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REFERÁTY

ČECH'S TOPOLOGICAL SEMINAR IN BRNO, 1936–1939

KAREL KOUTSKÝ, Brno

(Report on the author's lecture held on November 11th, 1963 in commemoration of the seventieth birthday anniversary of the late academician EDUARD ČECH)

If we want to deal with the history of Čech's Topological Seminar in Brno, it is necessary to make ourselves acquainted with the Čech's earlier activity in the field of topology.

The scientific work of prof. E. ČECH was originally in differential geometry, but after the year 1928 his interest turned to topology, both set (general) topology and combinatoric (algebraic) topology. The source of his scientific studies were first of all treatises published in "Fundamenta Mathematicae" by K. KURATOWSKI, W. SIERPIŃSKI, B. KNASTER, S. MAZURKIEWICZ and other authors. At the same time he followed any topological literature in other journals and was particularly interested in works by P. S. ALEXANDROV, L. LEFSCHETZ, R. L. WILDER and their pupils, referrig to combinatoric topology.

His first topological paper was published in the year 1930. In the year 1931 he discontinued his studies in differential geometry and he devoted his time exclusively to scientific work in topology. What a lot of energy he spent in it one can see from the fact that by the year 1935, when he went to America, he had published 23 treatises, 9 of which belonged to the field of set topology and 14 to the field of combinatoric topology. The closer connection of these two fields was the basic part of the scientific programme he had erected.

Čech's works on set topology are related to various special questions and, on the whole, there is only little connection among them. On the other hand, his works on combinatoric topology form an organic unit and represent important progress in the most notable parts of this theory. They are particularly concerned with the theory of homology and general varieties. Contrary to the conviction of topologists that combinatoric methods can be solely applied to the so called compact spaces, in one of his papers in 1932 Čech developed the theory of homology in quite general topological spaces, based on final open coverings. This theory of homology belongs nowadays to the best known of Čech's results and even in the world literature it is currently denoted by his name.

The second of Čech's pioneer papers appeared in the year 1933 and refers to the general theory of varieties and theorems of duality. This paper together with several others forms a remarkable chapter of combinatoric topology and belongs to the most successful works of Czech mathematics.

In his other works, Čech not only extended and improved the achieved results, but also started to study some further topological notions as e.g. local Betti's numbers, local connections (local acyclicities) of higher orders defined by means of the theory of homology, particular cases of the theorem of duality or the so called separation theorems which refer to the problem of how many pieces a space will split into by the removal of a given part, etc.

From the accummulation of such works, Čech was ranked among the outstanding world connoisseurs of combinatoric topology. In September 1935 he was invited to Moscow to a special conference on combinatoric topology, in which only a limited number of specialists from Europe and America took part. At this conference his results drew such great attention that he was invited to America for the purpose of lecturing at the American research centre "Institute for Advanced Study" in Princeton.

Čech's visit to America was of indisputable significance for the origin and development of the late topological seminar in Brno. At Princeton Čech not only met many well known mathematicians, but he also gained many valuable pieces of knowledge concerning the collective organization of scientific work. According to the report he published after his return home in "Naše věda" his scientific contacts were mainly with S. LEFSCHETZ, J. W. ALEXANDER and N. E. STEENROD. He particularly remembered L. ZIPPIN who at that time was the only one at Princeton who was working on non-combinatoric topology. Further, he mentions E. CHITTENDEN, who was especially interested in very general abstract spaces and who in this sense exerted upon him quite a great influence. Besides these, he shared interest with many other mathematicians as well.

In the above mentioned report Čech also describes the form of lectures held at the Institute. These lectures differed entirely from the usual university lectures and referred mostly to results not published earlier. They were often interrupted by a debate so that, on the whole, there was an impression of a permanent mathematical congress except for the fact that only one language was spoken.

Čech liked the social environment at Princeton very much. He especially appreciated the opportunity the scientists had for improving their contacts with each other. He said that the most important meetings of this type were afternoon teas, served daily at half past four. Here anyone could make himself acquainted with anyone else without any formality and discuss problems of mutual interest; here finished results as well as unfinished could be discussed. Problems could be proposed, dates fixed, or the recent literature could be discussed. And it was there that Čech clearly realized the significance and the advantage of collective scientific work and of frequent personal contacts among scientists.

