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Commentationes Mathematicae Universitatis Carolinae, Vol. 26 (1985), No. 1, 169--172

Persistent URL: http://dml.cz/dmlcz/106353

COMMENTATIONES MATHEMATICAE UNIVERSITATIS CAROLINAE

26,1 (1985)

GLOBAL (IN TIME) SOLUTION OF THE APPROXIMATE NON-LINEAR STRING EQUATION OF G. F. CARRIER AND R. NARASIMHA

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Abstract: Known results and open problems for the local/global solvability of the nonlinear string equation proposed by G.F. Carrier and R. Narasimha.

Key words: Approximate nonlinear string equation, global/local solution.

Classification: 73D35, 34G20, 45K05

We give a brief survey on the initial-boundary value problem for a non-linear integrodifferential equation introduced by S. Bernstein [B]:

$$(1)\quad \begin{cases} u_{tt} = m\left(\int_\Omega |u_x(x,t)|^2 dx\right) \Delta_x u \text{ for } x \in \Omega ,\ t \geq 0 \\ u(x,t) = 0 \quad \text{for } x \in \partial\Omega ,\ t \geq 0, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x). \end{cases}$$

Here Ω is an open subset of $\mathbb{R}^n$ and $m(r)$ is a continuous function such that $m(r) \geq 0$. For the choice $\Omega =]0,L[$ and

$$(2)\qquad m(r) = c_0^2 + \varepsilon r \quad (r \geq 0),$$

(1) gives the approximate model of G.F. Carrier [C] and R. Narasimha [N] for the free transversal vibration of a string

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This paper was presented in written form on the International Spring School on Evolution Equations, Dobřichovice by Prague, May 21-25, 1984.

clamped at its ends ($c_o = \sqrt{T_o/\rho_o}$ and $\varepsilon = E/2L\,\rho_o$, where $T_o \geq 0$ is the tension in the rest position, ρ_o is the linear density of the string and E is Young's modulus).

Known results

I) Uniqueness/local existence: there exists a unique solution of (1) on some interval $[0,T[$ provided that

$$(3)\quad \begin{cases} m \text{ is a lipschitzian function with } m(r) \geq \nu > 0; \\ u_o \in H_o^1(\Omega) \cap H^{2,2}(\Omega) \text{ and } u_1 \in H_o^1(\Omega) \\ \text{(cf. [B],[D],[M],[R]).} \end{cases}$$

II) Global existence: there exists a solution of (1) on $[0,+\infty[$ in each one of the following cases:

$$(4)\quad \begin{cases} \Omega = \mathbb{R}^1;\ m \text{ is as in (2) with } c_o > 0;\ u_o \text{ and } u_1 \text{ are small} \\ (O(c_o/\sqrt{\varepsilon}\,)) \text{ in a suitable weighted } H^{2,2} \text{ space with po-} \\ \text{lynomial weight functions (cf. [GH]).} \end{cases}$$

$$(5)\quad \begin{cases} \int_0^{+\infty} m(r)\,dr \text{ is divergent; for } i = 0,1,\ \Delta_x^j u_i \in H^1(\Omega) \\ \text{for each } j \in \mathbb{N} \text{ and there exists } s > 0 \text{ such that } u_i \text{ is} \\ \text{extendable to a holomorphic function on the complex} \\ \text{neighbourhood} \\ \Omega_s = \{z \in \mathbb{C}^n : d(z,\Omega) \leq s\}, \text{ with } u_i \in L^2(\Omega_s) \\ \text{(cf. [B],[P],[L],[AS],[A],[S]).} \end{cases}$$

Open questions

A) It would be interesting to exhibit a counterexample to global existence for (1) (at the present, no blow up phenomenon is explicitly known).

B) One could investigate existence/uniqueness for arbitrary initial data of finite energy, i.e. merely $u_0 \in H_0^1(\Omega)$ and $u_1 \in L^2(\Omega)$ (in such a generality no result is known but for the trivial case when m = const.).

References

[A] A. AROSIO: Portugaliae Math. 41(1982), to appear.

[AS] A. AROSIO, S. SPAGNOLO: Global solutions of the Cauchy problem for a nonlinear hyperbolic equation, in "Non-linear Partial Differential Equations and their Applications. Collège de France Seminar, Vol. 109", H. Brezis and J.L. Lions ed., Research Notes in Mathematics, Pitman, Boston, 1983.

[B] S. BERNSTEIN: Izv. Akad. Nauk SSSR, Ser. Mat. 4(1940), 17-26 (MR 2 # 102).

[C] G.F. CARRIER: Quart. Appl. Math. 3(1945), 157-165 and 7(1949), 97-101.

[D] R.W. DICKEY: Proc. Amer. Math. Soc. 23(1969), 459-468; Proc. Roy. Soc. Edinburgh Sect. A 82(1978/79), No. 1-2, 19-26.

[GH] J.M. GREENBERG, S.C. HU: Quart. Appl. Math. (1980), 289-311.

[L] J.L. LIONS, in "Contemporary developments in continuum mechanics and partial differential equations (G.M. De la Penha and L.A. Medeiros eds.)", North-Holland, London, 1978.

[M] G.P. MENZALA: Nonlinear Anal. 3(1979), No. 5,613-627.

[N] R. NARASIMHA: J. Sound Vibration 8(1968), 134-146.

[P] S.I. POHOŽAEV: Mat. USSR Sbornik 25,No. 1(1975), 145-158 (transl.).

[R] P.H. RIVERA RODRIGUEZ: Appl. Anal. 10(1980), 93-104.

[S] S. SPAGNOLO: Séminaire Vaillant (1982/83), Hermann, Paris.

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(Oblatum 25.5. 1984)