{BMOA}^p$$

for all $f \in BMOA$ if and only if there exists a constant $C' > 0$ such that

$$(1.4) \quad \mu(B_\delta(\xi)) \leq C'\delta^{n+|\alpha|p}$$

for all $\xi \in S$ and all $\delta(0 < \delta \leq 2)$.

2. Preliminary lemmas

Lemma 1. For $0 < r < 1$ let a be a point in B with $1 - |a| < 2 \left(1 + \sqrt{\frac{2}{1-r}}\right)^{-2}$.

Then $E(a, r) \subset B_\delta(\xi)$, where $\xi = a/|a|$ and $\delta = (1 - |a|) \left(1 + \sqrt{\frac{2}{1-r}}\right)^2$.

PROOF: By the identity ([Ru])

$$1 - |\varphi_a(z)|^2 = \frac{(1 - |a|^2)(1 - |z|^2)}{|1 - \langle z, a \rangle|^2}$$

for $a \in B$ and $z \in E(a, r)$ we have (see [Je])

$$(2.1) \quad \frac{1 - r}{1 + r} \leq \frac{1 - |a|^2}{1 - |z|^2} \leq \frac{1 + r}{1 - r},$$

and

$$(2.2) \quad |1 - \langle z, a \rangle|^2 = \frac{(1 - |a|^2)(1 - |z|^2)}{1 - |\varphi_a(z)|^2} \leq 4 \left(\frac{1 - |a|}{1 - r}\right)^2.$$

Let $\xi = a/|a|$. We obtain

$$\begin{aligned} |1 - \langle z, \xi \rangle|^{\frac{1}{2}} &\leq |1 - \langle z, a \rangle|^{\frac{1}{2}} + |1 - \langle a, \xi \rangle|^{\frac{1}{2}} \\ &\leq (1 - |a|)^{\frac{1}{2}} \left(1 + \sqrt{\frac{2}{1 - r}}\right). \end{aligned}$$

Taking $\delta = (1 - |a|) \left(1 + \sqrt{\frac{2}{1-r}}\right)^2$ we get that $E(a, r) \subset B_\delta(\xi)$. □

Lemma 2. Let $f \in BMOA$ and let $|\alpha|$ be a positive integer. Then there exists constant $C > 0$ such that

$$(2.3) \quad \left| \frac{\partial^{|\alpha|} f(a)}{\partial z^\alpha} \right| \leq C \|f\|_{BMOA} (1 - |a|)^{-|\alpha|}$$

for all $a \in B$.

PROOF: It is known that $BMOA \subset \mathcal{B}(B)$, where $\mathcal{B}(B)$ is the Bloch space of holomorphic functions f on B with $\|f\|_B = \sup\{(1 - |z|^2)|\nabla f(z)| : z \in B\} < \infty$, where $\nabla f(z) = (\partial f/\partial z_1, \dots, \partial f/\partial z_n)$ is the analytic gradient of f . From Theorem A in [Zh], for $f \in BMOA$ and positive integer $|\alpha|$, we have

$$\left| \frac{\partial^{|\alpha|} f(a)}{\partial z^\alpha} \right| \leq C \|f\|_B (1 - |a|)^{-|\alpha|} \leq C \|f\|_{BMOA} (1 - |a|)^{-|\alpha|}$$

for all $a \in B$. Here we used the estimate $\|f\|_B \leq C \|f\|_{BMOA}$. □

Lemma 3 ([Wu]). *Let μ be a finite positive measure on B , $0 < r < 1$ and $\beta > 0$. Then*

$$(2.4) \quad \sup_{a \in B} \int_B \left(\frac{1 - |a|^2}{|1 - \langle z, a \rangle|^2} \right)^{n+\beta} d\mu(z) < \infty$$

if and only if there exists a constant $C > 0$ such that

$$(2.5) \quad \mu(E(a, r)) \leq C(1 - |a|)^{n+\beta}$$

is fulfilled for all $a \in B$.

