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ON NATURAL OPERATORS ON SECTORFORM FIELDS

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Summary. We determine all natural operators of the following types: firstly from 1-sectorform bundle to 2-sectorform bundle, secondly from 1-sectorform bundle to 3-sectorform bundle and thirdly from 2-sectorform bundle to 3-sectorform bundle. We deduce that the fundamental operator here is the differential of sectorform fields.

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The concept of a k-sectorform introduced by J. E. White is generalization of the classical 1-form to the case of the k-times iterated tangent bundle, [7]. The aim of this paper is to determine all natural operators of the following types: firstly from 1-sectorform bundle to 2-sectorform bundle, secondly from 1-sectorform bundle to 3-sectorform bundle and thirdly from 2-sectorform bundle to 3-sectorform bundle. We deduce that the fundamental operator here is the differential of sectorform fields introduced by J. E. White, [7], and I. Kolář, [2]. In the paper, we use a general method for finding all natural operators of certain types developed by I. Kolář in [3].

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1. Let M be a smooth manifold.

Let

(1.1) $$p_M\colon TM \to M,\ p_{TM}\colon T(TM) \to TM, \ldots, p_{T_{k-1}M}\colon T(T_{k-1}M) \to T_{k-1}M$$

be the tangent bundles. Consider an iterated tangent bundle

(1.2) $$T_kM := \underbrace{T(T(\ldots TM))}_{k-\text{times}}.$$

There exist k vector bundle structures on T_kM over $T_{k-1}M$

(1.3) $$T_r p_{T_{k-r-1}M}\colon T_kM \to T_{k-1}M\,, \quad r = 0, 1, \ldots, k - 1$$

with projections $p_{T_{k-1}M}, Tp_{T_{k-2}M}, \ldots, T_{k-1}p_M$.

A classical 1-form on the manifold M can be interpreted as a linear map $TM \to R$ with respect to the vector bundle structure $p_M\colon TM \to M$. This concept can be generalized as follows [7].

Definition 1. A map $\Omega_{k,x}\colon (T_kM)_x \to R$ linear with respect to all k vector bundle structures (1.3) is called a *k-sectorform on M at x*.

Elements of the iterated tangent bundle T_kM are called k-sectors and can be expressed in the form

$$(1.4)\qquad A = \left.\frac{\partial}{\partial t^k}\right|_0 \cdots \left.\frac{\partial}{\partial t^1}\right|_0 \zeta(t^1, \ldots, t^k)$$

for a suitable smooth local map $\zeta\colon R^k \to M$.

The coordinate functions of a local chart $\varphi = (x^i)_{i=1,\ldots,n}$ on M induce the coordinate functions of a local chart $\psi_k = (x^i_{\lambda_1\ldots\lambda_k})$, $i = 1, \ldots, n$, $\lambda_l \in \{0, 1\}$, $l = 1, \ldots$ $\ldots, k$, on T_kM defined by

$$(1.5)\qquad x^i_{\lambda_1\ldots\lambda_k}(A) = \left.\frac{\partial^{|\lambda|}(x^i \circ \zeta(t^1, \ldots, t^k))}{(\partial t^1)^{\lambda_1} \ldots (\partial t^k)^{\lambda_k}}\right|_{(0,\ldots,0)}$$

with $|\lambda| = \lambda_1 + \ldots + \lambda_k$.

Let

$$(1.6)\qquad q_k\colon T^*_kM \to M$$

denote the fibre bundle of all k-sectorforms on M. Then a k-sectorform field on M is a section $\Omega_k\colon M \to T^*_kM$ and the value Ω_k at a point x can be considered as a map $\Omega_{k,x}\colon (T_kM)_x \to R$. Any k-sectorform Ω_k on M has the following form in the induced coordinates on T_kM for $k = 1, 2, 3$:

$$(1.7)\qquad \Omega_1 = e_i x^i_1\,,$$

$$(1.8)\qquad \Omega_2 = c_i x^i_{11} + b_{ij} x^i_{10} x^j_{01}\,,$$

$$(1.9)\qquad \Omega_3 = E_i x^i_{111} + B_{ij} x^i_{110} x^j_{001} + C_{ij} x^i_{101} x^j_{010} + D_{ij} x^i_{011} x^j_{100} + A_{ijk} x^i_{100} x^j_{010} x^k_{001}\,.$$

