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Luděk Jokl

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Using the result of [1] concerning the convexity of $\overline{R(A)}$ we get

Corollary 1. Let X be a reflexive rotund (H)-Banach space which is uniformly Gâteaux smooth (or equivalently X^* is weakly* uniformly rotund), $A:X \to 2^X$ an m-accretive mapping with $D(A) \subset X$. Then $\lim_{\lambda \to +\infty} \frac{1}{\lambda} J_\lambda(u) = -a^0$ for each $u \in D(A)$, where a^0 is a unique point of $\overline{R(A)}$ with the minimum norm.

As a further consequence of Thm. 2 we obtain the result of [6] concerning maximal monotone mappings in Hilbert spaces.

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MINIMAL CONVEX-VALUED WEAK USCO CORRESPONDENCES

Luděk Jokl (ČVUT Praha, Thákurova 7, 16629 Praha 6, Czechoslovakia),

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We say that a function $f:V \to R$ defined on a vector space V is rotund if it is convex and $f((u+v)/2) < t$ whenever $u,v \in V$, $u \neq v$ and $f(u)=t=f(v)$. In what follows X will be a real Banach space.

Theorem 1. If there exists a weak* lower semicontinuous rotund function $f:X^* \to R$, then X belongs to the Stegall class $\mathcal{S}$.

We denote by $w*$ the weak* topology for any dual Banach space. Let D be a topological space. Then we write $F \in USCOC(F,(X^*,w^*))$ if and only if, using the weak* topology, F is a convex-valued usco correspondence from D into X^*. The set $USCOC(D,(X^*,w^*))$ is partially ordered with order $\leq$, where $E \leq F$ iff $E(d) \subset F(d)$ for each $d \in D$. We denote by $uscoc(D,(X^*,w^*))$ the set of all minimal elements of $USCOC(D,(X^*,w^*))$.

Theorem 2. Let $T:X \to X^*$ be a maximal monotone operator and D be an open subset of X. If $Tx \neq \emptyset$ for all x in D then $T|D \in uscoc(D,(X^*,w^*))$.

If F is a correspondence from D into X^* then we define the set $C(F,D,X^*)$ as follows: $d \in C(F,D,X^*)$ if and only if $d \in D$ and, using the norm topology, F is upper semicontinuous and single-valued at d. In the following theorem X will be regarded as a closed vector subspace of X^{**}.

Theorem 3. Let K be a closed convex subset of X. Then K has the Radon-Nikodým property if and only if the set $C(F,D,X^{**})$ is dense in D whenever D is a Baire space, $F \in uscoc(D,(X^{**},w^*))$ and the set $F^{-1}(K)$ is dense in D.

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