HYPERGRAPHS AND INTERVALS, III

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(Received December 18, 1985)

Summary. Similarly to author's papers „Hypergraphs and intervals" and Hypergraphs and intervals, II" a projectoid means an ordered pair $(V, \mathscr{E})$, where V is a finite nonempty set, $\mathscr{E}$ is a set of nonempty subsets of V, and V can be ordered as a sequence $(v_1, \ldots, v_{|V|})$ in such a way that for each $E \in \mathscr{E}$, there exist $i, j \in \{1, \ldots |V|\}$ such that $i \leqq j$ and $E = \{v_i, \ldots, v_j\}$. In the present paper special kinds of projectoids (called Σ-projectoids and active Σ-projectoids) are studied.

Keywords: hypergraph, sequence of vertices.

AMS Classification: 05C65.

The present paper is a free continuation of papers [1] and [2]. However, the results of Parts 1 and 2 of the present paper are independent of the results of [1] and [2].

0. Let X and X' be arbitrary sets. If at least one of the sets $X - X'$, $X \cap X'$, and $X' - X$ is empty, we write $X \sim X'$. Otherwise, we write $X \nsim X'$.

By a nonempty sequence we shall mean an arbitrary finite sequence $(u_1, \ldots, u_m)$, where $m \geqq 1$. If $\alpha = (v_1, \ldots, v_n)$ is an arbitrary nonempty sequence $(n \geqq 1)$, we define

$$\langle \alpha \rangle = \{v;\ \text{there exists } i \in \{1, \ldots, n\} \text{ such that } v = v_i\} .$$

If $\alpha_1 = (v_{11}, \ldots, v_{1n_1}), \ldots, \alpha_k = (v_{k1}, \ldots, v_{kn_k})$ are nonempty sequences (where $k \geqq 2$ and $n_1, \ldots, n_k \geqq 1$), then the sequence

$$(v_{11}, \ldots, v_{1n_1}, \ldots, v_{k1}, \ldots, v_{kn_k})$$

will be denoted by $\alpha_1 \ldots \alpha_k$. Moreover, we introduce the empty sequence ω satisfying $\alpha\omega = \alpha = \omega\alpha$ for any nonempty sequence α, and $\omega\omega = \omega$. By a sequence we shall mean either a nonempty sequence or the empty one.

Let V be a finite nonempty set with n elements. We denote by $V^{\#}$ the set of all sequences $(v_1, \ldots, v_n)$ such that

$$\langle (v_1, \ldots, v_n) \rangle = V .$$

Obviously, $|V^{\#}| = n!$ (note that if X is a finite set, $|X|$ denotes the number of its elements). Let $\alpha \in V^{\#}$; we say that a set I is an interval set in α if there exists a nonempty sequence ι and sequences β and γ such that $\alpha = \beta\iota\gamma$ and $I = \langle \iota \rangle$; we denote by Int (α) the set of all interval sets in α. If $A \subseteq V^{\#}$ and $A \neq \emptyset$, then we denote

$$\mathrm{Int}(A) = \bigcap_{\alpha \in A} \mathrm{Int}(\alpha) .$$

Similarly to [1] and [2], by a hypergraph we mean an ordered pair $(V, \mathscr{E})$, where V is a finite nonempty set and $\mathscr{E}$ is a set of nonempty subsets of V. If $H = (V, \mathscr{E})$ is a hypergraph, we write $V(H) = V$ and $\mathscr{E}(H) = \mathscr{E}$. If $H = (V, \mathscr{E})$ is a hypergraph, then we denote

$$\Pi(H) = \{\alpha \in V^{\#};\ \mathscr{E} \subseteq \mathrm{Int}(\alpha)\} .$$

We say that a hypergraph H is a projectoid if $\Pi(H) \neq \emptyset$.

The following definition can be motivated by some results of papers [1] and [2]. Let H be a projectoid. We shall say that H is a Σ-projectoid if the following conditions hold:

(1) $V(H) \in \mathscr{E}(H)$,

(2) if $v \in V(H)$, then $\{v\} \in \mathscr{E}(H)$, and

(3) if $E, E' \in \mathscr{E}(H)$ and $E \nsim E'$, then $E \cup E'$, $E \cap E'$, $E - E' \in \mathscr{E}(H)$.