A coordinate change $x^i = x^i(\bar{x}^j)$ on M induces a coordinate change of the induced coordinates $(x^i_{\lambda_1\ldots\lambda_k})$ on T_kM. In this way we obtain the coordinate changes on T_kM in the following forms for $k = 1, 2, 3$:

$$(1.10)\qquad \bar{e}_i = e_j \tilde{a}^j_i\,,$$

$$(1.11)\qquad \bar{c}_i = c_j \tilde{a}^j_i\,,\quad \bar{b}_{ij} = b_{kl} \tilde{a}^k_i \tilde{a}^l_j + c_k \tilde{a}^k_{ij}\,,$$

$$(1.12)\qquad \begin{aligned} \bar{E}_i &= E_l \tilde{a}^l_i\,,\\ \bar{B}_{ij} &= B_{lm} \tilde{a}^l_i \tilde{a}^m_j + E_l \tilde{a}^l_{ij}\,,\\ \bar{C}_{ij} &= C_{lm} \tilde{a}^l_i \tilde{a}^m_j + E_l \tilde{a}^l_{ij}\,,\\ \bar{D}_{ij} &= D_{lm} \tilde{a}^l_i \tilde{a}^m_j + E_l \tilde{a}^l_{ij}\,,\\ \bar{A}_{ijk} &= A_{lmn} \tilde{a}^l_i \tilde{a}^m_j \tilde{a}^n_k + B_{lm} \tilde{a}^l_{ik} \tilde{a}^m_j + C_{lm} \tilde{a}^l_{jk} \tilde{a}^m_i + D_{lm} \tilde{a}^l_{ij} \tilde{a}^m_k + E_l \tilde{a}^l_{ijk} \end{aligned}$$

provided

$$(1.13)\qquad \tilde{a}^i_j = \frac{\partial x^i}{\partial \bar{x}^j}, \quad \tilde{a}^i_{jk} = \frac{\partial^2 x^i}{\partial \bar{x}^j\, \partial \bar{x}^k}, \quad \tilde{a}^i_{jkl} = \frac{\partial^3 x^i}{\partial \bar{x}^j\, \partial \bar{x}^k\, \partial \bar{x}^l}.$$

The differential of a real valued function $f\colon M \to R$ is the second component of the tangent map $Tf\colon TM \to TR = R \times R$, i.e. $\delta f = pr_2 \circ Tf$, where $pr_2\colon R \times R \to R$ is the projection on the second factor. The differential $\delta\omega$ of the 1-form ω on M, interpreted as a linear map $TM \to R$, is a 2-sectorform field $\delta\omega = pr_2 \circ T\omega$ interpreted as a map $\delta\omega\colon T_2M \to R$. In general, we have

Definition 2. ([2], [7]). The second component $\delta\Omega_k\colon T_{k+1}M \to R$ of the tangent map $T\Omega_k\colon T_{k+1}M \to TR$,

$$(1.14)\qquad \delta\Omega_k = pr_2 \circ T\Omega_k$$

is called the *differential of the k-sectorform field* $\Omega_k\colon M \to T^*_kM$.

If sectorform fields $\Omega_1 \in C^\infty T^*M$, $\Omega_2 \in C^\infty T^*_2M$ are of the form $\Omega_1 = e_i x^i_1$, $\Omega_2 = c_i x^i_{11} + b_{ij} x^i_{10} x^j_{01}$, then their differentials $\delta\Omega_1 \in C^\infty T^*_2M$, $\delta^2\Omega_1 \in C^\infty T^*_3M$, $\delta\Omega_2 \in C^\infty T^*_3M$ are of the form

$$(1.15)\qquad \delta\Omega_1 = e_i x^i_{11} + e_{ij} x^i_{10} x^j_{01},$$

$$(1.16)\qquad \delta^2\Omega_1 = e_i x^i_{111} + e_{ij} x^i_{110} x^j_{001} + e_{ij} x^i_{101} x^j_{010} + e_{ij} x^i_{100} x^j_{011} + e_{ijk} x^i_{100} x^j_{010} x^k_{001},$$

$$(1.17)\qquad \delta\Omega_2 = c_i x^i_{111} + c_{ij} x^i_{110} x^j_{001} + b_{ij} x^i_{101} x^j_{010} + b_{ij} x^i_{100} x^j_{011} + b_{ijk} x^i_{100} x^j_{010} x^k_{001}.$$

