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REMARKS ON CERTAIN UNIFORM COVERING PROPERTIES

Jan FRIED and Aarno HOHTI

__Abstract:__ A uniform space is called σ-discretely refinable if every open cover of the space has a σ-uniformly discrete refinement. This paper deals with some special covering properties connected with σ-discrete refinability. We give a characterization of σ-discrete refinability and show that σ-discretely refinable, uniformly countably paracompact spaces are supercomplete.

__Key words:__ σ-uniformly discrete, paracompact, uniformly paracompact, hypercozero-set, supercomplete.

Classification: 54E15

1. __Introduction.__ As paracompactness is one of the most important and fruitful concepts of general topology, it is natural to consider paracompactness in uniform spaces. (See [5],[6],[10], and [18].) In this paper we shall consider σ-discretely refinable spaces that were studied in [6] and [9]. The results concern the relation of σ-discrete refinability to other covering properties.

We refer the reader to [14] for information on uniform spaces. If μ and ν are uniformities on X, then μ/ν denotes the collection of all covers of X with a refinement of the form $\{U_i \cap V^i_j\}$ where $\{U_i\} \in \mu$ and $\{V^i_j\} \in \nu$ for each

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i. The precompact reflection of μX will be denoted by $p\mu X$. The fine uniformity (resp. the fine space) associated with a completely regular space X is denoted by $\mathcal{F}(X)$ (resp. $\mathcal{F}X$). A uniform space is uniformly paracompact if every open cover $\mathcal{V}$ of X has a uniformly locally finite open refinement, or, equivalently, the cover $\mathcal{V}^{<\omega}$ consisting of all unions of finite subsets of $\mathcal{V}$ is always uniform. (See [18].) Analogously a space μX is uniformly countably paracompact if every countable open cover of X has a uniformly locally finite open refinement. A space is uniformly para-Lindelöf if every open cover of the space has a uniformly locally countable open refinement.

A collection $\mathcal{U}$ of subsets of X is called σ-uniformly discrete if it is a countable union of its uniformly discrete subcollections. A uniform space μX is called σ-discretely refinable if every open cover of X has a σ-uniformly discrete refinement.

The symbol $C(\mu X)$ (resp. $C(X)$) denotes the set of all uniformly continuous (resp. continuous) real-valued functions on μX (resp. on X) and $C\mu$ (resp. C) denotes the uniformity on X with the sub-basis

$$\{f^{-1}(\mathcal{U}) : \mathcal{U} \text{ is a uniform cover of } R,\ f \in C(\mu X),$$
$$(\text{resp. } f \in C(X))\}.$$

A hypercozero-set of the first class has the form $\bigcup \mathcal{H}$, $\mathcal{H}$ being a σ-uniformly discrete family ranging in $\operatorname{coz}(\mu X) = \{\operatorname{coz} f : f \in C(\mu X)\}$. (See [8], page 56.)
The collection of all hypercozero-sets of the first class will be denoted by $h^{(1)}\operatorname{coz}(\mu X)$.

2. Two characterizations. Z. Frolík proved that a uniform space μX is σ-discretely refinable if and only if, any two closed and disjoint subsets of $X \times \beta X$ can be separated by members of $h^{(1)}coz(\mu X \times \mathcal{T} \beta X)$. Our first remark is concerned with replacing $h^{(1)}coz$ - normality by an equivalent covering condition. In the following lemma, m denotes the metric-fine coreflection and $\nu^{(1)}$ denotes the Ginsberg-Isbell derivative of a uniformity ν, $\nu^{(1)} = \nu/\nu$.

Lemma 2.1.: Let μX be a uniform space. Then

$$h^{(1)}coz(\mu X) = coz((m\mu)^{(1)}X).$$

Proof: First recall that $m\mu$ is a point-finite uniformity and hence $(m\mu)^{(1)}$ is a uniformity. To show that $h^{(1)}coz(\mu X) \subset coz((m\mu)^{(1)}X)$, it is enough to show that $H = \bigcup\{H_a : a \in A\} \in coz((m\mu)^{(1)}X)$, provided $\{H_a\}$ is a uniformly discrete family ranging in $coz(\mu X)$. Take for each $a \in A$ $f_a \in C(\mu X)$ such that $H_a = coz\ f_a$. Clearly $f = \sum f_a$ is uniformly continuous on $(m\mu)^{(1)}X$. Thus, $H \in coz((m\mu)^{(1)}X)$, since $H = coz\ f$.

