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A CONCENTRATION OF CATEGORIES WITH NON-INJECTIVE MONOMORPHISMS

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In [1] certain categories of algebraic systems are studied in which in contrast to the majority of other concrete categories non-injective monomorphisms may exist. The purpose of this contribution is to deal with this naturally arising problem:

Does there exist an isomorphic embedding of these categories to the category of sets carrying monomorphisms to injective mappings?

The answer to this problem is positive. Before we start the construction of such an embedding, we recall some definitions and results from [1].

A Brief Description of the Considered Categories

Let k, l be arbitrary but fixed ordinals and let Σ denote the class of all algebraic systems $\langle \mathbf{A}, \Omega_{\mathbf{F}}, \Omega_{\mathbf{P}} \rangle$, where $\Omega_{\mathbf{F}} = \{\mathbf{F}_i^{(n_i)} \mid i < k\}$ is an operator domain and $\Omega_{\mathbf{P}} = \{\varrho_j \mid j < l\}$ a non-empty predicate domain with binary relations only (n_i are fixed and denote the arity). We denote further $\Omega = \{\Delta\} \cup \Omega_{\mathbf{P}} \cup \{\varrho^{-1} \mid \varrho \in \Omega_{\mathbf{P}}\}$, where Δ is the identity relation on $\mathbf{A}$. Because of the simplicity the same symbols $\Omega_{\mathbf{F}}, \Omega_{\mathbf{P}}, \Omega$ are used for separate algebraic systems. That is why we shall write instead of $\langle \mathbf{A}. \Omega_{\mathbf{F}}, \Omega_{\mathbf{P}} \rangle$ only briefly $\mathbf{A}$ whenever it is clear from the context that $\mathbf{A}$ stands for an algebraic system of Σ.

An algebraic system $\mathbf{A} \in \Sigma$ is called *directed* if to each non-empty finite subset $\mathbf{M}$ of $\mathbf{A}$ there exist $a_0 \in \mathbf{A}$, $\varrho \in \Omega$ such that $a\varrho a_0$ holds for every $a \in \mathbf{M}$.

We define a category $\mathbf{U}_\Sigma(\mathfrak{A})$ in the following way:

a) Objects of the category $\mathbf{U}_\Sigma(\mathfrak{A})$ are exactly those algebraic systems $\mathbf{A} \in \Sigma$ satisfying the following three conditions:

I. $\mathbf{A}$ satisfies a prescribed collection $\mathfrak{A}$ of axioms for operations and relations. These axioms are hereditary to subsystems and direct products of systems,

II. $\mathbf{A}$ is *connected* (for each pair $a, a' \in \mathbf{A}$ is $a\varrho_1, \ldots, \varrho_n a'$ for some $\varrho_1, \ldots, \varrho_n \in \Omega$),

III. for arbitrarily chosen n-ary $\mathbf{F} \in \Omega_{\mathbf{F}} (n \geqq 1)$; $\varrho_1, \ldots, \varrho_n \in \Omega$ and $\mathscr{M} = \{\mathbf{M}_{\mathbf{A}}^p\}_{p=1,\ldots,v}$, where for every $p\mathbf{M}_{\mathbf{A}}^p$ is a family of elements $a_1^p, \ldots, a_n^p, a_1'^p, \ldots, a_n'^p$ of $\mathbf{A}$ with $a_i^p \varrho_i a_i'^p$ for $i = 1, \ldots, n$, there exist

α) terms $\mathbf{T}_0, \mathbf{T}_1, \ldots, \mathbf{T}_r$ containing some of the object variables $x_1, \ldots, x_n$, $x'_1, \ldots, x'_n, v_1, \ldots, v_q$.

