Commentationes Mathematicae Universitatis Carolinae

Svatopluk Poljak

Maximum rank of a power of a matrix of a given pattern

Commentationes Mathematicae Universitatis Carolinae, Vol. 29 (1988), No. 1, 195

Persistent URL: http://dml.cz/dmlcz/106609

ANNOUNCEMENTS OF NEW RESULTS

(of authors having an address in Czechoslovakia)

MAXIMUM RANK OF A POWER OF A MATRIX OF A GIVEN PATTERN

Svatopluk Poljak (Dept. of Applied Mathematics, Charles University, Malostranské nám. 25, 11800 Praha 1, Czechoslovakia), received 17.10. 1987.

Let G be a digraph with possible loops and without multiple edges. A t-walk is a sequence $w=(v_0,e_1,v_1,\dots,v_{t-1},e_t,v_t)$ of (not necessarily distinct) vertices and edges of G such that $e_i=v_{i-1}v_i$ is a directed edge for each i. We say that two t-walks w and w′ are vertex (edge) independent if $v_i \neq v_i'$ for $i=0,\dots,t$ ($e_i \neq e_i'$ for $i=1,\dots,t$). A path is a walk with $v_i \neq v_j$ for $i \neq j$, and a cycle is a walk with distinct vertices but $v_0=v_t$. We denote by $|P|$ and $|C|$ the number of vertices of a path or a cycle.

Theorem. For every digraph G and a positive integer p there are mutually vertex disjoint cycles $C_1,\dots,C_l$ and paths $P_1,\dots,P_k$ such that the maximum number of vertex independent p-walks equals $\sum_{i=1}^{l}|C_i| + \sum_{i=1}^{k}(|P_i|-p)$.

The above theorem may be interpreted as follows. Let m be the maximum number of people who could simultaneously walk in digraph G for p time units traversing one edge per a unit so that two or more people never meet in a vertex. Then the optimal schedule can always be organized as follows. The people are divided into several subgroups and each subgroup either walks round a cycle or along a path.

Let $A=(a_{ij})$ be a real matrix of size n by n. The pattern of G is the digraph on vertices $\{1,2,\dots,n\}$ and with an edge ij if $a_{ij} \neq 0$. For a digraph G let $\mathcal{A}(G)$ be the class of matrices of pattern G.

Corollary 1. Maximum possible rank of the p-th power A^p of a matrix A of a given pattern G equals maximum number of vertex independent p-walks in G. For a symmetric digraph G, we denote by $\mathcal{S}(G)$ the class of symmetric matrices of pattern G.

Corollary 2. $\max_{A\in\mathcal{S}(G)} rA^p = \max_{A\in\mathcal{A}(G)} rA^p$ for every symmetric digraph and a positive integer p.

The following Corollary 3 answers a question by J. Holenda who proved the case p=2.

Corollary 3. $\max rA^p = \max r(A_1,\dots,A_p)$ where $A,A_1,\dots,A_p$ are matrices of pattern G.

I thank J. Kratochvíl for a valuable discussion about the problem.

ORDINAL TYPES IN RAMSEY THEORY AND WELL-PARTIAL-ORDERING THEORY

Igor Kříž (matematicko-fyzikální fakulta UK, Malostranské nám. 25, 11800 Praha 1, Czechoslovakia), Robin Thomas (Matematicko-fyzikální fakulta UK, Sokolovská 83, 18600 Praha 8, Czechoslovakia), received 21.1. 1988.

There is a big gap between the infinite Ramsey theorem

(+) $$\omega \longrightarrow (\omega)^n_k$$

and its finite version

$$R(n;l_1,\dots,l_k) \longrightarrow (l_1,\dots,l_k)^n_k$$