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### ON ACCRETIVE MULTIVALUED MAPPINGS

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Let  $X$  be a real normed linear space,  $X^*$  its dual,  $\langle , \rangle$  the pairing between  $X$  and  $X^*$ ,  $J$  a duality mapping from  $X$  into  $2^{X^*}$  defined by  $J(u) = \{u^* \in X^* : \langle u^*, u \rangle = \|u\|^2, \|u^*\| = \|u\|\}$ ,  $u \in X$ .

Recall that a multivalued mapping  $A: X \rightarrow 2^X$  is said to be: (i) accretive on  $D(A) = \{u \in X : A(u) \neq \emptyset\}$  if for each  $u, v \in D(A)$  and each  $x \in A(u)$  and  $y \in A(v)$  there exists an element  $x^* \in J(u-v)$  such that  $\langle x-y, x^* \rangle \geq 0$ ; (ii) maximal accretive on  $D(A)$ , if  $A$  is accretive on  $D(A)$  and its graph  $G(A) = \{(u, x) \in X \times X : u \in D(A), x \in A(u)\}$  is not properly contained in the graph of any other accretive mapping defined on  $D(A)$ .

Theorem. Let  $X$  be a reflexive Fréchet smooth Banach space,  $A: X \rightarrow 2^X$  a multivalued maximal accretive mapping such that  $\text{int } D(A) \neq \emptyset$ . Then  $A$  is single-valued and norm-to-norm upper semicontinuous on a dense  $G_\delta$  subset of  $\text{int } D(A)$ .

In comparison with maximal monotone operators (see for instance [1], [2], [3]), the single-valuedness and the continuity properties of maximal accretive mappings ([4]) deeply rely on the structure of Banach spaces.

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