β) relations $\sigma_1, \ldots, \sigma_r \in \Omega$,

γ) if necessary elements $e_1, \ldots, e_q \in \mathbf{A}$

such that

(1) $\mathbf{T}_0 = \mathbf{F}(x_1, \ldots, x_n)$ and $\mathbf{T}_r = \mathbf{F}(x'_1, \ldots, x'_n)$,

(2) if $\mathbf{A}$ is not a directed system and $\Omega_{\mathbf{F}}$ does not contain any 0-ary operation, then the object variables $v_1, \ldots, v_q$ do not occur in $\mathbf{T}_0, \ldots, \mathbf{T}_r$,

(3) for every $p = 1, 2, \ldots, v$ it holds

$$\mathbf{T}_0^p \sigma_1 \mathbf{T}_1^p \sigma_2 \ldots \sigma_r \mathbf{T}_r^p,$$

where $\mathbf{T}_i^p$ is the value of the term $\mathbf{T}_i$ with object variables x_i replaced by a_i^p, x'_i by $a_i'^p$ for $i = 1, \ldots, n$ and v_j be e_j for $j = 1, \ldots, q$.

b) Morphisms in the category $\mathbf{U}_\Sigma(\mathfrak{A})$ are exactly all strong (with respect to all relations of $\Omega_{\mathbf{P}}$) *surjective homomorphisms* of algebraic systems.

$\mathbf{U}_\Sigma^{fin}(\mathfrak{A})$ will denote the full subcategory of all finite systems.

The condition III. in the preceding definition seems to be rather comp licated But it expresses nothing else but a natural relationship between relations and operations in connected algebraic systems. In the most special cases it is nearly trivially satisfied. For example it is easy to show that each directed algebraic system of Σ satisfies III. (for, see Lemma 1.1 in [1]). Similarly for many categories obtained by a special choice of Σ and $\mathfrak{A}$ (e.g. connected partially ordered semigroups or grupoids) the truth of the condition III. is an immediate consequence of the defining axioms $\mathfrak{A}$.

In further considerations the following terms will be of importance:

1. Paths. Let $\mathbf{A} \in \Sigma$; $a_0, a_1, \ldots, a_n \in \mathbf{A}$ and $\varrho_1, \ldots, \varrho_n \in \Omega$. The pair $\alpha = [(a_0, a_1, \ldots, a_n), (\varrho_1, \ldots, \varrho_n)]$ is called a *path* in $\mathbf{A}$ if $a_{i-1}\varrho_i a_i$ holds for every $i = 1, \ldots, n$. We write $\alpha : a_0 \to a_n$ or only $\alpha : a_0 \to$.

By $\alpha^{-1} : a_n \to a_0$ we denote the path $[(a_n, a_{n-1}, \ldots, a_0), (\varrho_n^{-1}, \ldots, \varrho_1^{-1}])$. If $\mathbf{f} : \mathbf{A} \to \mathbf{B}$ is a morphism in $\mathbf{U}_\Sigma(\mathfrak{A})$, then $\mathbf{f}(\alpha)$ denotes the path $[(\mathbf{f}(a_0), \mathbf{f}(a_1), \ldots, \mathbf{f}(a_n)), (\varrho_1, \ldots, \varrho_n)]$ in $\mathbf{B}$. $(x\varrho y) \subseteqq \alpha$ means that $x = a_{i-1}$, $y = a_i$ and $\varrho = \varrho_i$ for some $i = 1, 2, \ldots, n$.

If $\alpha : a \to b$, $\beta : b \to c$ are paths in $\mathbf{A}$, then they can be joined in a natural way to a path $\alpha\beta : a \to c$.

