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ON RACIAL INSUFFICIENCY: WHITE MESSENGERS CANNOT SIMULATE COLOURED ONES

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Abstract: At the Colloquium on Computation Theory 1984 in Poznań the question arised, whether for any family of mice with coloured (e.g. mutually distinguishable) pebbles there exists a family (of equal size) of mice with (the same number and distribution of) white (e.g. undistinguishable) pebbles which simulates the first one. A negative answer is presented.

Key words: Automata, labyrinths, simulation.

Classification: 68D99

Construction: We will describe a pair of mice called She and He, each of them able to recognise and manipulate red and green pebble in 1-dimesional labyrinths (i.e. indeed in segments of integral line.)

He (See Fig 1.) moves to the left end, turns there and

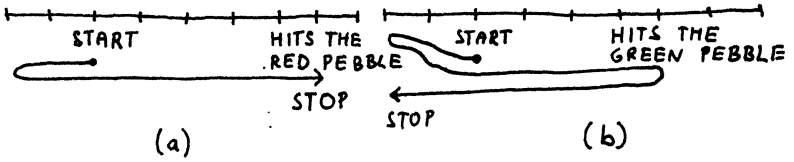

Fig 1.

moves to the right; if He hits a red pebble He stops (Fig 1a), if He hits a green one He returns to the left end and stops there (Fig 1b).

She (See Fig 2.) moves to the right end. There She puts a pebble to the left from Herself, the green one if the distance from the start to the left point was even, the red one otherwise. The other pebble is deposited to the right. Then She alternately

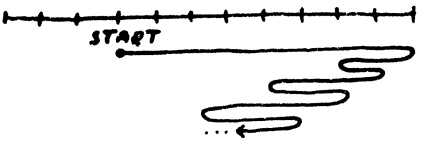

Fig 2. Her trajectory.

visits Her pebbles. Each time She visits the left or right pebble She moves it two steps or one step, respectively, to the left.

Test: Our tests consist at starting She and He simultaneously at some point of some finite 1-dimensional labyrinth. At the start She has both the red and the green pebble and He has none.

Simulation: A pair Simshe, Simhe of mice, each being able to recognize and manipulate white pebbles in 1-dimensional labyrinths is said to be simulating if there exists a natural number r such that if Simhe, Simshe are started in the same labyrinth and in the same point as She and He, Simshe having two white pebbles and Simhe none, they are in time rt in positions where She and He are in time t.

Theorem: There is no simulating pair.

Proof: Let us suppose there is. Then we take our test to guarantee that when He approaches the left pebble of Her the pebble is sufficiently distant from the left end of the labyrinth and from Her as well (this is always possible). Now let us have a look at Simhe simulating this moment. Simhe is moving rightwards in periodical loops. We choose our test in such a way that all irregularities are distant enough. The only thing to change Simhis behaviour is a white pebble waiting there. But indeed a reaction of Simhe is completely coded in Simhe and does not depend on the position of the white pebble within diameter r (which is enough). Thus Simhe must either stop or go back regardless the parity of distance from the start to the right end and the message is lost.

Remarks: We showed that coloured messengers are more powerful than white ones even in 1-dimension and when only

2 pebbles are present. This is, evidently, a border case. If one wants to define simulation regardless the number (r in our case) of steps simulating one step of an original mouse one enters a variety of problems. First, our example can be modified to deal with the question; however, 2-dimensional and infinite labyrinths are needed for a straightforward construction. What happens in finite labyrinths only seems to be a rather tricky problem. For the simulation in the case when the pebbles are not to be messengers, each of the mice uses only its own pebbles and the problem consists in avoiding misinterpretation see [Kriegel/Kříž/Úlehla] and [Pultr/Úlehla].

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