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ON THE G-SPACES HAVING AN $\mathcal{S}$-G-CW-APPROXIMATION BY A G-CW-COMPLEX OF FINITE G-TYPE

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Abstract: Let G be a compact Lie group and $\mathcal{S}$ a set of G-isotropy types, i. e. a set of conjugacy classes of closed subgroups of the group G.

In the paper the notion of an $\mathcal{S}$-G-CW-approximation of a G-space is given and the existence theorem is proved.

Key words: Compact Lie Group, G-CW-complex, $\mathcal{S}$-G-n-equivalence, $\mathcal{S}$-G-CW-approximation, G-Whitehead Theorem, Finite G-type, Isotropy Type.

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§ 1. Introduction. Let G be a compact Lie group in the paper.The terminology and notations,used in the paper, follow closely [4].By a G-CW-complex we mean a G-CW-complex in the sense of [7].The special case of G-CW-complexes with a finite group G is studied in [1], [3] and [9].

Let $\mathcal{S}$ be a set of G-isotropy types (a set of conjugacy classes of closed subgroups of the group G).Results of [7,§ 5] suggest the following definition.

Definition 1.1. A G-space X is said to have a G-isotropy type $\mathcal{S}$,if the conjugacy class (G_x) of the isotropy group G_x belongs to $\mathcal{S}$ for each $x \in X$.

Now,let us denote $\tilde{\mathcal{S}} = \{(H_1 \cap \ldots \cap H_n) \mid (H_i) \in \mathcal{S}, n = 1,2,\ldots\}$. The system $\mathcal{S}$ is said to be closed with respect to finite intersections,if $\tilde{\mathcal{S}} = \mathcal{S}$.

Lemma 1.2. Let $\mathcal{S}$ be a set of G-isotropy types.
Then:

i) If the set $\mathcal{S}$ is finite, the set $\tilde{\mathcal{S}}$ is finite, too.

ii) If the set $\mathcal{S}$ is closed with respect to finite intersections, then it is closed with respect to arbitrary intersections.

Proof: Let H be a closed subgroup of the group G. The maximal torus T of the group G acts locally smoothly on the compact manifold G/H (see [4], chapter IV). So the set $\{T_{gH} = gHg^{-1} \cap T \mid g \in G\} = \{H' \cap T \mid (H') = (H)\}$ is finite by Theorem IV.1.2 in [4]. Let M be a finite system, $M = \{H^1, \dots, H^k\}$, containing one group from each class in $\mathcal{S}$. The system $\{H^1_1 \cap \dots \cap H^1_{i_1} \cap \dots \cap H^k_1 \cap \dots \cap H^k_{i_k} \cap T \mid (H^i_j) = (H^i)\}$ must be finite by the previous note. By the Mostow Theorem [2, page 94] the system $\{(H^1_1 \cap \dots \cap H^1_{i_1} \cap \dots \cap H^k_1 \cap \dots \cap H^k_{i}) \mid (H^i_j) = (H^i)\}$ is finite, too. This proves the part i) of our lemma.

Let $(H_\alpha) \in \mathcal{S}, \alpha \in A$. Then there is a sequence $\alpha_1, \dots, \alpha_n \in A$ such that $\dim(\bigcap_{\alpha \in A} H_\alpha) = \dim(\bigcap_1^n H_{\alpha_i})$. Denote $L = \bigcap_1^n H_{\alpha_i}$. Then for $\alpha \in A$ the number of components of $L \cap H_\alpha \leqslant$ the number of components of L.

So there is a sequence $\beta_1, \dots, \beta_m \in A$ such that $\bigcap_{\alpha \in A} H_\alpha = L \cap (\bigcap_1^m H_{\beta_i})$.

q.e.d.

Definition 1.3. Let n be a positive integer. An equivariant map $f: X \longrightarrow Y$ of G-spaces is called an $\mathcal{S}$-G-n-equivalence, if the induced map $f^H: X^H \longrightarrow Y^H$ is an n-equivalence in the sense of [11, page 404] for each H with $(H) \in \mathcal{S}$. An equivariant map of G-spaces is called an $\mathcal{S}$-G-weak homotopy equivalence, if it is an $\mathcal{S}$-G-n-equivalence for each n.