2. Coverings. Let $\mathbf{A} \in \Sigma$ and $a \in \mathbf{A}$. The well-ordered set $\mathscr{W}_a = (\mathbf{W}_a, \alpha_0, \ldots, \alpha_i, \ldots)$ is called *covering* in $\mathbf{A}$ if $\mathbf{W}_a = \{\alpha \mid \alpha : a \to \}$ is a set of paths in $\mathbf{A}$ with the property: to each $x, y \in \mathbf{A}$, $\varrho \in \Omega_{\mathbf{P}}$ such that $x \varrho y$ there exists $\alpha \in \mathscr{W}_a$ with $(x\varrho y) \subseteqq \alpha$ and $\alpha_i : a \to \mathbf{F}_i(a, \ldots, a)$ are paths in $\mathbf{A}$ for every basic operation $\mathbf{F}_i \in \Omega_{\mathbf{F}}$. As far as $\Omega_{\mathbf{F}} = \emptyset$, we set $\mathscr{W}_a = \mathbf{W}_a$.

3. f-symmetric elements. Let $\mathbf{f} : \mathbf{A} \to \mathbf{B}$ be a morphism in the category $\mathbf{U}_\Sigma(\mathfrak{A})$.

The elements $a, a' \in \mathbf{A}$, $a \neq a'$, $\mathbf{f}(a) = \mathbf{f}(a')$ are called *f-symmetric* if the following two conditions hold:

(i) To each $x, y \in \mathbf{A}$ and $\varrho \in \Omega_{\mathbf{P}}$ such that $x\varrho y$ there exist paths $\alpha, \beta : a \to$ and $\alpha', \beta' : a' \to$ such that $(x\varrho y) \subseteqq \alpha, \beta'$ and $\mathbf{f}(\alpha) = \mathbf{f}(\alpha')$, $\mathbf{f}(\beta) = \mathbf{f}(\beta')$.

(ii) To each basic operation $\mathbf{F}_i \in \Omega_{\mathbf{F}}$ there exist paths

$$\alpha_i : a \to \mathbf{F}_i(a, \ldots, a) \qquad \text{and} \qquad \alpha_i' : a' \to \mathbf{F}_i(a', \ldots, a')$$

such that $\mathbf{f}(\alpha_i) = \mathbf{f}(\alpha_i')$.

The Most Important Results about $\mathbf{U}_\Sigma(\mathfrak{A})$.

We shall prove a stronger form of Lemma 1.2. in [1].

Lemma. *Let* $\mathbf{f} : \mathbf{A} \to \mathbf{B}$ *be a morphism in* $\mathbf{U}_\Sigma(\mathfrak{A})$. *If there are in* $\mathbf{A}$ $\alpha_a : a \to b$ *and* $\alpha_{a'} : a' \to b$ *paths with* $a \neq a'$ *and* $\mathbf{f}(\alpha_a) = \mathbf{f}(\alpha_{a'})$, *then* a, a' *are f-symmetric elements.*