Under the influence of that example, after his return from America in 1936, he started organizing his own mathematical school in Brno. However, he had dwelt on this idea of having his own school before his departure to Princeton. Convinced that the organized scientific work of younger mathematicians could be of much more persistent significance for our nation-wide culture than the best isolated scientific activity, he had experimented in this respect during the whole periode of his activity in Brno. Especially, when starting his research in topology he tried to draw the attention of younger mathematicians to this discipline. For this reason, he worked out in the Czech language some simple themes, chosen in such a way as to make the reader acquainted with the basic topological notions. His five Czech papers on connection, dimension and homology which he had published before 1935, belong to this part of his work. Because of his purpose he used a quite elementary form of explanation, not assuming any mathematical knowledge. It was not by chance that this was possible. In the preface to his second paper on the theory of dimension he wrote in 1933: "*The modern topology at its intensive analysis of space draws in no way from the fine ramified supply of notions of classical mathematics, but it copes successfully with its problems by means of a weapon quite simple, that is by using the elementary rules of logical thinking applied just to simple notions gained from opinion and axiomatically precised.*"

And so before the year 1935 he succeeded in drawing the attention of some of the younger mathematicians in Brno, first, of his assistant nowadays academician JOSEF NOVÁK, second, of the gifted student BEDŘICH POSPÍŠIL and also of the teachers at a grammar school MILOŠ NEUBAUER and KAREL KOUTSKÝ. In individual talks he proposed various special topological problems and some of them started to publish their own topological articles.

It was surely a satisfactory result but Čech, after his return from America, bore in mind a far higher aim. He wanted to create permanent, systematic and organized cooperation of younger mathematicians in order to develop a real mathematical centre in Brno which would have a good reputation and which, independently of him, could last in full vigour even when his own influence would pass away. The decision of realizing such an attempt was ripening in his mind during his sojourn at Princeton. There he had a great opportunity of watching how the experienced scientists helped their younger collegues to overcome their initial difficulties. He himself cooperated in this work and he often consulted outstanding scientists concerning his intentions. This decision of his, as he stated later, was influenced by the fact that he would not like to return to mathematical isolation after a stay of many months in one of the most busy mathematical centres.

Mathematics, however, is a very ramified science and it was necessary to confine oneself to some more definite field. In harmony with his scientific direction at that time, Čech elected the theory of sets and particularly (but not exclusively) topology. With respect to this fact he named the seminar, he meant to realize, the ***Topological Seminar***.

Since most of Čech's earlier work had been in the field of combinatoric topology, it seemed reasonable that the seminar should stay with this subject. However, Čech himself had many serious objections against this choice. First, his papers referring to combinatoric topology were, in the sense of his own words, a mere continuation of the work the preceeding generation had been occupied with. For a deeper comprehension it would be necessary to study too long, whereas he wanted to engage the young scientific workers in active scientific work. Besides, the scientific research in combinatoric topology supposed not only a knowledge of sets, but also a profound algebraic knowledge, which for the initial stage of the seminar work would be disadvantageous. Finally, he did not believe that it was right to engage the participants in a field he had worked in for such a long time. He used to say that the purpose of the seminar meetings was not to make the participants admire himself, but that all participants as well as he himself might gain from these seminar meetings impulses to new scientific work.

After a careful contemplation he came to the conclusion that it would be the best thing if the seminar covered what could be called a strictly axiomatic direction in topology, the founder and the principal supporter of which was the French mathematician M. Fréchet. He saw, too, that up to that time only little systematic work had been done in this field. Consequently, he decided to concentrate his main research work on this field.

Čech wrote from America about his intention to open a topological seminar to persons who might be interested. In Brno his news was welcomed with great pleasure and he was asked to start immediately after his return. Everybody felt that a great thing was being prepared. Therefore the mathematicians interested joined together in a small study circle and tried to deepen their topological knowledge. They used to meet onece a week in order to inform each other of the progress of their studies and to advice each other about various interesting things, and they even tried to solve some of the less significant questions they came across. As a result Čech found upon his return home that the soil was at least partly prepared for the development of his plans.

The first seminar meeting took place on May, 11th, 1936. The following meetings were held at weekly intervals (except for vacations) usually in the evenings in the lecture-room of the Mathematical Institute of the Faculty of Science at the University of Brno, Kotlářská 2. In May and June 1936 there were 8 participants besides prof. Čech in the seminar. In September 1936 two more members joined (one of them was J. Novák who had returned from one year of study with Prof. Menger in Vienna), but one left for Bratislava and one died, so that there were again 8 of them. All members of the seminar had finished their studies and most of them held a degree of doctor and many of them had worked scientifically in mathematics. By natural selection some participants fell away in 1937 till only 5 members took regularly part in meetings (E. Čech, B. Pospíšil, J. Novák, M. Neubauer and K. Koutský).