Theorem 2 in [1] can be reformulated as follows:

Lemma 0. *If H is a Σ-projectoid, then $\mathscr{E}(H) = \mathrm{Int}(\Pi(H))$.*

Let V be a finite nonempty set, and let $A \subseteq V^{\#}$, $A \neq \emptyset$. It is obvious that $(V, \mathrm{Int}(\alpha))$ is a Σ-projectoid for each $\alpha \in A$. Combining this fact with (1)–(3) we can easily get that $(V, \mathrm{Int}(A))$ is also a Σ-projectoid. This observation together with Lemma 0 gives the following result:

Theorem 0. *Let V be a finite nonempty set, and let H be a hypergraph such that $V(H) = V$. Then H is a Σ-projectoid if and only if there exists a nonempty subset A of $V^{\#}$ such that $\mathscr{E}(H) = \mathrm{Int}(A)$.*

1. Let H be a Σ-projectoid. We denote

$$\mathscr{F}(H) = \{F \in \mathscr{E}(H);\ F \sim E \text{ for each } E \in \mathscr{E}(H)\} .$$

For every $F \in \mathscr{F}(H)$, we denote by $\mathscr{N}_H(F)$ the set of $F' \in \mathscr{F}(H)$ such that F' is a proper subset of F and if $F'' \in \mathscr{F}(H)$ and $F' \subseteq F'' \subseteq F$, then either $F' = F''$ or $F'' = F$. Clearly, if $F \in \mathscr{F}(H)$, then $\mathscr{N}_H(F) \neq \emptyset$ if and only if $|F| \geqq 2$. Moreover, we denote by $\mathscr{F}^*(H)$ the set of all $F \in \mathscr{F}(H)$ with the property that there exists a proper subset $\mathscr{M}$ of $\mathscr{N}_H(F)$ such that $|\mathscr{M}| \geqq 2$ and

$$\bigcup_{F' \in \mathscr{M}} F' \in \mathscr{E}(H) .$$

Let $F \in \mathscr{F}(H)$ such that $\mathscr{N}_H(F) \neq \emptyset$, and let $\alpha \in \Pi(H)$. There exists exactly one sequence $(F_1, \ldots, F_n) \in (\mathscr{N}_H(F))^{\#}$ such that there exist sequences $\varphi_1, \ldots, \varphi_n$, β, and γ satisfying

$$\langle \varphi_1 \rangle = F_1, \ldots, \langle \varphi_n \rangle = F_n , \quad \text{and} \quad \alpha = \beta \varphi_1 \ldots \varphi_n \gamma .$$

We denote the sequence $(F_1, \ldots, F_n)$ by $S_H(F, \alpha)$ and the set

$$\{F_i \cup \ldots \cup F_j;\ 1 \leqq i < j \leqq n,\ j - i < n - 1\}$$

by $\mathscr{P}_H(F, \alpha)$.

The following theorem shows that if H is a Σ-projectoid, then $\mathscr{E}(H)$ can be derived from $\mathscr{F}(H)$, $\mathscr{F}^*(H)$, and one arbitrary $\alpha \in \Pi(H)$.

Theorem 1. *Let H be a Σ-projectoid, and let $\alpha \in \Pi(H)$. Then*

$$\mathscr{E}(H) - \mathscr{F}(H) = \bigcup_{F \in \mathscr{F}^*(H)} \mathscr{P}_H(F, \alpha).$$

Proof. If H_0 is a Σ-projectoid and $\alpha_0 \in \Pi(H_0)$, then we denote

$$\mathscr{Q}(H_0, \alpha_0) = \bigcup_{F_0 \in \mathscr{F}^*(H_0)} \mathscr{P}_{H_0}(F_0, \alpha_0).$$

We wish to prove that $\mathscr{E}(H) - \mathscr{F}(H) = \mathscr{Q}(H, \alpha)$.

We proceed by induction on $|V(H)|$. The case when $|V(H)| = 1$ is obvious. Let $|V(H)| > 1$. Assume that for every Σ-projectoid H' such that $|V(H')| < |V(H)|$ and every $\alpha' \in \Pi(H')$, it has been proved that $\mathscr{E}(H') - \mathscr{F}(H') = \mathscr{Q}(H', \alpha')$.