3. Proof of Theorem 1

We first consider the case $|\alpha| = 0$. Suppose that (1.4) holds for all $\xi \in S$ and all $\delta(0 < \delta \leq 2)$, that is, μ is a Carleson measure on B . For a holomorphic function f on B and $0 < p < \infty$, from [Ch] we have that $\|f\|_{BMOA} < \infty$ implies

$$(3.1) \quad \sup_{a \in B} \left\{ \int_S |f(\xi) - f(a)|^p \frac{(1 - |a|^2)^n}{|1 - \langle \xi, a \rangle|^{2n}} d\sigma(\xi) \right\}^{1/p} < \infty.$$

Thus for each $a \in B$ and each $f \in BMOA$ we have

$$F_a(z) = (f(z) - f(a)) \left(\frac{(1 - |a|^2)^n}{(1 - \langle z, a \rangle)^{2n}} \right)^{1/p} \in H^p, \quad 0 < p < \infty.$$

By Hörmander’s result we have

$$\int_B |F_a(z)|^p d\mu \leq C_p \int_S |F_a(\xi)|^p d\sigma(\xi),$$

it follows that

$$\begin{aligned} & \sup_{a \in B} \left\{ \int_B |f(z) - f(a)|^p \frac{(1 - |a|^2)^n}{|1 - \langle z, a \rangle|^{2n}} d\mu(z) \right\}^{1/p} \\ & \leq C' \sup_{a \in B} \left\{ \int_S |f(\xi) - f(a)|^p \frac{(1 - |a|^2)^n}{|1 - \langle \xi, a \rangle|^{2n}} d\sigma(\xi) \right\}^{1/p} \\ & \leq C' \|f\|_{BMOA}. \end{aligned}$$

To prove that (1.4) follows from (1.3) we only need to prove that (1.4) is valid for all $\xi \in S$ and all $\delta(0 < \delta \leq \frac{1}{4})$ since μ is finite. For each $\xi \in S$ and each $\delta(0 < \delta \leq \frac{1}{4})$ we take $a' = (1 - 2\delta)\xi \in B$. For $z \in B_\delta(\xi)$ we have

$$(3.2) \quad \begin{aligned} 2\delta & \leq |1 - \langle z, a' \rangle| \leq (|1 - \langle z, \xi \rangle|^{\frac{1}{2}} + |1 - \langle \xi, a' \rangle|^{\frac{1}{2}})^2 \\ & \leq \sqrt{5}\delta < 3\delta < 4\delta(1 - \delta) = 1 - |a'|^2. \end{aligned}$$

For fixed $a' \in B$ above we choose a function $f(z) = (1 - \langle z, a' \rangle)^{-n} \in BMOA$. From (3.2) we have

$$\begin{aligned}
 & \sup_{a \in B} \int_B |f(z) - f(a)|^p \frac{(1 - |a|^2)^n}{|1 - \langle z, a \rangle|^{2n}} d\mu(z) \\
 & \geq \int_B |f(z) - f(a')|^p \frac{(1 - |a'|^2)^n}{|1 - \langle z, a' \rangle|^{2n}} d\mu(z) \\
 (3.3) \quad & \geq \int_{B_\delta(\xi)} \left(|1 - \langle z, a' \rangle|^{-n} - (1 - |a'|^2)^{-n} \right)^p \frac{(1 - |a'|^2)^n}{|1 - \langle z, a' \rangle|^{2n}} d\mu(z) \\
 & \geq \left(\frac{1}{\sqrt{5}} - \frac{1}{3} \right)^{np} \delta^{-np} \int_{B_\delta(\xi)} \frac{(1 - |a'|^2)^n}{|1 - \langle z, a' \rangle|^{2n}} d\mu(z) \\
 & \geq C \delta^{-np-n} \mu(B_\delta(\xi)).
 \end{aligned}$$

On the other hand,

$$(3.4) \quad \|f\|_{BMOA}^p \leq C(1 - |a'|)^{-np} \leq C\delta^{-np}.$$

Therefore, from (1.3), (3.3) and (3.4) there exists a constant $C' = C(n, p)$ such that

$$\mu(B_\delta(\xi)) \leq C' \delta^n,$$

this shows that μ is a Carleson measure on B since μ is finite.