Let $\xi\colon R \to M$ or $\zeta\colon R^2 \to M$ be a suitable smooth local map defining an element of TM or TTM respectively, by

$$(1.18)\qquad \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_0 \xi(t) \in TM,$$

$$(1.19)\qquad \left.\frac{\partial}{\partial t^1}\right|_{(0,0)} \zeta(t^1, t^2), \quad \left.\frac{\partial}{\partial t^2}\right|_{(0,0)} \zeta(t^1, t^2), \quad \left.\frac{\partial^2}{\partial t^1\, \partial t^2}\right|_{(0,0)} \zeta(t^1, t^2).$$

Consider a canonical injection $k\colon TM \to TTM$ defined by

$$(1.20)\qquad k\colon \left.\frac{\partial}{\partial t^1}\right|_0 \xi(t^1) \mapsto \left.\frac{\partial^2}{\partial t^1\, \partial t^2}\right|_{(0,0)} \xi(t^1 . t^2).$$

In local coordinates on TM or T_2M the canonical injection $k\colon TM \to T_2M$ is of the form

$$k\colon (x^i_0, x^i_1) \mapsto (x^i_{00} = x^i_0,\ x^i_{10} = 0,\ x^i_{01} = 0,\ x^i_{11} = x^i_1).$$

For a 2-sectorform $\Omega_2 \in T^*_2M$ we get an underlying 1-sectorform defined by $\Omega_1 =$

$= \Omega_2 \circ k$. In local coordinates, for the 2-sectorform $\Omega_2 = c_i x^i_{11} + b_{ij} x^i_{10} x^j_{01}$ the underlying 1-sectorform is $\Omega_1 = c_i x^i_1$.

Consider the projections

$$(1.21) \qquad p_{TM} \circ p_{TTM},\ Tp_M \circ p_{TTM},\ Tp_M \circ T_2 p_M \colon T_3 M \to TM\,,$$

$$(1.22) \qquad T_2 p_M,\ Tp_{TM},\ p_{TTM} \colon T_3 M \to T_2 M\,.$$

Then, for a given 1-sectorform $\Omega_1 \in T^*M$ and a 2-sectorform $\Omega_2 \in T^*_2 M$ of the form (1.7) and (1.8), we obtain the 3-sectorforms

$$(1.23) \qquad \Omega_1 \circ p_{TM} \circ p_{TTM} \otimes \Omega_2 \circ T_2 p_M = \left(e_i x^i_{100}\right)\left(c_j x^j_{011} + b_{jk} x^j_{010} x^k_{001}\right),$$

$$(1.24) \qquad \Omega_1 \circ p_{TM} \circ p_{TTM} \otimes \Omega_1 \circ Tp_M \circ p_{TTM} \otimes \Omega_1 \circ Tp_M \circ T_2 p_M = \\ = \left(e_i x^i_{100}\right)\left(e_j x^j_{010}\right)\left(e_k x^k_{001}\right).$$

Thus, we can define the following inclusions of tensor products of a 1-sectorform bundle and a 2-sectorform bundle to a 3-sectorform bundle or of three copies of a 1-sectorform bundle to a 3-sectorform bundle

$$(1.25) \qquad T^*M \otimes T^*_2 M \to T^*_3 M \qquad \text{by} \quad (1.23)\,,$$

$$(1.26) \qquad T^*M \otimes T^*M \otimes T^*M \to T^*_3 M \quad \text{by} \quad (1.24)\,.$$

2. In this part we determine all natural operators from a 1-sectorform bundle to a 2-sectorform bundle.

Theorem 1. *All natural operators $T^*M \to T^*_2 M$ form a 3 parameter family*

$$(2.1) \qquad c_i = (\mu + \nu)\, e_i\,, \quad b_{ij} = \mu e_{ij} + \nu e_{ji} + \lambda e_i e_j\,, \quad \mu, \nu, \lambda \in R\,,$$

where (e_i, e_{ij}) or (c_i, b_{ij}) denote the canonical coordinate on the first jet prolongation $J^1 T^*M$ or $T^*_2 M$, respectively.