On the other hand, let $H \in coz((m\mu)^{(1)}X)$. By [8] there is a sequence $\{\mathcal{U}_n\}$ of elements of $(m\mu)^{(1)}$ such that

$$H = \bigcup\{\bigcup\{U : U \in \mathcal{U}_n \text{ and } St(U,\mathcal{U}_n) \subset H\} : n \in \omega\}.$$

We may and shall suppose that $\mathcal{U}_n$ are of the form

$$\mathcal{U}_n = \{W \cap V : W \in \mathcal{W}_n, V \in \mathcal{V}_W\},$$

$\mathcal{W}_n$, $\mathcal{V}_W$ being σ-uniformly discrete completely $coz(\mu X)$-additive covers, since such covers form a basis in $m\mu X$.

Define for $W \in \mathcal{W}_n$

$$\hat{W} = \bigcup\{W \cap V : V \in \mathcal{V}_W, W \cap V \subset H\}.$$

It is clear that for each n $\{\hat{W} : W \in \mathcal{W}_n\}$ is a σ-uniformly discrete family ranging in $coz(\mu X)$. Thus, $H \in h^{(1)}coz(\mu X)$, since $H = \bigcup\{\bigcup\{\hat{W}: W \in \mathcal{W}_n\}: n \in \omega\}$.

We say that a uniform space μX satisfies the condition $(*)$ if for every pair A, B of closed and disjoint subsets of X there is a sequence $\{\mathcal{U}_n\}$ in $\mu^{(1)}$ such that

$$\bigcap\{St(A, \mathcal{U}_n): n \in \omega\} \cap \bigcap\{St(B, \mathcal{U}_n): n \in \omega\} = \emptyset.$$

Remark: The spaces satisfying the condition $(*)$ are normal. Indeed, if A, B are closed disjoint subsets of X and there exists a normal sequence $\{\mathcal{U}_n\}$ with the intersection property given above, then A and B have disjoint closures in the pseudometric space corresponding to this normal sequence.

Lemma 2.2.: Let μX be a uniform space. Then X satisfies the condition $(*)$ if and only if, any two disjoint closed subsets of X can be separated by hypercozero-sets of the first class.

Proof: For sufficiency, one can easily see that X is a normal space and $h^{(1)}coz(\mu X) = coz(\mathcal{T}X)$. Denote by $D\mu X$ the distal modification of μX, generated by all finite-dimensional uniform covers of X. Since μX and $D\mu X$ have the same uniformly discrete families and $coz(\mu X) = coz(D\mu X)$, it is clear that $h^{(1)}coz(\mu X) = h^{(1)}coz(D\mu X)$. Following the proof of Lemma 2.1 one can prove that $h^{(1)}coz(\mu X) \subset coz((D\mu)^{(1)}X)$. Let A and B be closed disjoint subsets of X. It is easily seen that the cover $\mathcal{U} = \{X - A, X - B\} \in m(D\mu)^{(1)}X$. Thus there exists a uniformly continuous mapping $f: (D\mu)^{(1)}X \to (M, \varphi)$ into some metric space and an open cover $\mathcal{V}$ of M such that $f^{-1}(\mathcal{V}) < \mathcal{U}$. Let $\mathcal{V}_n$ be a cover of (M, φ) by 1/n balls. Then the desired sequence is $\{f^{-1}(\mathcal{V}_n)\}$.

On the other hand, suppose $(*)$. For any disjoint closed sets A and B we may, of course, construct a normal sequence $\{\mathcal{V}_n\}$ of covers in $(m\mu)^{(1)}$ satisfying the separation property from $(*)$. Thus, A and B have disjoint closures in the uniformly continuous pseudometric corresponding to this sequence. Hence, A and B can be separated by sets belonging to $coz(m\mu)^{(1)}X = h^{(1)}coz(\mu X)$.

We obtain the following corollary.

Theorem 2.3.: Let μX be a uniform space. The following statements are equivalent:

(i) X is σ-discretely refinable;

(ii) $X \times \mathcal{T} \beta X$ satisfies the condition $(*)$.

Proof: The claim follows immediately from the results of [8], where it was we proved μX is σ-discretely refinable iff any two disjoint closed subsets of $\mu X \times \mathcal{T} \beta X$ can be separated by a hypercozero-set of the first class.