Of course, to make possible the development of the scientific activity of the seminar as quickly as possible, it was first of all necessary to make all participants acquianted with the basic topological notions. All of them had great interest but their knowledge was much varied and for this reason during the initial stage of the seminar all of the lectures were given by Prof. Čech. The other participants, on the whole, played only a passive role. At that time it was the custom that every participant had the right to interrupt the lecture whenever he did not understand anything. This profitable arrangement, a consequence of Čech's experinence in Princeton, was preserved during the whole existence of the seminar.

As to the contents of these introducing lectures, Čech first discussed the concept of topological space in the sense of the article by K. Kuratowski "Sur l'opération $\bar{A}$ de l'Analysis Situs", published in Fundamenta Mathematicae *3* (1922), pp. 182–199, where closures of sets in the space P satisfied the following four axioms: (1) $\bar{\emptyset} = \emptyset$, (2) $M \subset P \Rightarrow M \subset \bar{M}$, (3) $M_1 \subset P$, $M_2 \subset P \Rightarrow \overline{M_1 \cup M_2} = \bar{M}_1 \cup \bar{M}_2$, (4) $M \subset P \Rightarrow \bar{\bar{M}} = \bar{M}$. E. Čech also introduced the notion of a neighbourhood and defined such a space by means of complete systems of the neighbourhoods of its points. He discussed various axioms of separability and introduced a series of example of special topological spaces (e.g. the space of Kolmogorov, Riesz, Hausdorff, regular, completely regular, normal, hereditary normal, perfectly normal etc.). He explained and proved the necessary and sufficient conditions under which a topological space was metrizable.

According to his own words, Čech had no precise idea what program he would follow in further seminar meetings. In his report, one year later, he wrote on the topological seminar in "Naše věda": "*What was really my intention at that time is illustrated by my article* "*On Bicompact Spaces*" (published in 1937 in Annals of Mathematics 38, pp. 823–844), *which had originated in close connection with my lectures in the topological seminar and which contained, besides the series of achieved results, some unsolved problems also. My intention at that time was to put these and similar problems to the participants of the seminar.*"

With the initial vagueness of the program of the seminar work in future, there was closely connected even the question of what spaces were to be the object of corresponding topological considerations. The originally introduced notion of a topological space proved to be rather inadequately general, because certain important spaces of real continuous functions did not fulfil axiom (4) and for this reason they were excluded from further investigation. To remove this trouble, Čech introduced a new notion of topological space where closures of sets were subjected only to axiom (1) till (3), but in no way to axiom (4). He supposed that this transition to the more general concept would create no difficulties for the participants of the seminar and therefore without any hesitation he went on with his lectures. However, there were some difficulties. The concept of closures of sets supposed by axiom (4) had been meanwhile so deeply rooted in the considerations of some members that misunderstandings occurred too frequently.

The summer holidays break came in handy to Čech, because he wanted to make full use of it to consider not only the course of the seminar up to this date, but also the directions that were indicated by the experience gained. In the report mentioned on the topological seminar he says: "I realize that I hardly can expect a real understanding of problems I am bearing in mind, from listeners to whom the things I lectured about were new and quite unusual. Besides, in trying to get to these problems as quickly as possible, I hurried too quickly over the basic notions."

The result of Čech's contemplation was that after the reopening of the seminar in September 1936, he started again from basic notions, however this time, these were gone through in a much more detailed way. Simultaneously, under the impression of Chittenden's opinion, he generalized the notion of topological space in such a way that closures of sets were assumed to satisfy the three following axioms only:

$$\text{(I)}\ \bar{\emptyset} = \emptyset\,, \quad \text{(II)}\ M \subset P \Rightarrow M \subset \bar{M}\,, \quad \text{(III)}\ M_1 \subset M_2 \subset P \Rightarrow \bar{M}_1 \subset \bar{M}_2\,,$$

(I) and (II) being identical with axioms (1) and (2) and condition (III) being the simple consequence of axiom (3).

Topological spaces where axiom (3) was fulfilled were called by Čech *A*-spaces, and to those where axiom (4) was fulfilled he gave the name *U*-spaces. Then in his terminology, topological spaces with axioms (1) through (4) were *AU*-spaces.

He published his interpretations tpgether with further details and supplements in an article "*Topologické prostory*" (Topological Spaces), Časopis pro pěstování matematiky a fysiky, 66, pp. D 225 – D 267, Praha 1937, which became one of the basic sources for the work of the members of the seminar. Let us remark that topological spaces with axioms (I) through (III) are still often quoted in the current literature as "*Čech's topological spaces*".