Proof. Let $x, y \in \mathbf{A}$ be elements with $x\varrho y$ for some $\varrho \in \Omega$. Because of the connectedness of $\mathbf{A}$, there exists a path $\gamma : b \to x$. Hence it is clear that $\alpha = \beta = \alpha_a\gamma[(x, y), (\varrho)]$ and $\alpha' = \beta' = \alpha_{a'}\gamma[(x, y), (\varrho)]$ are the desired paths from (*i*). It remains to check (ii). Let $\mathbf{F}_i \in \Omega_{\mathbf{F}}$ be an arbitrary n-ary basic operation. For $n = 0$ we lay $\alpha_i = \alpha_a\gamma$, $\alpha_i' = \alpha_{a'}\gamma$ where $\gamma : b \to \mathbf{F}_i(a, \ldots, a) = \mathbf{F}_i(a', \ldots, a')$ is a path. For the case $n \geqq 1$ we shall prove (ii) by means of III. from the definition $\mathbf{U}_\Sigma(\mathfrak{A})$. For, let us denote $\alpha_a = [(a_0^1, \ldots, a_m^1), (\varrho_1, \ldots, \varrho_m)]$, $\alpha_{a'} = [(a_0^2, \ldots, a_m^2), (\varrho_1, \ldots, \varrho_m)]$. For every $k = 1, \ldots, m$ we apply III. in this way: For $\mathbf{F}_i, \varrho_k$ (n times) and $\mathcal{M}_k = \{\mathbf{M}_{k,\mathbf{A}}^1, \mathbf{M}_{k,\mathbf{A}}^2\}$, where for $j = 1, 2$ $\mathbf{M}_{k,\mathbf{A}}^j$ is a family of elements a_{k-1}^j (n times) and a_k^j (n times) with $a_{k-1}^j\varrho_k a_k^j$ (n times), it holds by (3) for $j = 1, 2$ and some terms $\mathbf{T}_{k,i}$ ($i = 1, 2, \ldots, r_k$) and $\sigma_{k,i} \in \Omega$: $\mathbf{F}_i(a_{k-1}^j, \ldots, a_{k-1}^j) = \mathbf{T}_{k,0}^j\sigma_{k,1}\mathbf{T}_{k,1}^j\sigma_{k,2}, \ldots, \sigma_{k,r_k}\mathbf{T}_{k,r_k}^j = \mathbf{F}_i(a_k^j, \ldots, a_k^j)$. Thus for every $k = 1, 2, \ldots, m$ we have got paths $\gamma_k^j = [(\mathbf{T}_{k,0}^j, \ldots, \mathbf{T}_{k,r_k}^j), (\sigma_{k,1}, \ldots, \sigma_{k,r_k})] : \mathbf{F}_i(a_{k-1}^j, \ldots, a_{k-1}^j) \to \mathbf{F}_i(a_k^j, \ldots, a_k^j)$ for $j = 1, 2$.

By $\mathbf{f}(\alpha_a) = \mathbf{f}(\alpha_{a'})$ we have $\mathbf{f}(a_k^1) = \mathbf{f}(a_k^2)$ and hence $\mathbf{f}(\mathbf{T}_{k,j}^1) = \mathbf{f}(\mathbf{T}_{k,j}^2)$ for $j = 0, 1, \ldots, r_k$ which says that $\mathbf{f}(\gamma_k^1) = \mathbf{f}(\gamma_k^2)$. The same holds for the joined paths

$$\gamma_1 = \gamma_1^1 \ldots \gamma_m^1 : \mathbf{F}_i(a, \ldots, a) \to \mathbf{F}_i(b, \ldots, b)$$
$$\gamma_2 = \gamma_1^2 \ldots \gamma_m^2 : \mathbf{F}_i(a', \ldots, a') \to \mathbf{F}_i(b, \ldots, b).$$

If $\delta : b \to \mathbf{F}_i(b, \ldots, b)$ is an arbitrary path, then $\alpha_i = \alpha_a\delta\gamma_1^{-1}$, $\alpha_i' = \alpha_{a'}\delta\gamma_2^{-1}$ are the searched paths satisfying $\mathbf{f}(\alpha_i) = \mathbf{f}(\alpha_i')$.

Theorem 1. (cf. Theorem 2.3. in [1]). *A morphism* $\mathbf{f} : \mathbf{A} \to \mathbf{B}$ *is a monomorphism in the category* $\mathbf{U}_\Sigma(\mathfrak{A})$ ($\mathbf{U}_\Sigma^{fin}(\mathfrak{A})$) *if and only if there are not any f-symmetric elements in* $\mathbf{A}$.

By means of this characterization theorem another interesting result was obtained for $\mathbf{U}_\Sigma(\mathfrak{A})$, namely that $\mathbf{U}_\Sigma(\mathfrak{A})$ is *locally small* (cf. Theorem 2.6. in [1]).