Now we consider the case $|\alpha| > 0$. Assume that μ satisfies (1.4) and let $f \in BMOA$. By Lemma 2 and the elementary inequality

$$(a + b)^p \leq 2^p(a^p + b^p), \quad 0 < p < \infty, \quad a > 0, \quad b > 0,$$

we have

$$\begin{aligned}
 & \int_B \left| \frac{\partial^{|\alpha|} f(z)}{\partial z^\alpha} - \frac{\partial^{|\alpha|} f(a)}{\partial z^\alpha} \right|^p \frac{(1 - |a|^2)^{n + \frac{3|\alpha|p}{2}} (1 - |z|^2)^{\frac{|\alpha|p}{2}}}{|1 - \langle z, a \rangle|^{2n+2|\alpha|p}} d\mu(z) \\
 (3.5) \quad & \leq C \|f\|_{BMOA}^p \int_B \frac{(1 - |a|^2)^{n + \frac{3|\alpha|p}{2}} (1 - |z|^2)^{-\frac{|\alpha|p}{2}}}{|1 - \langle z, a \rangle|^{2n+2|\alpha|p}} d\mu(z) + \\
 & + C \|f\|_{BMOA}^p \int_B \frac{(1 - |a|^2)^{n + \frac{|\alpha|p}{2}} (1 - |z|^2)^{\frac{|\alpha|p}{2}}}{|1 - \langle z, a \rangle|^{2n+2|\alpha|p}} d\mu(z) \\
 & \leq C \|f\|_{BMOA}^p \int_B \left(\frac{1 - |a|^2}{|1 - \langle z, a \rangle|^2} \right)^{n + \frac{|\alpha|p}{2}} d\nu(z),
 \end{aligned}$$

where $d\nu(z) = (1 - |z|^2)^{-\frac{|\alpha|p}{2}} d\mu(z)$. For a fixed $r(0 < r < 1)$ and $a \in B$ with $1 - |a| < 2 \left(1 + \sqrt{\frac{2}{1-r}}\right)^{-2}$, we set $\xi = a/|a|$ and $\delta = (1 - |a|) \left(1 + \sqrt{\frac{2}{1-r}}\right)^2$. By (1.4), (2.1) and Lemma 1 we have

$$(3.6) \quad \nu(E(a, r)) = \int_{E(a,r)} (1 - |z|^2)^{-\frac{|\alpha|p}{2}} d\mu(z) \leq C(1 - |a|)^{n + \frac{|\alpha|p}{2}}.$$

Since μ is finite, we see that (3.6) holds for all $a \in B$. Using Lemma 3 for the case $|\alpha| > 0$ and $0 < p < \infty$ we obtain

$$(3.7) \quad \sup_{a \in B} \int_B \left(\frac{1 - |a|^2}{|1 - \langle z, a \rangle|^2} \right)^{n + \frac{|\alpha|p}{2}} d\nu(z) < \infty.$$

Therefore, from the estimates (3.5) and (3.7) we get (1.3).

Conversely, suppose that (1.3) holds for all $f \in BMOA$. Let $a \in B$ with $|a| > 191/192$ and take $a' = (32|a| - 31)a/|a|$, then $Q_a(a') = 0$ and $|\varphi_a(a')| > 176/197$. Given $r, 0 < r < 1/33$, then $a' \notin E(a, r)$. By Lemma 1 and (2.2) for $z \in E(a, r)$ we have

$$(3.8) \quad |1 - \langle z, a' \rangle| \leq (|1 - \langle z, a \rangle|^{\frac{1}{2}} + |1 - \langle a, a' \rangle|^{\frac{1}{2}})^2 \leq \frac{825}{16}(1 - |a|),$$

and

$$(3.9) \quad \frac{176}{3}(1 - |a|) \leq 1 - |a'|^2 \leq 64(1 - |a|).$$

Combining (3.8) with (3.9) we have

$$(3.10) \quad |1 - \langle z, a' \rangle| \leq \left(\frac{15}{16}\right)^2 (1 - |a'|^2), \quad z \in E(a, r).$$

For fixed a' above we take $f(z) = (1 - \langle z, a' \rangle)^{-n}$. It is easy to see that $f \in BMOA$ and for any positive integer $|\alpha|$ we have

$$(3.11) \quad \frac{\partial^{|\alpha|} f(z)}{\partial z^\alpha} = n(n+1) \cdots (n + |\alpha| - 1) \overline{a'}^{|\alpha|} (1 - \langle z, a' \rangle)^{-n - |\alpha|}.$$