The procedure, Čech had chosen at the reopening of the topological seminar, proved to be very convenient. Noting every detail in his lectures he observed that there was a whole series of questions, the answer to which was unknown to him. Later on, these questions were given to the participants of the seminar as problems. He gave no instructions for solution, being satisfied with the fact that the set problems were a consequence of his interpretation so that they seemed to be quite natural to the participants of the seminar. Besides, he himself did not know the solution of any of these problems, so that he could not judge whether they were difficult or easy.

At the same time it turned out that the participants would need more notions and theorems from transfinite arithmetic than were covered in JARNÍK's "Úvod do teorie množství" (Introduction to set theory) which appeared as a supplement to the 2nd edition of PETR's "Počet integrální" (Integral calculus) Praha 1931, pp. 655 – 725. As no convenient text-book for set theory corresponding to the intended purpose was at their disposal in Brno at that time, Čech asked M. Neubauer to hold a lecture in the seminar on transfinite numbers and to publish it later on in a little more extended form for the use of all persons interested. The article of Neubauer appeared in Časopis pro pěstování matematiky a fysiky 67 (1938), pp. D 101 – D 120, under the title "*Úvod do transfinitní aritmetiky*" (Introduction to Transfinite Arithmetic) and it fulfilled ist purpose in all respects.

During the course of the topological seminar Čech proposed 125 problems. In addition he had formulated 8 other problems in his above mentioned article „Topologické prostory" (Topological Spaces). One of the main sources of these problems was (especially at the beginning) the stimulative treatise of the Soviet mathematicians P. ALEXANDROV and P. URYSOHN "*Mémoire sur les espaces topologiques compacts*" (Verhandelingen der Kininklijke Akademie van Wetenschappen te Amsterdam, Afdecling Natuurkunde, Deel XIV, No 1. Amsterdam 1929) that had been discussed in the seminar continuously for a long time. Later on, some other sources joined it.

The work environment and the personality of Čech successfully influenced the members of the seminar who were keen on solving such problems and who really solved a great deal of them. And so at the beginning of the year 1937 Čech wrote in "Naše věda":

"*The up to day course of the topological seminar, I think, guarantees that it will mean for Czech mathematics a permanent and substantial profit. I did not expect that the initial difficulties would be overcome so soon, and, first of all, that the members of the seminar would be able so soon to cooperate actively, and I was very surprised by it.*"

The consequence of this activity soon brought a substantial change in the character of the seminar meetings. The introductory lectures of Čech became remarkably scarcer and instead of them there were reports of individual members of the seminar on the results achieved. Sometimes, of course, more members of the seminar participated in solving some problems and their solutions were identical. At other times the partial answers of the members complemented each other so conveniently that being summed up they gave the total solution. Usually, however, the answers, both partial or complete, were such that they led to the formation of other problems so that the final solution was only attained after several interchanges of questions and answers.

The problems as well as the date of their proposal Čech put down in a special notebock where he also recorded the names of the persons who eventually gave the solution according to the fact how the results were announced to him at the beginning of the meeting. The very precise evidence of all problems was also recorded by K. Koutský who besides the formulations of problems, date of their setting and names of persons giving the solution, put down for each problem a short summary of results gained, as they were reported in the seminar. These two notebooks are now valuable historical documents of the development of topological research in our country at that time.

It stands to reason that the contents of the proposed problems were rather varied and different. It can be said only very roughly and very inaccurately that most problems belonged to the theory of characters and pseudocharacters of points and sets of topological spaces and to other related questions. Here, for a topological space P the character $\chi(x)$ of a point $x \in P$ was defined as the smallest of the cardinality of complete systems of the neighbourhoods of the point x in a sapce P and the pseudocharacter $\psi(x)$ of a point $x \in P$ as the smallest of the cardinality of all such systems of the neighbourhoods of the point x, the intersection of which equals the intersection of all neighbourhoods of the point x in a space P. Quite analogously characters and pseudocharacters of sets were defined. A lot of problems referred to L-spaces, especially to spaces of continuous functions, where the convergence was introduced in various ways. The subject of a further series of problems was compact and locally compact spaces and their different generalizations, eventually spaces containing a dense subset of a given cardinality, as well as Cartesians products of topological spaces. There were, of course, many problems (approximately one fifth) which it was not possible to place in any of the preceeding groups as far as the contents are concerned, and which for the most part belonged to isolated topological and set questions.