The Isomorphic Embedding of $\mathbf{U}_\Sigma(\mathfrak{A})$ in Set Carrying Monomorphisms to Injective Mappings

We associate with every object $A \in \mathbf{U}_\Sigma(\mathfrak{A})$ the set $\mathscr{P}(A)$ of all coverings in A. As A is connected, we can find to each $a \in A$ at least one covering $\mathscr{W}_a$ which means that $\mathscr{P}(A) \neq \emptyset$. Evidently $A_1 \neq A_2 \Rightarrow \mathscr{P}(A_1) \neq \mathscr{P}(A_2)$. Indeed, for instance to $a \in A_1 - A_2$ we can find $\mathscr{W}_a \in \mathscr{P}(A_1)$. But all paths in $\mathscr{W}_a$ are starting from a and as $a \bar{\in} A_2$, the covering $\mathscr{W}_a$ cannot belong to $\mathscr{P}(A_2)$.

Also every morphism $\mathbf{f}: A \to B$ defines a mapping $\mathscr{P}(\mathbf{f}): \mathscr{P}(A) \to \mathscr{P}(B)$ in this way: for $\mathscr{W}_a = (W_a, \alpha_0, \ldots, \alpha_i, \ldots) \in \mathscr{P}(A)$ it is $\mathscr{P}(\mathbf{f})(\mathscr{W}_a) = (\mathbf{f}(W_a), \mathbf{f}(\alpha_0), \ldots, \mathbf{f}(\alpha_i), \ldots)$ where $\mathbf{f}(W_a) = \{\mathbf{f}(\alpha) \mid \alpha \in W_a\}$. It remains to verify that $\mathscr{P}(\mathbf{f})(\mathscr{W}_a) \in \mathscr{P}(B)$.

For, let $b, b' \in B$, $\varrho \in \Omega_P$ be such that $b\varrho b'$.

As $\mathbf{f}$ is a strong homomorphism of algebraic system, there exist $a, a' \in A$ satisfying $(a, a') \in \varrho$ and $\mathbf{f}(a) = b$, $\mathbf{f}(a') = b'$. Hence for some $\alpha \in W_a$ $(a\varrho a') \subseteqq \alpha$ and thus also $(b\varrho b') \subseteqq \mathbf{f}(\alpha)$.

It is easy to see that $\mathscr{P}$ describes a functor $\mathbf{U}_\Sigma(\mathfrak{A}) \to Set$ injective on objects and morphisms. In other words $\mathscr{P}$ is an isomorphic embedding $\mathbf{U}_\Sigma(\mathfrak{A}) \to Set$.

Theorem 2. $\mathscr{P}(\mathbf{f})$ *is an injective mapping whenever* $\mathbf{f}: A \to B$ *is a monomorphism in* $\mathbf{U}_\Sigma(\mathfrak{A})$.

Proof. Let $\mathbf{f}: A \to B$ be a morphism in $\mathbf{U}_\Sigma(\mathfrak{A})$ and $\mathscr{W}_a, \mathscr{W}'_{a'} \in \mathscr{P}(A)$, $\mathscr{W}_a \neq \mathscr{W}'_{a'}$ such that $\mathscr{P}(\mathbf{f})(\mathscr{W}_a) = \mathscr{P}(\mathbf{f})(\mathscr{W}'_{a'})$. We denote $\mathscr{W}_a = (W_a, \alpha_0, \alpha_1, \ldots, \alpha_i, \ldots)$ and $\mathscr{W}'_{a'} = (W'_{a'}, \alpha'_0, \alpha'_1, \ldots, \alpha'_i, \ldots)$. We must distinguish two cases.

(1) Let $a \neq a'$. By definition of $\mathscr{W}_a$ to each $x, y \in A$, $\varrho \in \Omega_P$ where $x\varrho y$ we can find a path $\alpha \in W_a$ such that $(x\varrho y) \subseteqq \alpha$. As $\mathbf{f}(\alpha) \in \mathbf{f}(W_a) = \mathbf{f}(W'_{a'})$, we can write $\mathbf{f}(\alpha) = \mathbf{f}(\beta)$ where $\beta \in W'_{a'}$. Considering the symmetry, we have just proved (i) for a, a'. By $\mathscr{P}(\mathbf{f})(\mathscr{W}_a) = \mathscr{P}(\mathbf{f})(\mathscr{W}'_{a'})$ (ii) holds, too. Thus a, a' are f-symmetric elements in A which means by Theorem 1 that $\mathbf{f}$ is not a monomorphism.