From (2.1), (3.9), (3.10) and (3.11) we get

$$\begin{aligned}
 & \sup_{a \in B} \int_B \left| \frac{\partial^{|\alpha|} f(z)}{\partial z^\alpha} - \frac{\partial^{|\alpha|} f(a)}{\partial z^\alpha} \right|^p \frac{(1 - |a|^2)^{n + \frac{3|\alpha|p}{2}} (1 - |z|^2)^{\frac{|\alpha|p}{2}}}{|1 - \langle z, a \rangle|^{2n+2|\alpha|p}} d\mu(z) \\
 & \geq \int_B \left| \frac{\partial^{|\alpha|} f(z)}{\partial z^\alpha} - \frac{\partial^{|\alpha|} f(a')}{\partial z^\alpha} \right|^p \frac{(1 - |a'|^2)^{n + \frac{3|\alpha|p}{2}} (1 - |z|^2)^{\frac{|\alpha|p}{2}}}{|1 - \langle z, a' \rangle|^{2n+2|\alpha|p}} d\mu(z) \\
 (3.12) \quad & \geq C(n, |\alpha|, p) \int_{E(a,r)} \left(|1 - \langle z, a' \rangle|^{-n-|\alpha|} - (1 - |a'|^2)^{-n-|\alpha|} \right)^p \times \\
 & \times \frac{(1 - |a'|^2)^{n + \frac{3|\alpha|p}{2}} (1 - |z|^2)^{\frac{|\alpha|p}{2}}}{|1 - \langle z, a' \rangle|^{2n+2|\alpha|p}} d\mu(z) \\
 & \geq C(1 - |a'|^2)^{-np-|\alpha|p} \int_{E(a,r)} \frac{(1 - |a'|^2)^{n + \frac{3|\alpha|p}{2}} (1 - |z|^2)^{\frac{|\alpha|p}{2}}}{|1 - \langle z, a' \rangle|^{2n+2|\alpha|p}} d\mu(z) \\
 & \geq C(1 - |a|)^{-n-np-|\alpha|p} \mu(E(a, r)).
 \end{aligned}$$

Also, we have

$$(3.13) \quad \|f\|_{BMOA}^p \leq C(1 - |a|)^{-np}.$$

Hence, from the estimates (3.12), (3.13) above and (1.3), we obtain

$$(3.14) \quad \mu(E(a, r)) \leq C'(1 - |a|)^{n+|\alpha|p}$$

for $\frac{191}{192} < |a| < 1$. In fact, we can get that (3.14) is fulfilled for all $a \in B$ since μ is finite. Since $|\alpha| > 0$ and $0 < p < \infty$, then by Lemma 3 we know that (3.14) implies

$$(3.15) \quad K = \sup_{a \in B} \int_B \left(\frac{1 - |a|^2}{|1 - \langle z, a \rangle|^2} \right)^{n+|\alpha|p} d\mu(z) < \infty.$$

For each $\xi \in S$ and each $\delta(0 < \delta \leq 2)$, we set $a = (1 - \frac{\delta}{2})\xi$. For $z \in B_\delta(\xi)$, we have

$$|1 - \langle z, a \rangle| \leq (|1 - \langle z, \xi \rangle|^{\frac{1}{2}} + |1 - \langle \xi, a \rangle|^{\frac{1}{2}})^2 \leq 4\delta.$$

This implies that

$$\begin{aligned}
 (3.16) \quad K & \geq \int_B \left(\frac{1 - |a|^2}{|1 - \langle z, a \rangle|^2} \right)^{n+|\alpha|p} d\mu(z) \\
 & \geq \int_{B_\delta(\xi)} \left(\frac{1 - |a|^2}{|1 - \langle z, a \rangle|^2} \right)^{n+|\alpha|p} d\mu(z) \\
 & \geq C(p, n, |\alpha|) \delta^{-n-|\alpha|p} \mu(B_\delta(\xi)).
 \end{aligned}$$

Therefore, from (3.15) and (3.16), we have

$$\mu(B_\delta(\xi)) \leq C\delta^{n+|\alpha|p}$$

for all $\xi \in S$ and all $0 < \delta \leq 2$. Thus the proof of Theorem 1 is complete. \square

From the second part of the proof of Theorem 1, we can get the following result:

Theorem 2. *Let μ be a finite positive measure on B , $0 < r < 1$ and $\alpha > n$. Then the following statements are equivalent:*

- (i) $\mu(B_\delta(\xi)) \leq C\delta^\alpha$ for all $\xi \in S$ and all $0 < \delta \leq 2$;
- (ii) $\mu(E(a, r)) \leq C(1 - |a|)^\alpha$ for all $a \in B$.

Notice that (i) implies (ii), but the converse fails if $\alpha = n$ (see [Lu] for case $n = 1$), that is, the Carleson region $B_\delta(\xi)$ cannot be replaced by the pseudohyperbolic ball $E(a, r)$ for case $n = \alpha$.

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