We distinguish two cases:

Case 1. Assume that there exists no $F \in \mathscr{F}(H)$ such that $1 < |F| < |V(H)|$. If $\mathscr{F}^*(H) = \emptyset$, then $\mathscr{E}(H) - \mathscr{F}(H) = \emptyset = \mathscr{Q}(H, \alpha)$. Let $\mathscr{F}^*(H) \neq \emptyset$. Then $\mathscr{F}^*(H) = \{V(H)\}$. It is obvious that $\mathscr{E}(H) - \mathscr{F}(H) \subseteq \mathscr{Q}(H, \alpha)$. We shall assume that $\mathscr{Q}(H, \alpha) - (\mathscr{E}(H) - \mathscr{F}(H)) \neq \emptyset$. Consider such $X \in \mathscr{Q}(H, \alpha) - (\mathscr{E}(H) - \mathscr{F}(H))$ that for each $X' \in \mathscr{Q}(H, \alpha) - (\mathscr{E}(H) - \mathscr{F}(H))$, $|X'| \leqq |X|$. Denote

$$\alpha = (v_1, \ldots, v_n).$$

There exist f, $h \in \{1, \ldots, n\}$ such that $1 \leqq f \leqq h \leqq n$ and that $X = \{v_f, \ldots, v_g\}$. Since $X \notin \mathscr{F}(H)$, we have $f < h$ and $h - f < n - 1$.

Assume that $1 < f$ and $h < n$. As follows from the maximality of $|X|$, $\{v_1, \ldots, v_h\}$, $\{v_f, \ldots, v_n\} \in \mathscr{E}(H)$. Since H is a Σ-projectoid, it follows from (3) that

$$\{v_1, \ldots, v_h\} \cap \{v_f, \ldots, v_n\} \in \mathscr{E}(H),$$

which is a contradiction. This means that either $f = 1$ or $h = n$. Without loss of generality we assume that $f = 1$.

According to (2), $\{v_1\} \in \mathscr{E}(H)$. We denote by g the maximum integer not exceeding h such that $\{v_1, \ldots, v_g\} \in \mathscr{E}(H)$. Since $\mathscr{F}^*(H) = \{V(H)\}$, there exists $E \in \mathscr{E}(H)$ such that $E \nsim E'$ for at least one $E' \in \mathscr{E}(H)$. By the assumption of Case 1, there exists no $F \in \mathscr{F}(H)$ such that $1 < |F| < |V(H)|$. The fact that H is a Σ-projectoid implies that there exist $E_1, E_2 \in \mathscr{E}(H)$ such that $E_1 \nsim E_2$ and $E_1 \cup E_2 = V$. Therefore, either $g \geqq 2$ or $h + 1 \leqq n - 1$. Moreover, we get $\{v_{g+1}, \ldots, v_{h+1}\} \in \mathscr{E}(H)$. Since $\{v_{g+1}, \ldots, v_{h+1}\} \notin \mathscr{F}(H)$, there exists $E_0 \in \mathscr{E}(H)$ such that $E_0 \nsim \{v_{g+1}, \ldots, v_{h+1}\}$. Clearly, either (i) $E_0 \nsim \{v_1, \ldots, v_{h+1}\}$ or (ii) $E_0 \subseteq \{v_1, \ldots, v_{h+1}\}$ and $v_g, v_{g+1} \in E_0$.

Therefore, there exists k, $g + 1 \leqq k \leqq h - 1$, such that $\{v_1, \ldots, v_k\} \in \mathscr{E}(H)$, which is a contradiction. Thus, $\mathscr{E}(H) - \mathscr{F}(H) = \mathscr{Q}(H, \alpha)$.

Case 2. Assume that there exists $F_0 \in \mathscr{F}(H)$ such that $1 < |F_0| < |V(H)|$. Then there exists $F \in \mathscr{F}(H)$ such that $1 < |F| < |V(H)|$ and that for every $F' \in \mathscr{F}(H)$ the inequality $|F'| < |F|$ implies $|F'| = 1$. There exist sequences β, γ, and φ such that $\alpha = \beta\varphi\gamma$ and $\langle\varphi\rangle = F$.