(2) Let $a = a'$. Owing to $\mathscr{W}_a \neq \mathscr{W}'_a$ it holds $W_a \neq W'_a$ or $\alpha_i \neq \alpha'_i$ for some i. In case $W_a \neq W'_a$ at least one of these sets contains a path not belonging to the other one. Without loss of generality we can suppose $\alpha \in W_a - W'_a$. But $f(\alpha) \in \mathbf{f}(W'_a)$ in view of $\mathbf{f}(W_a) = \mathbf{f}(W'_a)$ and therefore there exists a path $\alpha' \in W'_a$ such that $\mathbf{f}(\alpha) = \mathbf{f}(\alpha')$. $\alpha \bar{\in} W'_a \Rightarrow \alpha \neq \alpha'$. In case $W_a = W'_a$ and $\alpha_i \neq \alpha'_i$ for some i, the paths α_i, α'_i play the role of α, α'. As α, α' are different paths starting from the same element a and $\mathbf{f}(\alpha) = \mathbf{f}(\alpha')$, then by Lemma there exist f-symmetric elements in A. This means again that $\mathbf{f}$ is not a monomorphism.

Restriction of $\mathscr{P}$ on $\mathbf{U}^{fin}_\Sigma(\mathfrak{A})$ gives also an isomorphic embedding $\mathbf{U}^{fin}_\Sigma(\mathfrak{A}) \to Set$ carrying monomorphisms to injective mappings. As we can see card $\mathscr{P}(A) \geqq \aleph_0$ for each $A \in \mathbf{U}^{fin}_\Sigma(\mathfrak{A})$. We ask:

Does there exist an embedding $\mathbf{U}^{fin}_\Sigma(\mathfrak{A}) \to Set$ *carrying monomorphisms to injective mappings and satisfying* card $\mathscr{P}(A) < \aleph_0$ *for each* $A \in \mathbf{U}^{fin}_\Sigma(\mathfrak{A})$?

The answer to this question is generally negative. We show this in the category $\mathbf{O}^{fin}(\leqq)$ of all *finite connected partially ordered sets*. For, let us consider a sequence of morphisms $\mathbf{f}_n: A_n \to B$ $(n = 1, 2, 3, \ldots)$ where A_n, B are connected partially ordered sets from fig. 1 and $\mathbf{f}_n$ are defined in this way:

$$\mathbf{f}_n(a_i^j) = b_i \qquad \text{for } i = 1, 2, \ldots, 5 \text{ and } j = 1, 2, \ldots, n$$