Proposition 2.4.: Let μX be a uniform space. Then the following statements are true:

(i) X satisfies the condition $(*)$ iff for every pair A and B of closed disjoint subsets of X there exist a closed cover $\{F_n\}$ of X and a sequence $\{\mathcal{U}_n\}$ of covers in $\mu^{(1)}$ such that for each n
$St(A, \mathcal{U}_n) \cap St(B\ \mathcal{U}_n) \cap F_n = \emptyset$

(ii) μX is σ-discretely refinable iff for every open cover $\mathcal{V}$ there exists a closed countable cover $\{F_n\}$ of X such that for each n the restriction
$\mathcal{V} \restriction F_n$ is a uniform cover of F_n.

Proof: Exercise.

Remark: A uniform space μX is C-normal [10] if every finite open cover of X belongs to $\mu^{(1)}$. The above proposition and Theorem 2.3 show that the condition (*) is related to C-normality as σ-refinability is related to uniform paracompactness. (Recall that a topological space X is called subnormal [2] if any two disjoint closed subsets of X can be separated by G_δ-sets.)

$$\begin{array}{ccc} \sigma\text{-discretely refinable} & \longrightarrow & h^{(1)}\text{coz-normal} \\ \vdots & & \vdots \\ \text{uniformly paracompact} & \longrightarrow & \text{C-normal} \end{array}$$

Lemma 2.5.: Let μX be a uniformly countably paracompact, σ-discretely refinable metric-fine uniform space. Then μX is uniformly paracompact.

Proof: it is enough to show that $\mathcal{V}^{<\omega}$ is a uniform cover, provided $\mathcal{V}$ is a σ-uniformly discrete open cover. Let $\mathcal{V} = \bigcup_\omega \mathcal{V}_n$, each $\mathcal{V}_n$ being uniformly discrete. Let $\mathcal{U}$ be a uniform cover such that $\mathcal{U} \overset{*}{<} \{\cup \mathcal{V}_n\}^{<\omega}$. Take for any n a uniform cover $\mathcal{W}_n$ such that $\{St(W, \mathcal{W}_n): W \in \mathcal{W}_n\}$ witnesses discreteness of $\mathcal{V}_1, \dots, \mathcal{V}_n$. Take for each x $n_x \in \omega$ such that $St(x, \mathcal{U}) \subset \bigcup_{i=1}^{n_x} \cup \mathcal{V}_i$. Obviously, the cover $\mathcal{G} = \{St(x, \mathcal{U} \wedge \mathcal{W}_{n_x}): x \in X\}$ refines $\mathcal{V}^{<\omega}$. Since μX is metric-fine, $\mathcal{G}$ is a uniform cover.

A family $\{V_a\}$ of subsets of a uniform space μX is called σ-uniformly discretely refinable if there exists a σ-uniformly discrete collection $\mathcal{U}$ refining $\{V_a\}$ such that $\cup \mathcal{U} = \cup \{V_a\}$.

Lemma 2.6.: Let μX be a uniform space, let ν be a compatible uniformity on X such that every uniformly discrete family in νX is σ-uniformly discretely refinable in μX. Then μX is σ-discretely refinable, provided $(\mu \wedge \nu)X$ is σ-discretely refinable.

Proof: It is enough to show that any uniformly discrete family in $(\mu \wedge \nu)X$ is σ-uniformly discretely refinable in μX.

Let $\mathcal{H} = \{H_a : a \in A\}$ be a uniformly discrete family in $(\mu \wedge \nu)X$. Then there exist covers $\mathcal{W}$ and $\mathcal{V}$ σ-uniformly discrete in μX and νX respectively such that $\mathcal{W} \wedge \mathcal{V}$ witnesses the discreteness of $\mathcal{H}$. Obviously, $\mathcal{V}$ has a σ-uniformly discrete (in μX!) refinement $\mathcal{V}'$. Let $\mathcal{W} = \bigcup \mathcal{W}_n$, $\mathcal{V}' = \bigcup \mathcal{V}_m$, $\mathcal{W}_n$, $\mathcal{V}_m$ being uniformly discrete families in μX. Then for every n, m

$$\mathcal{G}_{n,m} = \{H_a \cap V \cap U : a \in A,\ U \in \mathcal{W}_n, V \in \mathcal{V}_m\}$$

is a uniformly discrete family in μX. Thus,

$\mathcal{G} = \bigcup_{n,m \in \omega} \mathcal{G}_{n,m}$ is a σ-uniformly discrete refinement of $\mathcal{H}$.

Corollary 2.7.: Let μX be a uniformly countably paracompact uniform space. Then the following statements are equivalent:

i) μX is σ-discretely refinable;

ii) $m\mu X$ is uniformly paracompact.