If the system $\mathcal{S}$ contains conjugacy classes of all closed subgroups, an $\mathcal{S}$-G-weak equivalence will be called simply a G-weak homotopy equivalence.

Lemma 1.4. Let X and Y be G-spaces having a G-isotropy type $\mathcal{S}$. Let an equivariant map $f: X \longrightarrow Y$ be a $\tilde{\mathcal{S}}$-G-weak homotopy equivalence.

Then it is a G-weak homotopy equivalence.

Proof: Let us denote $L = \bigcap\{H' \in \mathcal{S} \mid H' \supset H\}$. Then $L \in \tilde{\mathcal{S}}$ and we have the following commutative diagram:

$$\begin{array}{ccc} X^H & \xrightarrow{f^H} & Y^H \\ \| & & \| \\ X^L & \xrightarrow{f^L} & Y^L \end{array}$$

q.e.d.

Analysing proofs in [7], we obtain the following two theorems.

Theorem 1.5. Let $f: X \to Y$ be an equivariant map of G-spaces. Then f is an $\mathcal{S}$-G-n-equivalence if and only if the induced map

$$f_{\#}: [K;X]_G \longrightarrow [K;Y]_G$$

is bijective for every G-CW-complex K of the G-isotropy type $\mathcal{S}$ with $\dim_G(K) < n$ and surjective for every G-CW-complex K of the G-isotropy type $\mathcal{S}$ with $\dim_G(K) \leq n$.

Theorem 1.6. Let $f: X \to Y$ be an equivariant map of G-spaces, where both X and Y have G-homotopy type of a G-CW-complex of the G-isotropy type $\mathcal{S}$.

Then f is a G-homotopy equivalence if and only if it is an $\mathcal{S}$-G-weak homotopy equivalence.

Theorem 1.6 and Lemma 1.4 give rise the following equivariant version of J.H.C. Whitehead Theorem.

Theorem 1.7. Let X and Y be G-spaces having a G-isotropy type $\mathcal{S}$. Let X and Y have G-homotopy type of a G-CW-complex.

Then an equivariant map $f: X \to Y$ is an $\tilde{\mathcal{S}}$-G-weak homotopy equivalence if and only if it is a G-homotopy equivalence.

§ 2. Main Theorems. In the view of the previous note it seems to be natural to introduce the following notions.

Definition 2.1. Let Y be a G-space. An $\mathcal{S}$-G-CW-approximation of the space Y is a G-CW-complex X of the G-isotropy type $\mathcal{S}$

together with an $\mathcal{P}$-G-weak homotopy equivalence $f:X \longrightarrow Y$.

If $\mathcal{P}$ consists of all conjugacy classes of closed subgroups of the group G, an $\mathcal{P}$-G-CW-approximation will be called simply a G-CW-approximation. The following proposition follows from the definition.

Proposition 2.2. Let $f:X \longrightarrow Y$ and $f':X' \longrightarrow Y$ be two $\mathcal{P}$-G-CW-approximations of the space Y.

Then there is a G-homotopy equivalence $g:X \longrightarrow X'$ such that f is G-homotopic with $f' \circ g$. In addition, the G-homotopy class of g is uniquely determined by the G-homotopy classes of f and f'.

We are going to prove the following theorem in the paper.