Proof. I. The first order operators $F\colon T^*M \to T^*_2 M$ are in bijection with the natural transformations $F\colon J^1 T^*M \to T^*_2 M$ and the L^2_n-equivariant maps of standard fibres $F\colon (J^1 T^* R^n)_0 \to (T^*_2 R^n)_0$. The group L^2_n acts on the fibre $(J^1 T^* R^n)_0$ in the form

$$(2.2) \qquad \bar{e}_i = e_k \tilde{a}^k_i\,, \quad \bar{e}_{ij} = e_{kl} \tilde{a}^k_i \tilde{a}^l_j + e_k \tilde{a}^k_{ij}\,.$$

We denote by $(\tilde{a}^i_j, \tilde{a}^i_{jk})$ the coordinates of the inverse element a^{-1} to an element $a \in L^2_n$ with coordinates (a^i_j, a^i_{jk}). The group L^2_n acts on the fibre $(T^*_2 R^n)_0$ by the formula (1.11).

First, consider equivariancy with respect to homotheties: $\tilde{a}^i_j = k\delta^i_j$, $\tilde{a}^i_{jk} = 0$ for a map $F\colon (J^1 T^* R^n)_0 \to (T^*_2 R^n)_0$ of the form

$$F\colon c_i = g_i(e_i, e_{ij})\,, \quad b_{ij} = f_{ij}(e_i, e_{ij})\,.$$

This gives a homogeneity condition:

(2.3) $$k\, g_i(e_i, e_{ij}) = g_i(ke_i, k^2 e_{ij}),$$
$$k^2 f_{ij}(e_i, e_{ij}) = f_{ij}(ke_i, k^2 e_{ij}).$$

We need the following

Lemma [4]. *Let $g(x^i, y^p, \ldots, z^t)$ be a smooth function defined on $R^m \times R^n \times \ldots \times R^p$ and let $a > 0$, $b > 0, \ldots, c > 0$, d be real numbers such that $k^d g(x^i, y^p, \ldots, z^t) = g(k^a x^i, k^b y^p, \ldots, k^c z^t)$ for every real number k. Then g is the sum of polynomials of degrees ξ in x^i, η in $y^p, \ldots, \zeta$ in z^t satisfying $a\xi + b\eta + \ldots + c\zeta = d$.*

By this lemma, functions g_i or f_{ij} must be polynomials of degrees r_1, r_2 or s_1, s_2 with respect to e_i, e_{ij} satisfying

(2.4) $$1 = r_1 + 2r_2, \quad 2 = s_1 + 2s_2.$$

The first equation (2.4) has the only solution $r_1 = 1$, $r_2 = 0$, i.e. g_i is linear in e_i and is independent of e_{ij}. The second equation (2.4) has two solutions $s_1 = 0$, $s_2 = 1$ and $s_1 = 2$, $s_2 = 0$, i.e. f_{ij} is the sum of a polynomial of degree 1 in e_{ij} and of a polynomial of degree 2 with respect to e_i.

Now we shall use the classical description of all invariant tensors, i.e. those elements of $\otimes^p R^n \otimes \otimes^q R^{n*}$ which are invariant with respect to all linear isomorphims of R^n [1], [3], [5]. The non-zero invariant tensors exist only for $p = q$ and are of the form

(2.5) $$\sum_{\sigma \in S(p)} k_\sigma I^1_{\sigma(1)} \otimes \ldots \otimes I^p_{\sigma(p)}, \quad k_\sigma \in R,$$

where $I^\alpha_\beta = [\delta^i_j]$ denotes the identity tensor in $R^n_\alpha \otimes R^{n*}_\beta$ and R^n_α or R^{n*}_β is the α-th or β-th component in $\otimes^p R^n$ or $\otimes^q R^{n*}$, respectively. Using invariant tensors we deduce that g_i and f_{ij} are of the form