Corollary 2.8.: Let μX be a uniform space. Then the following statements are equivalent:

i) μX is σ-discretely refinable;

ii) $m(C \wedge \mu)X$ is uniformly paracompact.

Remark: It should be noted that the term C cannot be omitted from ii) in Corollary 2.8. To see this, let X be the set $\omega_1 \times \omega$ and define a uniformity μ on X by the basic covers

$$\mathcal{U}_\alpha = \{\{\beta\} \times \omega : \alpha \leq \beta \leq \omega_1\} \cup \{(\beta, n) : \beta < \alpha, n \in \omega\},$$

where $\alpha < \omega_1$. Then μX is a σ-uniformly discrete metric-fine space which is not uniformly paracompact. Indeed, the cover of X by one-point sets does not have a uniformly locally finite refinement. In this context it should be noted that μX is uniformly para-Lindelöf.

3. Supercompleteness. A uniform space is supercomplete ([13]) if the hyperspace $H(\mu X)$ of all closed subsets of X ([14], p. 28) is complete. By [13], μX is supercomplete if and only if, X is paracompact and $\lambda \mu X = \mathcal{F} X$, where λ is the Ginsberg-Isbell locally fine coreflection. Uniformly paracompact spaces are supercomplete since by [18] the equation $p\mu/\mu X = \mathcal{F} X$ holds for every uniformly paracompact space μX. On the other hand, complete σ-discretely refinable spaces need not be supercomplete. The second author has shown in [12] that a fine paracompact p-space X has the property that X x Y is supercomplete for any fine separable metrizable space Y if and only if, the space X is C-scattered.

Let X be a completely regular space. Then by Theorem 2 in [5], X is Lindelöf if and only if, CX is uniformly paracompact. Indeed, one can show that X is Lindelöf if and only if, CX is supercomplete. A paracompact space μX is uniformly countably paracompact if and only if, every $f \in C(X)$ is uniformly locally bounded. (See [10].) Thus, if X is a paracompact non-Lindelöf space, then CX is a uniformly countably paracompact space which is not supercomplete. Moreover, we obtain the following result:

<u>Proposition 3.1.</u>: Let μX be a uniformly countably paracompact, σ-discretely refinable space. Then every open cover of X belongs to $p\mu/\mu^{(1)}$.

Proof: Let $\mathcal{V}$ be an open cover of X. For each $x \in X$, choose $V_x \in \mathcal{V}$ and an open $\mathcal{U}_x \in \mu$ such that $St^2(x, \mathcal{U}_x) \subset V_x$. Now $\mathcal{W} = \{St(x, \mathcal{U}_x) : x \in X\}$ is an open cover of X and hence it has a σ-uniformly discrete open refinement $\mathcal{W}'$. Write $\mathcal{W}' = \bigcup \mathcal{W}_n$, where each $\mathcal{W}_n$ is uniformly discrete relative to $\mathcal{U}_n \in \mu$. Put $G_n = \bigcup \mathcal{W}_n$. Then $\mathcal{G} = \{G_n : n \in \omega\}$ is a countable open cover of X. By uniform countable paracompactness the cover $\mathcal{G}^{<\omega}$ is uniform. For each $n \in \omega$ define $\hat{\mathcal{U}}_n = \mathcal{U}_1 \wedge \wedge \ldots \wedge \mathcal{U}_n$. Let

$$\mathcal{H}_n = \hat{\mathcal{U}}_n \restriction (\bigcup_{i=1}^{n} G_i)$$

and let $\mathcal{H} = \bigcup \{\mathcal{H}_n : n \in \omega\}$. Then $\mathcal{H} \in \mu^{(1)}$ and each member of $\mathcal{H}$ is contained in the union of a finite subfamily of $\mathcal{W}$. Given $H \in \mathcal{H}$, let $F_H \subset X$ be a finite subset with $H \subset \bigcup\{St(x, \mathcal{U}_x) : x \in F_H\}$. Put $\mathcal{U}_H = \bigwedge \{\mathcal{U}_x : x \in F_H\}$. If $x \in H$, then there is a $y \in F_H$ such that $x \in St(y, \mathcal{U}_y)$ and consequently $St(x, \mathcal{U}_H) \subset St^2(y, \mathcal{U}_y)$. Thus,

$\{St^2(y, \mathcal{U}_y) : y \in F_H\} \restriction H$ and a fortiori

$\{V_y : y \in F_H\} \restriction H$ is a uniform cover of H. Put

$$\mathcal{B} = \{H \cap V_x : x \in F_H\} \cdot$$

Then $\mathcal{B} \in p\mu/\mu^{(1)}$ and $\mathcal{B} < \mathcal{V}$, as required.