We denote by H_F the hypergraph defined as follows:

$$V(H_F) = F \quad \text{and} \quad \mathscr{E}(H_F) = \{E \in \mathscr{E}(H);\ E \subseteq F\}\,.$$

It is easy to see that H_F is a Σ-projectoid, $\varphi \in \Pi(H_F)$, $\mathscr{F}(H_F) = \mathscr{F}(H) \cap \mathscr{E}(H_F)$, $\mathscr{F}^*(H_F) = \mathscr{F}^*(H) \cap \mathscr{E}(H_F)$, and $\mathscr{Q}(H_F, \varphi) = \mathscr{Q}(H, \alpha) \cap \mathscr{E}(H_F)$. Since $|F| < |V(H)|$, according to the induction hypothesis $\mathscr{E}(H_F) - \mathscr{F}(H_F) = \mathscr{Q}(H_F, \varphi)$; thus

$$(4) \qquad (\mathscr{E}(H) - \mathscr{F}(H)) \cap \mathscr{E}(H_F) = \mathscr{Q}(H, \alpha) \cap \mathscr{E}(H_F)\,.$$

Consider an element x such that $x \notin V(H)$. We denote by H^F the hypergraph defined as follows:

$$\begin{aligned} V(H^F) &= (V - F) \cup \{x\} \quad \text{and} \\ \mathscr{E}(H^F) &= \{E_1;\ E_1 \in \mathscr{E}(H),\ E_1 \cap F = \emptyset\} \cup \\ &\quad \cup \{(E_2 - F) \cup \{x\};\ E_2 \in \mathscr{E}(H),\ F \subseteq E_2\}\,. \end{aligned}$$

We can easily see that H^F is a Σ-projectoid and $\beta(x)\,\gamma \in \Pi(H^F)$. Moreover, we can see that

$$\begin{aligned} \mathscr{F}(H^F) &= \{F_1;\ F_1 \in \mathscr{F}(H);\ F_1 \cap F = \emptyset\} \cup \\ &\quad \cup \{(F_2 - F) \cup \{x\};\ F_2 \in \mathscr{F}(H),\ F \subseteq F_2\} \quad \text{and} \\ \mathscr{F}^*(H^F) &= \{F_1;\ F_1 \in \mathscr{F}^*(H),\ F_1 \cap F = \emptyset\} \cup \\ &\quad \cup \{(F_2 - F) \cup \{x\};\ F_2 \subseteq \mathscr{F}^*(H),\ F \subseteq F_2,\ F \neq F_2\}\,. \end{aligned}$$

Since $|V(H^F)| < |V(H)|$, it follows from the induction hypothesis that $\mathscr{E}(H^F) - \mathscr{F}(H^F) = \mathscr{Q}(H^F, \beta(x)\,\gamma)$. This implies

$$(5) \qquad (\mathscr{E}(H) - \mathscr{F}(H)) - \mathscr{E}(H_F) = \mathscr{Q}(H, \alpha) - E(H_F)\,.$$

Combining (4) and (5), we get $\mathscr{E}(H) - \mathscr{F}(H) = \mathscr{Q}(H, \alpha)$, which completes the proof of Theorem 1.

Corollary. *Let H_1 and H_2 be Σ-projectoids such that $V(H_1) = V(H_2)$ and $\Pi(H_1) \cap \Pi(H_2) \neq \emptyset$. Then $\mathscr{E}(H_1) = \mathscr{E}(H_2)$ if and only if $\mathscr{F}(H_1) = \mathscr{F}(H_2)$ and $\mathscr{F}^*(H_1) = \mathscr{F}^*(H_2)$.*

2. Let V be a finite nonempty set. If $A \subseteq V^\#$, then we denote by Stab (A) the set of all $X \in$ Int (A) which possess the following property:

(6) if φ_i, ξ_i, and ψ_i (for $i = 1$ and 2) are arbitrary sequences such that $\varphi_1\xi_1\psi_1$, $\varphi_2\xi_2\psi_2 \in A$ and $\langle\xi_1\rangle = X = \langle\xi_2\rangle$, then $\varphi_2\xi_1\psi_2 \in A$.