Thanks to the variety of its problems and to the results achieved, Čech's seminar in Brno took one of the foremost places in all Czech mathematics. As evidence of its great significance is the fact that in the short time of its duration — 3 years and 6 months — 27 scientific treatises had their source in it.*) Among them was Čech's above mentioned paper from 1938 on bicompact, or as it is usually called nowadays, compact spaces. This paper was the first to investigate the so called compactification $\beta(S)$ of a completely regular space S, i.e. a compact Hausdorff's space containing S as a dense subset and such that every bounded continuous function on S can be continuously extended on $\beta(S)$. The results of Čech attracted great attention of mathematicians from abroad and the space $\beta(S)$ was later on in honour of Čech called *Čech's bicompactification* (see e.g. P. S. ALEXANDROV, Úspěchi matematičeskich nauk 1960, T. 15, pp. 25—95). Some properties of the space $\beta(S)$ were investigated simultaneously by an American mathematician M. H. STONE, and for this reason this space is often denoted in literature as *Stone-Čech's compactification* (see e.g. J. E. KELLEY, General Topology, 1955, pp. 298). But it was Čech's paper which showed the real significance of β-compactification and the possibilities of its use. β-compactification has become one of the most important instruments of general topology even for some fields of functional analysis.

It is worth while mentioning the results of other members of the seminar. It would be of no use and it is not even possible to speak in this short essay about all the results achieved, and therefore I confine myself only to the most important of those that were published.

One of the most gifted members of the topological seminar was BEDŘICH POSPÍŠIL (1912—1944), whose scientific work aroused the interest and admiration of all workers interested in general topology and mathematical logic. Despite his youth he had an all-round knowledge from various branches of mathematics and during the six years which elapsed between the completion of his studies at the university and his ap-

*) Hereby I correct the false statement "26 treatises" which is mentioned both in the article to Čech's sixtieth birthday and in his obituary article.

prehension by the Nazis in 1941 he enriched mathematical science with really excellent results. If he had been permitted to go on working, he would surely have developped into the leading spirit of Czech mathematics. The rude treatment of the Nazis first broke his health and later deprived him of his young life, and shattered all the well-founded hopes laid in him.

Bedřich Pospíšil published altogether 19 of his own papers, 3 of which date back to his student years. The remaining ones originated mostly in the topological seminar in the years 1936–1939, although some of them were published a little later. Apart from these papers he published 2 papers in cooperation with Prof. Čech.

A very important role in the scientific career of B. Pospíšil was played by problem No 36, which Čech proposed (for $\mathfrak{m} = \aleph_0$) on January 25th 1937:

"*What may be the cardinality of the Hausdorff space P which contains a dense subset H of a given infinite cardinality* $\mathfrak{m}$?"

If we denote, according to Pospíšil, the cardinality of a system of all subsets of a set of cardinality $\mathfrak{m}$ by a sign $\exp \mathfrak{m}$, it is easy to see that the cardinality of the space P cannot be greater than $\exp \exp \mathfrak{m}$. It is far more difficult to prove that this estimation cannot be lowered. And here the merit of Pospíšil consists in elaborating a very ingenious construction of a Hausdorff space $P(H)$ of cardinality $\exp \exp \mathfrak{m}$, where the isolated set H of cardinality $\mathfrak{m}$ lies densely. It was found that this space $P(H)$ is of basic importance to both the theory of characters of topological spaces, and the theory of compact spaces and Boolean rings.

Another result of Pospíšil consits in the determination of the number of all possible topologies of Čech in the infinite set P of cardinality $\mathfrak{m}$. He arrives at the fact that this number equals $\exp \exp \mathfrak{m}$ and does not decrease if we confine ourselves to such topologies that fulfil some separate axiom (regularity, normality or hereditary normality). On the other hand, as he proved in cooperation with Prof. Čech, the number of L-topologies in the infinite set P of cardinality $\mathfrak{m}$ equals $\exp \mathfrak{m}^{\aleph_0}$, so e.g. for $\mathfrak{m} = \aleph_0$ it equals $\exp \exp \mathfrak{m}$, whereas for $\mathfrak{m} = \exp \aleph_0$ it equals exclusively $\exp \mathfrak{m} < \exp \exp \mathfrak{m}$. By means of the same method, based on the use of the above mentioned space $P(H)$, he succeeded in determining the number of topologies in the infinite set P of cardinality $\mathfrak{m}$, where the characters of the points of the space P do not exceed the given cardinal number $\mathfrak{a}$, for which $\aleph_0 \leqq \mathfrak{a} \leqq \mathfrak{m}$. This number is $\exp \mathfrak{a}$.