(2.6) $$g_i = \tau \delta^j_i e_j,$$
$$f_{ij} = \mu \delta^k_i \delta^l_j e_{kl} + \nu \delta^l_j \delta^k_j e_{kl} + \lambda \delta^k_i \delta^l_j e_k e_l.$$

Considering equivariancy with respect to the kernel of the projection $L^2_n \to L^1_n$ i.e. $\tilde{a}^i_j = \delta^i_j$ and $\tilde{a}^i_{jk}$ arbitrary, we get $\tau = \mu + \nu$. This yields (2.1).

II. An r-th order operator $F\colon T^*M \to T^*_2 M$ corresponds to an L^{r+1}_n-equivariant map $F\colon (J^r T^* R^n)_0 \to (T^*_2 R^n)_0$. Equivariancy of F with respect to homotheties $\tilde{a}^i_j = k\delta^i_j$, $\tilde{a}^i_{jk_1 \ldots k_s} = 0$, $s = 1, \ldots, r$, gives

(2.7) $$k g_i(e_i, e_{ij_1}, \ldots, e_{ij_1 \ldots j_r}) = g_i(ke_i, k^2 e_{ij_1}, \ldots, k^{r+1} e_{ij_1 \ldots j_r}),$$
$$k^2 f_{ij}(e_i, e_{ij_1}, \ldots, e_{ij_1 \ldots j_r}) = f_{ij}(ke_i, k^2 e_{ij_1}, \ldots, k^{r+1} e_{ij_1 \ldots j_r}).$$

By our lemma, functions g_i are independent of $e_{ij_1 \ldots j_s}$, $s = 1, \ldots, r$ and f_{ij} are independent of $e_{ij_1 \ldots j_p}$, $p = 2, \ldots, r$. Hence, we have the case I. By Slovak's theorem, [6], every natural operator on T^*M has finite order. This proves Theorem 1.

The geometrical interpretation of the 3-parameter family (2.1) of natural operators $F\colon T^*M \to T_2^*M$ is

$$(2.8)\qquad \begin{aligned} &F\colon \Omega_1 \mapsto \delta\Omega_1 && \text{for} && \mu = 1\,, \quad \nu = 0\,, \quad \lambda = 0\,,\\ &F\colon \Omega_1 \mapsto \delta\Omega_1 \circ \iota_M && \text{for} && \mu = 0\,, \quad \nu = 1\,, \quad \lambda = 0\,,\\ &F\colon \Omega_1 \mapsto \Omega_1 \circ p_{TM} \otimes \Omega_1 \circ Tp_M\,, && && \mu = 0\,, \quad \nu = 0\,, \quad \lambda = 1\,, \end{aligned}$$

where $\delta\colon T^*M \to T_2^*M$ is the differential defined by (1.15) and $\iota_M\colon T_2M \to T_2M$ is the canonical involution.

3. In this part we determine all natural operators from a 1-sectorform bundle T^*M to a 3-sectorform bundle T_3^*M.

Theorem 2. *All natural operators $F\colon T^*M \to T_3^*M$ form a 10-parameter family of the form*

$$(3.1)\qquad \begin{aligned} E_i &= (\mu_1 + \mu_2 + \mu_3)\, e_i\,,\\ B_{ij} &= (\mu_1 + \mu_2)\, e_{ij} + \mu_3 e_{ji} + (\nu_5 + \nu_6)\, e_i e_j\,,\\ C_{ij} &= (\mu_1 + \mu_3)\, e_{ij} + \mu_2 e_{ji} + (\nu_3 + \nu_4)\, e_i e_j\,,\\ D_{ij} &= \mu_1 e_{ij} + (\mu_2 + \mu_3)\, e_{ji} + (\nu_1 + \nu_2)\, e_i e_j\,,\\ A_{ijk} &= \mu_1 e_{ijk} + \mu_2 e_{jik} + \mu_3 e_{jik} + \nu_1 e_i e_{jk} + \nu_2 e_i e_{kj} +\\ &\quad + \nu_3 e_j e_{ik} + \nu_4 e_j e_{ki} + \nu_5 e_k e_{ij} + \nu_6 e_k e_{ji} + \lambda e_i e_j e_k\,, \end{aligned}$$