Lemma 1. *Let V be a finite nonempty set, and let $A \subseteq V^\#$. Then $(V, \operatorname{Stab}(A))$ is a Σ-projectoid.*

Proof. Denote $H_A = (V, \operatorname{Stab}(A))$. According to Theorem 0, H_A is a projectoid. It is obvious that $V \in \operatorname{Stab}(A)$ and that $\{v\} \in \operatorname{Stab}(A)$ for each $v \in V$. Let $X, Y \in \operatorname{Stab}(A)$, $X \nsim Y$, and let $Z \in \{X \cup Y, X \cap Y, X - Y\}$. Consider arbitrary sequences φ_i, ζ_i, and ψ_i for $i = 1$ and 2 such that $\varphi_1\zeta_1\psi_1$, $\varphi_2\zeta_2\psi_2 \in A$, and $\langle\zeta_1\rangle = Z = \langle\zeta_2\rangle$. We wish to show that $\varphi_2\zeta_1\psi_2 \in A$.

Since $X \cup Y$, $X \cap Y$, $X - Y$, $Y - X \in \operatorname{Int}(A)$, there exist sequences ϱ_i, β_i, γ_i, δ_i, and σ_i for $i = 1$ and 2 such that

$$\varrho_j\beta_j\gamma_j\delta_j\sigma_j = \varphi_j\zeta_j\psi_j\,, \quad \text{for} \quad j = 1 \quad \text{and 2, and}$$
$$\{\langle\beta_k\gamma_k\rangle, \langle\gamma_k\delta_k\rangle\} = \{X, Y\}\,, \quad \text{for} \quad k = 1 \quad \text{and } 2\,.$$

Without loss of generality we assume that $X = \langle\beta_1\gamma_1\rangle$. Therefore, $Y = \langle\gamma_1\delta_1\rangle$.

Let $X = \langle\gamma_2\delta_2\rangle$. Then $Y = \langle\beta_2\gamma_2\rangle$. Since $X \in \operatorname{Stab}(A)$, it follows from (6) that

$$\varrho_2\beta_2\beta_1\gamma_1\sigma_2 \in A\,.$$

We have $\langle\beta_2\rangle \cup \langle\gamma_1\rangle = (Y - X) \cup (Y \cap X) = Y$. Since $\langle\beta_1\rangle = X - Y$, β_1 is a nonempty sequence. Thus

$$Y \notin \operatorname{Int}(\varrho_2\beta_2\beta_1\gamma_1\sigma_2)\,,$$

which is a contradiction. This means that $X = \langle\beta_2\gamma_2\rangle$, and therefore, $Y = \langle\gamma_2\sigma_2\rangle$.

Recall that $\varrho_i\beta_i\gamma_i\delta_i\sigma_i \in A$ for $i = 1, 2$. Since $\langle\beta_j\gamma_j\rangle = X$ for $j = 1, 2$, it follows from (6) that

$$\varrho_2\beta_1\gamma_1\delta_2\sigma_2 \in A\,.$$

Analogously, since $\langle\gamma_k\delta_k\rangle = Y$ for $k = 1, 2$, it follows from (6) that

$$\varrho_2\beta_2\gamma_1\delta_1\sigma_2 \in A\,.$$

Since $\langle\beta_1\gamma_1\rangle = \langle\beta_2\gamma_1\rangle$, the fact that $\varrho_2\beta_1\gamma_1\delta_2\sigma_2$, $\varrho_2\beta_2\gamma_1\delta_1\sigma_2 \in A$ implies

$$\varrho_2\beta_1\gamma_1\delta_1\sigma_2 \in A\,.$$

Since $\langle\gamma_1\delta_1\rangle = \langle\gamma_1\delta_2\rangle$, the fact that $\varrho_2\beta_1\gamma_1\delta_2\sigma_2$, $\varrho_2\beta_2\gamma_1\delta_1\sigma_2 \in A$ implies

$$\varrho_2\beta_2\gamma_1\delta_2\sigma_2 \in A\,.$$

Finally, since $\langle\gamma_2\delta_2\rangle = \langle\gamma_1\delta_1\rangle$, the fact that $\varrho_2\beta_2\gamma_2\delta_2\sigma_2$, $\varrho_2\beta_1\gamma_1\delta_1\sigma_2 \in A$ implies

$$\varrho_2\beta_1\gamma_2\delta_2\sigma_2 \in A\,.$$

Since $Z \in \{X \cup Y, X \cap Y, X - Y\}$, we have $\varphi_2\zeta_1\psi_2 \in A$. Hence, H_A is a Σ-projectoid, which completes the proof of the lemma.