All $\mathbf{f}_n$ $(n = 1, 2, \ldots)$ *are monomorphisms:*

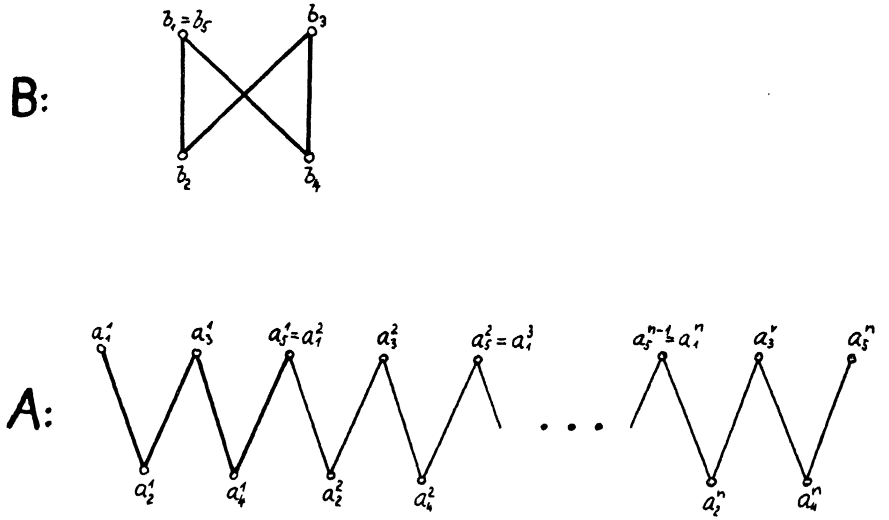

Fig. 1

By Theorem 1 it is sufficient to show that for each n there do not exist $\mathbf{f}_n$-symmetric elements $a, a' \in \mathbf{A}_n$. If this were the case, then it would have to be $a = a_i^j$, $a' = a_i^{j'}$ for some i, j, j' where $j \neq j'$ (let us suppose $j' > j$). Let

$$\alpha = \left[(a_0, \ldots, a_m), (\varrho_1, \ldots, \varrho_m)\right] : a \rightarrow$$

and

$$\alpha' = \left[(a'_0, \ldots, a'_m), (\varrho_1, \ldots, \varrho_m)\right] : a' \rightarrow$$

be paths in $\mathbf{A}_n$ satisfying $\mathbf{f}_n(\alpha) = \mathbf{f}_n(\alpha')$. In $\mathbf{O}^{fin}(\leqq)$ this means that $\varrho_k = \leqq$ or $\varrho_k = \geqq$ and thus $a_{k-1} \leqq a_k$, $a'_{k-1} \leqq a'_k$ or $a_{k-1} \geqq a_k$, $a'_{k-1} \geqq a'_k$ for $k = 1, 2, \ldots, m$. Since $\mathbf{f}_n(a_k) = \mathbf{f}_n(a'_k)$ for every $k = 0, 1, \ldots, m$, the a_k, a'_k are of the form $a_k = a_p^q$, $a'_k = a_p^{q'}$ where $q + j' - j = q'$ (i.e. $q < q'$). In view of this fact it cannot be $(a_2^1 \leqq a_3^1) \subseteqq \alpha'$ and so (i) is not true.

For each isomorphic embedding $\mathscr{P} : \mathbf{O}^{fin}(\leqq) \rightarrow Set$ carrying monomorphisms to injective mappings it holds for $n = 1, 2, \ldots$:

(1) card $\mathscr{P}(\mathbf{A}_n) \leqq$ card $\mathscr{P}(\mathbf{B})$ because $\mathscr{P}(\mathbf{f}_n)$ are injective,

(2) card Hom $(\mathbf{A}_n, \mathbf{B}) \leqq$ card Hom $(\mathscr{P}(\mathbf{A}_n), \mathscr{P}(\mathbf{B}))$ because $\mathscr{P}$ is injective on morphisms.

We define for each $n = 1, 2, \ldots$ and $j = 1, \ldots, n$ a strong surjective homomorphism $\mathbf{g}_{n,j} : \mathbf{A}_n \to \mathbf{B}$ in this way: $\mathbf{g}_{n,j}(a_i^j) = b_i$ and $\mathbf{g}_{n,j}(a_i^k) = b_1$ for $k \neq j$ ($i = 1, \ldots, 5$). Evidently $\mathbf{g}_{n,j} \neq \mathbf{g}_{n,j'}$ for $j \neq j'$ and thus card Hom $(\mathbf{A}_n, \mathbf{B}) \geqq n$. Together with (2) we have card Hom $(\mathscr{P}(\mathbf{A}_n), \mathscr{P}(\mathbf{B})) \geqq n$ for $n = 1, 2, \ldots$ which means by (1) that card $\mathscr{P}(\mathbf{B}) \geqq \aleph_0$.

REFERENCES

[1] Kopeček, I., Veselý, V.: *Monomorphisms in the categories of connected algebraic systems with strong homomorphisms*. Arch. Mat. XII (1976), 153—168.

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