<u>Corollary 3.2.</u>: If μX is a uniformly countably paracompact and uniformly para-Lindelöf space, then every open cover of X belongs to $p\mu/\mu^{(1)}$.

Proof: Trivial, since uniformly para-Lindelöf spaces are σ-discretely refinable.

Corollary 3.3.: Uniformly countably paracompact, σ-discretely refinable spaces are supercomplete. In particular, uniformly countably paracompact uniformly para-Lindelöf spaces are supercomplete.

Remark: By an argument more elaborate than that used in proving 2.5 and 2.6 one can establish the following: the locally fine coreflection of a uniform space μX is σ-discretely refinable iff m μX is supercomplete. (This follows from the fact that the metric-fine coreflection of a locally fine space is locally fine.)

In the following we shall consider a special class of supercomplete spaces. A uniform space μX is equinormal if any two two closed disjoint subsets of X are separated by μ-uniform neighbourhoods.

Proposition 3.4.: Let μX be an equinormal and uniformly locally connected space. Then every continuous real-valued function on X is uniformly continuous.

Proof: Let $f: X \to R$ be a continuous function. One can easily for $\varepsilon > 0$ construct two closed disjoint sets $A_1, A_2 \subset f(X)$ such that

1) $d(A_1, A_2) \geq \varepsilon/8$;
2) for each $r \subset f(X)$ $d(x, A_1 \cup A_2) < \varepsilon/8$
3) for each $a \in A_i$ if $|a' - a| < \varepsilon/4$, $a' \in A_i$, then $a = a'$.

Take $B_i = f^{-1}(A_i)$. Then B_1, B_2 are closed disjoint subsets of X. Then there is a uniform cover $\mathcal{U}$ of μX consisting of connected sets such that $St(B_1, \mathcal{U}) \cap B_2 = \emptyset$. Let for $U \in \mathcal{U}$ diam $f(U) \geq \varepsilon$. Then there are $r_1 < r_2 \in f(U)$ such that $r_2 - r_1 > 3\varepsilon/4$. Since U is connected, $f(U) \supset (r_1, r_2)$. Thus, obviously, $f(U) \cap A_1 \neq \emptyset \neq f(U) \cap A_2$. Contradiction.

Corollary 3.5.: Let μX be an equinormal and uniformly locally connected space. Then every family $\mathcal{F}$ of real-valued continuous functions on X with topologically discrete family of supports is equiuniformly continuous.

Proof: Let $\mathcal{F} = \{f_a\}$, let $\varepsilon > 0$. Since $f = \sum f_a$ is uniformly continuous, we can find a uniform cover $\mathcal{U}$ of μX consisting of connected sets such that for $U \in \mathcal{U}$ diam $f(U) < \varepsilon/2$. Let there be, for some a and some U, points x and y such that $|f_a(x) - f_a(y)| \geq \varepsilon$. We may suppose $x \in \operatorname{coz} f_a$, $f_a(y) = 0$. Then $f_a(x) \geq \varepsilon$. Thus, $f_a(U) \supset \langle 0, \varepsilon \rangle$, which is impossible.

Remark 1: The uniform spaces whose continuous real-valued functions are uniformly continuous, were characterized by Atsuji [1]. It seems that our proof of 3.5 cannot be simplified by the results proved therein.

Remark 2: In [16], J. Nagata stated that a locally complete, paracompact, equinormal and uniformly locally connected space is complete. However, if the cardinality of the space is non-measurable, then by Katětov-Shirota theorem the space is realcompact and consequently (by Proposition 3.4) $C\mu X = C\mathcal{F}X$ is complete. Hence, barring measurable cardinals it follows that $\mu < C\mathcal{F}X$ is complete even without the assumption that the space is locally complete. However, the full use of 3.5 give even more.

A family $\mathcal{F}$ of functions is called σ-equiuniformly continuous if it is the union of countable collection of equiuniformly continuous families.

Lemma 3.6.: Let μX be a uniform space such that every open cover of X has a σ-equiuniformly continuous partition of unity. Then $m\mu X$ is fine uniformly compact.