We shall say that a Σ-projectoid H is active if $\mathcal{N}_H(F) \cap \mathcal{F}^*(H) = \emptyset$ for each $F \in \mathcal{F}^*(H)$.

The statement of the next theorem is analogous to that of Theorem 0. In the proof Theorem 1 will be used.

Theorem 2. *Let V be a finite nonempty set, and let H be a hypergraph such that $V(H) = V$. Then H is an active Σ-projectoid if and only if there exists a nonempty subset A of $V^{\#}$ such that $\mathcal{E}(H) = \mathrm{Stab}(A)$.*

Proof. (I) Assume that H is an active Σ-projectoid. Consider an arbitrary $\alpha \in \Pi(H)$. For every $F \in \mathcal{F}(H)$, we introduce a set $A(F)$ as follows:

(i) Let $\mathcal{N}_H(F) = \emptyset$. Let x denote the only vertex of F. Then we put $A(F) = \{(x)\}$.

(ii) Let $\mathcal{N}_H(F) \neq \emptyset$. Let $(F_1, \ldots, F_n)$ denote $S_H(F, \alpha)$. If $F \in \mathcal{F}^*(H)$, we put

$$A(F) = \{\varphi_1 \ldots \varphi_n;\ \varphi_1 \in A(F_1), \ldots, \varphi_n \in A(F_n)\}\ ;$$

if $F \notin \mathcal{F}^*(H)$, we put

$$A(F) = \{\varphi_1 \ldots \varphi_n;\ \text{either}\ \varphi_1 \in A(F_1), \ldots, \varphi_n \in A(F_n)\ \text{or}$$
$$\varphi_1 \in A(F_n), \ldots, \varphi_n \in A(F_1)\}\ .$$

Moreover, we denote $A = A(V)$ and $H_A = (V, \mathrm{Stab}(A))$. According to Lemma 1, H_A is a Σ-projectoid.

The definition of A easily yields that

(7) $\mathcal{F}(H) \subseteq \mathcal{E}(H_A)$, and

(8) if $F \in \mathcal{F}(H)$, $\mathcal{N}_H(F) \neq \emptyset$, and there exists $Z \in \mathcal{E}(H_A)$ such that Z is the union of at least two but not all elements of $\mathcal{N}_H(F)$, then $F \in \mathcal{F}^*(H)$, $Z \in \mathcal{P}_H(F, \alpha)$ and $\mathcal{P}_H(F, \alpha) \subseteq \mathcal{E}(H_A)$.

We wish to show that $\mathcal{E}(H) = \mathrm{Stab}(A)$. To the contrary, let $\mathcal{E}(H) \neq \mathrm{Stab}(A)$. Combining Theorem 1 with (7) and (8), we get that $\mathcal{F}(H) - \mathcal{F}(H_A) \neq \emptyset$. Hence, there exist $X \in \mathrm{Stab}(A)$ and $F_0 \in \mathcal{F}(H)$ such that $X \nsim F_0$. Consider such $F \in \mathcal{F}(H)$ that $X \subseteq F$ and for any $F' \in \mathcal{F}(H)$, if $X \subseteq F' \subseteq F$, then $F' = F$. Obviously, $\mathcal{N}_H(F) \neq \emptyset$. We denote $S_H(F, \alpha)$ by $(G_1, \ldots, G_m)$. Since $X \nsim F_0$, there exist f and h, $1 \leqq f < h \leqq m$, such that

$G_f \cap X \neq \emptyset \neq G_h \cap X$,

$G_g \subseteq X$ for each $g, f < g < h$, and either $G_f \nsim X$ or $G_h \nsim X$.

Without loss of generality, let $G_f \nsim X$. Obviously, $\mathcal{N}_H(G_f) \neq \emptyset$. We denote $S_H(G_f, \alpha)$ by $(J_1, \ldots, J_n)$. Since $X \cap G_{f+1} \neq \emptyset$, there exists i, $1 \leqq i \geqq n$, such that

$J_i \cap X \neq \emptyset$,

$J_k \subseteq X$ for each k, $i < k \leqq n$, and if $i = 1$, then $J_i \nsim X$.