Soon afterwards he erected the systematic theory of the characters of points of topological spaces, where he obtains results, the generality of which was so surprising that it in itself would suffice for the author to be ranked among the great scientists. Here Pospíšil quite generally assigns to every point x of an infinite set P of cardinality $\mathfrak{m}$ an infinite cardinal number $\chi(x)$ liable only to the necessary condition $\chi(x) \leqq \exp \mathfrak{m}$ and in an ingenious way he constructs in P a topology where the character of every point x is just $\chi(x)$. At the same time there are in his method so many grades of freedom that he also succeeds in expressing the number of all topologies with the prescribed characters $\chi(x)$; this number is $\exp \sum \chi(x)$. Actually besides characters he also prescribes pseudocharacters $\psi(x)$ which are liable only to the trivial conditions $\psi(x) \leqq \mathfrak{m}$, $\psi(x) \leqq \chi(x)$, and proves that even for the pseudocharacters prescribed in this way the number of all topologies in the set P remains equal to $\exp \sum \chi(x)$ and that it does not decrease even in the case that we require these topologies to fulfil some other separative axiom. He also solves the same problem under the further supposition that it is prescribed for the definite subset of P to lie in the space P densely. Then he partly transferred these results in cooperation with Prof. Čech to L-spaces.

The above mentioned space $P(H)$ enabled Pospíšil to solve several important questions referring to the compactification $\beta(S)$ of the completely regular space S. So first of all, he showed that in the case of the infinite isolated space S of cardinality $\mathfrak{m}$, the cardinality of correspondent compactification $\beta(S)$ equals $\exp \exp \mathfrak{m}$. Now, if we remove from this space $\beta(S)$ all open sets of cardinality smaller than $\mathfrak{m}$, we get a compact space $\alpha(S)$ the cardinality of which remains still $\exp \exp \mathfrak{m}$. It can be easily seen that the characters of points both in a space $\alpha(S)$ and in $\beta(S)$ cannot exceed the cardinality $\exp \mathfrak{m}$. It is not so easy to show that in both these spaces there exist points, the character of which just equals $\exp \mathfrak{m}$. And here Pospíšil found that each of the spaces $\alpha(S)$ and $\beta(S)$ contains $\exp \exp \mathfrak{m}$ such points. Besides, for $\mathfrak{m} = \aleph_0$ he proved that every dense part of a space $\alpha(S)$ has the cardinality at least $\exp \mathfrak{m}$; whether the same also applies for $\mathfrak{m} > \aleph_0$ has not been known up to now.

As for further topological results of Pospíšil it is necessary to be reminded of his theorem on the Cartesian product of ennumerably many spaces each of which contains more than one point, saying that this Cartesian product is never hereditary normal. Hence Pospíšil deduced that at the infinite isolated space S, spaces $\alpha(S)$, $\beta(S)$ are not hereditary normal. The same result is then proved in a quite different way in one of his papers written in cooperation with Prof. Čech.

The research of Pospíšil on spaces $\alpha(S)$, $\beta(S)$ is of basic importance in the theory of Boolean rings which is one of the basic chapters of mathematical logic. But it would lead too far if we wanted to follow Pospíšil's work in this direction and therefore we say only briefly that his method among others enabled him to define in a topological way the number of prime ideals in the series of a current field of sets. These results attracted so much attention that the editors of the prominent journal Fundamenta Mathematicae asked him for a new elaboration in the sense that the topological formulation would be translated into an algebraic one, and so they would be accessible to a broader circle of readers. This appeal Pospíšil answered with a new treatise, which was not a mere transcription of the corresponding preceeding papers, but which contained a series of new remarkable results. The respective volume of Fundamenta Mathematicae, where this new treatise was published (Vol. 33), was issued, as a whole, in December 1945 after the end of the second world war. Notwithstanding the reprints of Pospíšil's articles had been published in 1939.

Let us in passing only remark that in the several papers following, regrettably his last ones, Pospíšil laid the foundations to the so called *theory of continuous distribu-*

tions that is closely connected with the study of Boolean rings and measurable functions. Unfortunately it is impossible to explain this theory in detail in a sufficiently brief form.

The preceeding description of the results of Pospíšil, though very incomplete and rather concise, gives, on the whole, a clear impression of the broad pallette of his work. His papers have been worth serious studies up to now, and the reading of them would surely bring many stimuli for further research in the branches he had dealt with. In addition, some unsolved problems are to be found here, too.

Another outstanding participant of the topological seminar was JOSEF NOVÁK. In 1925, the Soviet mathematician P. Urysohn had constructed a countable regular space, containing one point with an ennumerable character. For a long time this fact seemed to be only a paradox phenomenon. And when Novák in 1936 as the first to improve the result of Urysohn by means of an ingenious construction of a countable space, each point of which had an ennumerable character, it turned out that it was a question of far deeper significance than was originally guessed. This construction of Novák's can be considered as the beginning of continuous studies of the characters of points of topological spaces, that were carried out in the seminar and that later led to Pospíšil's general theory of characters mentioned above.