for any $\mu_p, \nu_a, \lambda \in R$, $p = 1, 2, 3$, $a = 1, 2, \dots, 6$ where (e_i, e_{ij}, e_{ijk}) and $(E_i, B_{ij}, C_{ij}, D_{ij}, A_{ijk})$ are the canonical coordinates on the second jet prolongation J^2T^*M and T_3^*M, respectively.

Proof. I. The second order operators $F\colon T^*M \to T_3^*M$ are in bijection with the natural transformations $F\colon J^2T^*M \to T_3^*M$ and the L_n^3-equivariant maps of standard fibres $F\colon (J^2T^*R^n)_0 \to (T_3^*R^n)_0$. The latter map is of the form

$$(3.2)\qquad \begin{aligned} E_i &= g_i(e_i, e_{ij}, e_{ijk})\,, \qquad B_{ij} = f_{ij}(e_i, e_{ij}, e_{ijk})\,,\\ C_{ij} &= h_{ij}(e_i, e_{ij}, e_{ijk})\,, \qquad D_{ij} = r_{ij}(e_i, e_{ij}, e_{ijk})\,,\\ A_{ijk} &= s_{ijk}(e_i, e_{ij}, e_{ijk})\,. \end{aligned}$$

The group L_n^3 acts on the standard fibre of the second jet prolongation $(J^2T^*R^n)_0$ in the form

$$(3.3)\qquad \begin{aligned} \bar{e}_i &= e_l \tilde{a}_i^l\,,\\ \bar{e}_{ij} &= e_{lm} \tilde{a}_i^l \tilde{a}_j^m + e_l \tilde{a}_{ij}^l\,,\\ \bar{e}_{ijk} &= e_{lmn} \tilde{a}_i^l \tilde{a}_j^m \tilde{a}_k^n + e_{lm} \tilde{a}_i^l \tilde{a}_{jk}^m + e_{lm} \tilde{a}_{ij}^l \tilde{a}_k^m + e_{lm} \tilde{a}_{ik}^l \tilde{a}_j^m + e_l \tilde{a}_{ijk}^l \end{aligned}$$

and on the fibre $(T_3^*R^n)_0$ by the formula (1.12).

Considering equivariancy of (3.2) with respect to the homotheties $\tilde{a}^i_j = k\delta^i_j$, $\tilde{a}^i_{jk} = 0$, $\tilde{a}^i_{jkl} = 0$ and using the lemma, we arrive at the following facts. Functions f_{ij}, h_{ij}, r_{ij} are linear in e_{ij} and quadratic in e_i. Functions s_{ijk} are linear in e_{ijk}, bilinear in e_i, e_{ij} and of the third degree in e_i. Using the classical description of invariant tensors, we obtain the components of F in the form

$$
\begin{aligned}
(3.4)\qquad g_i &= \alpha e_i\,,\\
f_{ij} &= \beta e_{ij} + \gamma e_{ji} + \varepsilon e_i e_j\,,\\
h_{ij} &= \beta_1 e_{ij} + \gamma_1 e_{ji} + \varepsilon_1 e_i e_j\,,\\
r_{ij} &= \beta_2 e_{ij} + \gamma_2 e_{ji} + \varepsilon_2 e_i e_j\,,\\
s_{ijk} &= \mu_1 e_{ijk} + \mu_2 e_{jik} + \mu_3 e_{kij} + \nu_1 e_i e_{jk} + \nu_2 e_i e_{kj} +\\
&\quad + \nu_3 e_j e_{ik} + \nu_4 e_j e_{ki} + \nu_5 e_k e_{ij} + \nu_6 e_k e_{ji} + \lambda e_i e_j e_k\,.
\end{aligned}
$$