If $i = 1$, we put $d = 2$; if $i \geqq 2$, we put $d = i$. According to (7), $\mathscr{F}(H) \subseteq \text{Stab}(A)$. Since H_A is a Σ-projectoid, it follows from (3) that

$$J_d \cup \ldots \cup J_n \cup G_{f+1} \cup \ldots \cup G_h \in \text{Stab}(A).$$

It follows from (6) that F, $G_f \in \mathscr{F}^*(H)$. This implies that H is not active, which is a contradiction. Hence, $\mathscr{E}(H) = \text{Stab}(A)$.

(II) Assume that there exists $A \subseteq V^\#$ such that $\mathscr{E}(H) = \text{Stab}(A)$. According to Lemma 1, H is a Σ-projectoid. We wish to show that H is active. To the contrary, we assume that there exist F, $G \in \mathscr{F}^*(H)$ such that $G \in \mathscr{N}_H(F)$. According to the definition, $\text{Stab}(A) \subseteq \text{Int}(A)$. It follows from the definition of a projectoid that $A \subseteq \Pi(H)$. Consider an arbitrary $\alpha \in A$. Denote $S_H(F, \alpha) = (F_1, \ldots, F_n)$. Since $F \in \mathscr{F}^*(H)$, $m \geqq 3$. Without loss of generality we assume that there exists k, $1 \leqq k \leqq m - 1$, such that $G = F_k$. Denote $S_H(F_k, \alpha) = (G_1, \ldots, G_n)$. Since F, $G \in \mathscr{F}^*(H)$, Theorem 1 implies

$$(9) \qquad S_H(F, \alpha') = (F_1, \ldots, F_m) \quad \text{and} \quad S_H(F_k, \alpha') = = (G_1, \ldots, G_n), \quad \text{for each} \quad \alpha' \in A.$$

Since G_n, $F_{k+1} \in \text{Stab}(A)$, it follows from (6) and (9) that $G_n \cup F_{k+1} \in \text{Stab}(A)$. Since $G \nsim G_n \cup F_{k+1}$, $G \notin \mathscr{F}(H)$, which is a contradiction. Thus, H is active, which completes the proof.

Remark. The subject of the present paper has its origin in the author's study of combinatorial properties of linguistic notions.

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Souhrn

HYPERGRAFY A INTERVALY, III

LADISLAV NEBESKÝ

Podobně jako v autorových článcích „Hypergraphs and intervals" a „Hypergraphs and intervals, II" se i v tomto článku projektoidem míní uspořádaná dvojice $(V, \mathscr{E})$, kde V je konečná neprázdná množina, $\mathscr{E}$ je množina nějakých neprázdných podmnožin množiny V a konečně kde V může být uspořádaná do posloupnosti $(v_1, \ldots, v_{|V|})$ takovým způsobem, že pro každé $E \in \mathscr{E}$ existují $i, j \in \{1, \ldots, |V|\}$, že $i \leqq j$ a přitom $E = \{v_i, \ldots, v_j\}$. V tomto článku se studují zvláštní druhy projektoidů (Σ-projektoidy a aktivní Σ-projektoidy).

Резюме

ГИПЕРГРАФЫ И ИНТЕРВАЛЫ, III

LADISLAV NEBESKÝ

Как и в статьях автора „Гиперграфы и интервалы“ и „Гиперграфы и интервалы, II“, так и в этой работе проектоидом называется упорядоченная пара $(V, \mathscr{E})$, где V — конечное непустое множество, $\mathscr{E}$ — некоторая система непустых подмножеств множества V и элементы множества V можно расположить в последовательность $(v_1, \ldots, v_{|V|})$ таким образом, что для каждого $E \in \mathscr{E}$ существуют $i, j \in \{1, \ldots, |V|\}$ такие, что $i \leqq j$ и $E = \{v_i, \ldots, v_j\}$. В настоящей статье изучаются специальные виды проектоидов (Σ — проектоиды и активные Σ — проектоиды).

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