The most successful paper of Novák during the time of the topological seminar is probably his treatise on L-spaces (1939), in which he published summarily his remarkable results referring to these spaces. In this paper he presented a total classification of L-spaces, the type of which were arranged in the following hierarchy: General L-spaces, HL-spaces, $\overline{H}L$-spaces, $\overline{\overline{H}}L$-spaces, regular L-spaces, completely regular L-spaces, normal L-spaces, hereditary normal L-spaces, perfectly normal L-spaces and metric spaces. Here, as in the sense of the above mentioned article of Čech "Topologické prostory", a space H, or $\overline{H}$, or $\overline{\overline{H}}$ is understood to be such a topological space, where there exist for every two of its points x, y neighbourhoods $V(x)$, $V(y)$ fulfilling the following conditions: $V(x) \cap V(y) = \emptyset$, or $\overline{V(x)} \cap V(y) = V(x) \cap \overline{V(y)} = \emptyset$, or $\overline{V(x)} \cap \overline{V(y)} = \emptyset$.

It is immediately evident that in this hierarchy an L-space of a certain type is simultaneously the L-space of all the preceeding types. But it is less easy to give an example of each type of L-space, which is not an L-space of the following type. From the older literature only isolated results were known in this direction. Novák completed these results by the construction of a series of further special L-spaces and by this he thoroughly answered the preceeding question with but one exception only. The unsolved problem was, whether there existed a regular L-space which is not completely regular. Let us mention that J. Novák solved this problem later in 1948 in his paper: "Regulární prostor, na němž je každá spojitá funkce konstantní" (On a Regular Space on which Every Continuous Function is Constant).

The treatise of Novák mentioned above also contains the complete solution of several problems of Čech, concerning the charakters and the pseudocharacters of points in L-spaces, as well as one important problem proposed by M. Fréchet in his book "Les espaces abstraits" (Paris 1928). Besides, it discusses certain important questions from the theory of Cartesian products of L-spaces.

As for the other results of Novák, let us mention here his theorem on the characters of sets in a metric space P, saying that the characters of a set $M \subset P$ is countable, if and only if this set M is a set theoretic sum of the compact and open set. Another interesting essay of his is on the so called Bernstein's ultracontinuum, i.e. on an ordered space, the elements of which are infinite sequences $[\alpha_n] = \alpha_1, \alpha_2, \ldots, \alpha_n, \ldots$ where α_n are ordinal numbers of the first and second ordinal class $(0 \leqq \alpha_n < \omega_1)$ and where the ordering is defined according to the following rule: A point $[\alpha_n]$ preceeds a point $[\beta_n]$ when $\alpha_i = \beta_i$ for $i = 1, 2, \ldots, k - 1$, whereas $\alpha_k < \beta_k$ for odd k and $\alpha_k > \beta_k$ for even k. Novák showed, among other things, that in this space there exist only lacks of types (0,1), (1,0) and (1,1) every type in number $\aleph_1$. It is impossible, however, to deal with the details here.

The third participant of the topological seminar in Brno Miloš Neubauer (1898 to 1959) occupied himself chiefly with the spaces of real continuous functions where the convergence was defined in various ways. He published his results in one paper which appeared in Fundamenta Mathematicae 31 (1938) and which belonged to the notable contributions to the theory of such spaces. Let us remark, too, that Neubauer had worked in this field before the establishment of the topological seminar and had published several good treatises on real functions. Rightly Čech wrote in his obituary article that Neubauer was an excellent connoisseur of this discipline.

The fourth participant of the seminar Karel Koutský ranked himself among those who were successful in solving some of Čech's problems. Let us recall briefly his informative results on topological spaces which have one of the following properties: If two sets are separated and non-empty, then one, eventually both of them, are open, resp. closed, resp. simultaneously open and closed. His essay on the so called *modifications* is also of some importance. If f is a given topological property and u a given topology in the set P, then we say of a topology w that it is a lower (upper) f-modification of the topology u if and only if w is the strongest (weakest) of all topologies in a set P which have the property f and are weaker (stronger) than the topology u. Evidently, when a topology u has property f, there exist both its lower and upper f-modification and these two topologies are identical with the topology u. If, however, a topology u does not have property f, then its lower, resp. upper f-modification need not exist or it exists under special presumptions only. In the paper mentioned, Koutský presents the complete solution of the question of existence and the construction of a lower and upper modification for a series of properties defined in Čech's paper "Topologické prostory". Let us remark that the question of modifications of a given topology proved to be important later on in the investigation of the structure of the lattice of Čech's topologies in a given set, and in 1960 was worked through by Koutský and his fellow workers in several directions.