Equivariancy of the operators (3.4) with respect to the kernel of the projection $L^3_n \to L^1_n$ i.e. $\tilde{a}^i_j = \delta^i_j$ and $\tilde{a}^i_{jk}$, $\tilde{a}^i_{jkl}$ are arbitrary, gives the following relations for parameters

$$
\begin{aligned}
(3.5)\qquad \alpha &= \mu_1 + \mu_2 + \mu_3\,,\\
\beta &= \mu_1 + \mu_2\,, \quad & \gamma &= \mu_3\,, \quad & \varepsilon &= \nu_5 + \nu_6\,,\\
\beta_1 &= \mu_1 + \mu_3\,, & \gamma_1 &= \mu_2\,, & \varepsilon_1 &= \nu_3 + \nu_4\,,\\
\beta_2 &= \mu_1\,, & \gamma_2 &= \mu_2 + \mu_3\,, & \varepsilon_2 &= \nu_1 + \nu_2\,.
\end{aligned}
$$

Thus, we get a 10-parameter family (3.1) of natural operators.

II. The r-th order operators $F\colon T^*M \to T^*_3M$ correspond to the L^{r+1}_n-equivariant maps $F\colon (J^rT^*R^n)_0 \to (T^*_3R^n)_0$. Equivariancy of the operator F with respect to the homotheties $\tilde{a}^i_j = k\delta^i_j$, $\tilde{a}^i_{jk_1\ldots k_s} = 0$, $s = 1, \ldots, r$, gives independency of F of the coordinates $e_{ij_1\ldots j_p}$, $p = 3, \ldots, r$. Hence, the r-th order operators are reduced to the case I for every $r > 2$. By Slovak's theorem, [6] every natural operator on T^*M has finite order. This proves Theorem 2.

The geometrical interpretation of the 10-parameter family (3.1) of natural operators $F\colon T^*M \to T^*_3M$ is

(3.6) $\qquad F\colon \Omega_1 \mapsto \delta^2\Omega_1$

for $\mu_1 = 1$ and all the others are 0,

(3.7) $\qquad F\colon \Omega_1 \mapsto \Omega_1 \circ p_{TM} \circ p_{TTM} \otimes \delta\Omega_1 \circ T_2p_M$

for $\nu_1 = 1$ and all the others are 0,

(3.8) $\qquad F\colon \Omega_1 \mapsto \Omega_1 \circ p_{TM} \circ p_{TTM} \otimes \Omega_1 \circ Tp_M \circ p_{TTM} \otimes \Omega_1 \circ Tp_M \circ T_2p_M$

for $\lambda = 1$ and all the others are 0.

The remaining cases are derived from (3.6) and (3.7) by applying the canonical action of the symmetric group S_3 of 3 letters on T_3M [7].

Theorem 3. *All natural operators $T_2^*M \to T_3^*M$ form a 22-parameter family containing the* 10*-parameter family* (3.1) *of natural operators $T^*M \to T_3^*M$ defined on the underlying* 1*-sectorform fields, a* 6*-parameter family consisting of the differential* (1.17) *combined with the action of S_3 on T_3M, and a* 6*-parameter family consisting of the tensor product* (1.23) *of the* 2*-sectorform with the underlying* 1*-sectorform combined with the action of S_3 on T_3M.*

Proof is quite similar to those of Theorems 1 and 2, and we will not perform it explicitly here.

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Souhrn

PŘIROZENÉ OPERÁTORY NA POLÍCH SEKTORFORM

JAN KUREK

Určují se všechny přirozené operátory s 1-sektorform bandlu do 2-sektorform bandlu a s 1-sektorform bandlu do 3-sektorform bandlu a také z 2-sektorform bandlu do 3-sektorform bandlu. Dokazuje se, že hlavním operátorem je v tomto případě diferenciál pole sektorform.

Резюме

НАТУРАЛЬНЫЕ ОПЕРАТОРЫ НА ПОЛЯХ СЕКТОРФОРМ

JAN KUREK

Определяются все натуральные операторы следующих типов: из расслоения 1-секторформ в расслоение 2-секторформ, из расслоения 1-секторформ в расслоение 3-сектороформ и из расслоения 2-секторформ в расслоение 3-секторформ. Показывается, что главным оператором в этом случае является дифференциал поля секторформ.

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