Proof: Suppose that $V_a = \operatorname{coz} f_a$, $\{f_a\}$ being an equiuniformly continuous partition of unity. Then the map $f = (f_a)_A :$ $: \mu X \longrightarrow \ell_\infty(A)$ is uniformly continuous. Define $B_a = \{x :$ $: x \quad f(X),\ x(a) > 0\}$. B_a is an open cover of $f(X)$, thus $\{V_a\}$ is a uniform cover of $m\mu X$ since $V_a = f^{-1}(B_a)$.

Now, let $\mathcal{F} = \cup \mathcal{F}_n$, each $\mathcal{F}_n$ being an equiuniformly continuous family. Then

$\sigma_n(x,y) = \sup\{|f_a(x) - f_a(y)| : f_a \in \mathcal{F}_n\}$ is uniformly continuous pseudometric, $\sigma_n(x,y) \leq 1$. Thus,

$\sigma(x,y) = \Sigma\, 2^{-n}\, \sigma_n(x,y)$ is uniformly continuous and all functions from $\mathcal{F}$ are Lipschitz with respect to σ. Replacing a function $f \in \mathcal{F}_n$ by 2^n copies of the function $2^{-n}f$ we get an equiuniformly continuous partition.

Lemma 3.7.: Let μX be an equinormal, uniformly locally connected, topologically paracompact space. Then $m\mu X$ is a fine uniformly paracompact space.

Proof: Since every open cover of a paracompact space has a partition of unity with a topologically discrete family of supports, the claim follows immediately from 3.4 and 3.6.

Corollary 3.8.: Let μX be an equinormal, uniformly locally connected, topologically paracompact space. Then μX is supercomplete.

Proof: By 3.4, μX is uniformly countably paracompact. Thus, the claim follows from 3.3 and 3.7.

4. Concluding remarks. We have not been able to solve the question whether a uniformly countably paracompact, uniformly para-Lindelöf spaces are uniformly paracompact. In a metric case, the answer is yes, since a uniformly countably paracompact space is uniformly paracompact [11]. In distal spaces [8], the answer is likewise yes, since each distal space has a basis of covers which are finite unions of uniformly discrete families. Obviously, the answer is affirmative for both locally fine and separable spaces. Distal, locally fine and separable spaces have one property in common: they admit a point-finite basis. In fact, we have the following simple result.

Proposition 4.1.: Let μX be a uniformly para-Lindelöf, uniformly countably paracompact uniform space with point-finite basis. Then μX is uniformly paracompact.

Proof: Let $\mathcal{U}$ be an open cover of X. As μX is uniformly para-Lindelöf, there is a uniform cover $\mathcal{V}$, uniformly locally finite with respect to the cover $\mathcal{W}$, such that for each $V \in \mathcal{V}$ there is a sequence $\{U_i^V\}$ such that $V \subset \bigcup U_i^V$, $U_i^V \in \mathcal{U}$.

Define $V_n = \bigcup \{U_n^V : V \in \mathcal{V}\}$. Since X is uniformly countably paracompact, there exists a uniform cover $\mathcal{B}$ of X, $\mathcal{B} < \mathcal{W}$, such that $\mathcal{B} < \{V_n\}^{<\omega}$. Take $B \in \mathcal{B}$. $B \subset V_1 \cup \dots \cup V_n$. B intersects just $V_1, \dots, V_k$ from $\mathcal{V}$. Thus

$$B \subset \bigcup_{j=1} \bigcup_{i=1} U_j^{V_i}$$

Thus, $\mathcal{B} < \mathcal{U}^{<\omega}$ and μX is uniformly paracompact.

A collection $\mathcal{V}$ of subsets of a topological space X is called a k-network if for each compact subset $C \subset X$ and each neighbourhood U of C there is a finite subfamily $\mathcal{V}' \subset \mathcal{V}$ such that $C \subset \bigcup \mathcal{V}' \subset U$. A regular space with a σ-locally finite k-network is called an $\aleph$-space [17]. The proof of the following lemma is straightforward.

Lemma 4.2.: A uniform space μX is a σ-discretely refinable $\aleph$-space if, and only, μX has a σ-uniformly locally finite k-network.

M. Kubo proved in [15] that if X is a paracompact $\aleph$-space, then the hyperspace K(X) of compact subsets of X is a paracompact $\aleph$-space. The same proof can be modified to establish the following proposition.

Proposition 4.3.: If μX is a σ-discretely refinable $\aleph$-space, then so is $K(\mu X)$.

M. Čoban noted in [3] that the hyperspace K(X) of a paracompact p-space is a paracompact p-space. It is not difficult to establish the following analogue.

Proposition 4.4.: If μX is a σ-discretely refinable p-space, then so is $K(\mu X)$.

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