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is quasi-continuous up to the boundary extended by the values of f. This function h coincides with the "Perron solution" of the considered Dirichlet problem.

Theorem B. Let U be a finely open set. Let u be a quasi-l.s.c. and finely l.s.c. function on U. Suppose that for every $x \in U$ there is a fundamental system of fine neighborhoods V of x with the property $\varepsilon_x^{CV}(u) \leq u(x)$. Then u is finely hyperharmonic on U.

The results of the dissertation are published in [2].

References:

[1] N. BOBOC, Gh. BUCUR, A. CORNEA: Order and Convexity in Potential Theory: H-Cones. Lecture Notes in Mathematics 853, Springer-Verlag, Berlin-Heidelberg-New York 1981.

[2] J. LUKEŠ, J. MALÝ, L. ZAJÍČEK: Fine Topology Methods in Real Analysis and Potential Theory. Lecture Notes in Mathematics 1189, Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo 1986.

SOME ASPECTS OF CONVEX ANALYSIS AND THE THEORY OF ASPLUND SPACES

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Theorem 1.28 and Corollary 2.3 in [1] form a mechanism in which Fréchet differentiability works. We show, using methods of convex analysis, that this differentiability can be replaced by any $\mathcal{A}$-differentiability having the property (m) defined below. For instance, Gâteaux differentiability on separable Banach spaces can be included in this mechanism.

We say that a family $\mathcal{A}$ of bounded subsets of a Banach space X is a generating system if (i) $A \in \mathcal{A}$ implies $-A \in \mathcal{A}$ and (ii) the span of the set $\bigcup\{A : A \in \mathcal{A}\}$ is dense in X. A function $f: X \longrightarrow R$ is said to be $\mathcal{A}$-differentiable at a point $x \in X$ if there exists an element x^* (called an $\mathcal{A}$-derivative of f at x and denoted by $\mathcal{A}$-df(x)) in the dual Banach space X^* such that the relation

$$\lim_{t \downarrow 0} \sup_{h \in A} |t^{-1}(f(x+th)-f(x)) - \langle h, x^* \rangle| = 0$$

is satisfied for all A in $\mathcal{A}$. We denote by $\mathcal{T}_{\mathcal{A}}$ the topology of uniform convergence on members of $\mathcal{A}$ for the set X^*. We say that $\mathcal{A}$ has the property (m) if the topology $\mathcal{T}_{\mathcal{A}}|M$ is metrizable for each set $M \subset X^*$.

Theorem 1. Let $\mathcal{A}$ be a generating system having the property (m). Then the following statements (a) and (b) are equivalent.

(a) $\{x \in X : \mathcal{A}\text{-}df(x) \text{ exists}\}$ is a dense G_δ-subset of X for every continuous convex function $f: X \longrightarrow R$.

(b) For every pair [M,V], where $M \subset X^*$ is bounded and nonempty and V is a $\mathcal{T}_{\mathcal{A}}$-neighbourhood of the point $0 \in X^*$, there exists a weak* open set $W \subset X^*$ such that $M \cap W \neq \emptyset$ and $M \cap W - M \cap W \subset V$.

We say that X is an almost Asplund space if there exists a generating system $\mathcal{A}$ having the property (m) so that (a) or

(b) holds.

Theorem 2. Let Y, Z be Banach spaces and $T: X \longrightarrow Y$ be a continuous linear operator with dense range. If X and Z are almost Asplund spaces then the same holds for Y and $X \times Z$.

Every Asplund and wcg Banach space is an almost Asplund spa- and every almost Asplund space is in the class $\mathcal{K}$ defined in [2]. The results communicated in [2] form a part of the defended work.

References:

[1] R.R. PHELPS: Differentiability of Convex Functions on Banach spaces, Lecture Notes, Univ .College London, 1978.

[2] L. JOKL: On a class of weak Asplund spaces which has some permanence properties, Comment.Math.Univ.C arolinae 27(1986), 205-206.