From this survey, however incomplete, of the results achieved in the topological seminar in Brno it is, I suppose, sufficiently evident that this seminar fulfilled its task and that it brought to Czech mathematical science a rich profit. In fact, its scientific value was greater than could be indicated here. Some results, and not just secondary ones, were not published at that time, and unfortunately it should be admitted that some of them are not known to the broader mathematical public even today. This is the case e.g. of a series of theorems referring both to the notion of $\mathfrak{m}$-compactness (similar problems were dealt with by Ju. M. SMIRNOV in 1950, too), to U-spaces with the smallest open basis, to other special topological spaces as well as to the theory of general topology in Cartesian products and to other interesting topological questions. And even though the vehement development of set topology has taken a rather different direction in the postwar years, yet it might be worth while to return to Čech's problems of that time, and to adapt them conveniently to the demands of modern time.

The significance of Čech's topological seminar in Brno does not consist only in the great enrichment of mathematical science with new pieces of knowledge, but it also hints in its consequence at the very substance of the modern conception of scientific work. Thanks to Čech's extraordinary scientific outlook and his pedagogical mastery, in our country the first and a very successful attempt at a new progressive form in mathematical research was carried through, the main sign of which was an organized and systematic collective cooperation. This type of scientific work in mathematics which, in our country, in the course of time has became current, has many advantages over individual research, as can be easily seen from the fact that the number of valuable publications of Czech mathematicians has increased to an unusual extent in the postwar years. And in this respect it is necessary to appreciate the pioneer work of Čech.

Finally I would like to add two small personal remembrances: The meetings of the topological seminar used to be followed by some amusement. After the serious seminar work the members used to go to a café to have a chat, and sometimes when their wives were present they would go to a dance. However, even at this congenial and sometimes very gay entertainement, the members of the seminar found time to talk over their unsolved results and eventually the difficulties which they had came across in solving the proposed problems. They often put various questions to one another, and it can be truly said that a series of ideas were developed which, later on, were of great use for their work. Čech used to say in jest that these meetings reminded him of Princeton's afternoon teas and that it was pity, they could not take place every day.

The second reminiscence is not so cheerful. The occupation of the Czechoslovak Republic affected the meetings but the work in the seminar went on under more difficult conditions. Then a think happened that nobody would believe. After the outbreak of the second world war all Czech universities were closed by the Nazis on November 17th 1939, and hereby the members of the seminar lost the opportunity of meeting on university grounds. In the Czech secondary schools in Brno lessons went on the whole day long, so that it was impossible to organize the meetings there. For that reason the further existence of the seminar was quite impossible. E. Čech, B. Pospíšil and J. Novák used to meet at Pospíšil's place for some time to work together, but the apprehension of Pospíšil in 1941 deprived them even of this possibility.

I find in my notebook that the last seminar meeting took place on November 16th 1939, i.e. on the very eve of the forced closing of the Czech universities. This meeting marked the end of the three and a half year's excellent scientific activity of Čech's topological seminar in Brno, having been nipped at the peak of its fruitful work. The great influence left by the seminar was so deep that it will constantly remind us of the greatness of its founder's personality.

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Výtah

ČECHŮV TOPOLOGICKÝ SEMINÁŘ Z LET 1936–1939

Karel Koutský, Brno

Autor popisuje vznik a vývoj Čechova topologického semináře v Brně z let 1936 až 1939, který byl u nás prvním a zdařilým pokusem o kolektivní vědeckou práci v matematice. Zmiňuje se o problémech, které byly v semináři položeny a uvádí vcelku vyčerpávající přehled výsledků, jež byly jednotlivými účastníky dosaženy. Jako jeden z přímých účastníků semináře připojuje též několik svých osobních vzpomínek.

Резюме

ТОПОЛОГИЧЕСКИЙ СЕМИНАР Э. ЧЕХА В 1936–1939 ГГ.

КАРЕЛ КОУТСКИЙ, (Karel Koutský), Брно

Автор описывает возникновение и развитие топологического семинара в г. Брно в 1936–1939 гг., который был у нас первой и в то же время успешной попыткой коллективной научной работы в математике. Описываются поставленные в семинаре проблемы, и приводится в общем исчерпывающий обзор результатов, полученных отдельными участниками. Как один из прямых участников семинара автор присоединяет также несколько своих личных воспоминаний.