Theorem 2.3. Let Y be a G-space of the G-isotropy type $\mathcal{P}$. Let the following two conditions be satisfied:

i) For $(H) \in \tilde{\mathcal{P}}$ the space Y^H has finitely many components, the groups $\pi_1(Y^H, *)$ are finitely generated Abelian or finite and the groups $\pi_i(Y^H, *)$ are finitely generated for $i = 2,3\ldots$

ii) There are finite sets $\mathcal{P}_0 \subset \mathcal{P}_1 \subset \ldots \mathcal{P}_i \subset \ldots \subset \tilde{\mathcal{P}}$ of G-isotropy types, closed with respect to finite intersections such, that:

For $n = 0,1,\ldots$ and $(H) \in \tilde{\mathcal{P}}$ there is $H' \supset H, (H') \in \mathcal{P}_n$ and, if we denote $L = \bigcap\{H' \mid (H') \in \mathcal{P}_n, H' \supset H\}$, the inclusion $Y^L \hookrightarrow Y^H$ is an n-equivalence of topological spaces.

Under these two conditions there is an $\tilde{\mathcal{P}}$-G-CW-approximation $f:X \longrightarrow Y$, where X is a G-CW-complex of finite G-type.

The theorem will be proved in the following paragraph.

We can show, making use of the Mostow Theorem, that the following two conditions are equivalent with the contition ii) of Theorem 2.3.

ii') There are finite sets $\mathcal{K}_0, \mathcal{K}_1, \ldots, \mathcal{K}_i, \ldots \subset \tilde{\mathcal{P}}$ of G-isotropy types, closed with respect to finite intersections, such, that:

For $n = 0,1,\dots$ and $(H)\in\tilde{\mathcal{P}}$ there is $H'\supset H, (H')\in\mathcal{K}_n$ and, if we denote $L = \bigcap\{H' \mid (H')\in\mathcal{K}_n, H'\supset H\}$, the inclusion $Y^L \hookrightarrow Y^H$ is an n-equivalence of topological spaces.

ii'') There are finite sets $\mathcal{J}_0, \mathcal{J}_1, \dots, \mathcal{J}_i, \dots \subset \tilde{\mathcal{P}}$ of G-isotropy types, closed with respect to finite intersections, such that:

I) If $(H)\in\tilde{\mathcal{P}}$, then there is $H'\supset H$ with $(H')\in\mathcal{J}_n$ such that $incl_{\#} : \pi_n(Y^{H'},*) \longrightarrow \pi_n(Y^H,*)$ is an epimorphism.

II) If $(H')\in\mathcal{J}_n$ and $(H)\in\tilde{\mathcal{P}}$ with $H'\supset H$ are such that $incl_{\#} : \pi_n(Y^{H'},*) \longrightarrow \pi_n(Y^H,*)$ is an epimorphism, then there is $(H'')\in\mathcal{J}_n$ with $H'\supset H''\supset H$ such that $Ker(incl_{\#} : \pi_n(Y^{H'},*) \longrightarrow \pi_n(Y^H,*)) = Ker(incl_{\#} : \pi_n(Y^{H'},*) \longrightarrow \pi_n(Y^{H''},*))$.

Corrolary 2.4. Let X be a space, having the G-isotropy type $\mathcal{P}$. Let the space X satisfy the conditions i) and ii) of Theorem 2.3 with $\pi_1(X^H,*)$ finite. Let the G-space X have a G-homotopy type of G-CW-complex. Let (x,y) be a point of XxX and let us denote $L = G_x \cap G_y$.

Then the loop space $\Omega_{(x,y)}(X) = \Omega(X)$ is endowed with the natural structure of the L-space of the L-isotropy type $\tilde{\mathcal{P}}\cap L$, where $\tilde{\mathcal{P}}\cap L$ denotes the set $\{(H\cap L) \mid (H)\in\tilde{\mathcal{P}}\}$. The space $\Omega(X)$ has L-homotopy type of an L-CW-complex of a finite L-type.

Proof: Theorem 2.3 gives rise an $\tilde{\mathcal{P}}$-G-CW-approximation $f:Y\to X$ of the space X, where Y is a G-CW-complex of finite G-type. For the space X has the G-homotopy type of a G-CW-complex, the equivariant map f is, by G-Whitehead Theorem, a G-homotopy equivalence. So X has G-homotopy type of a G-CW-complex of finite G-type.

For $L\supset H$ we have $(\Omega(X))^H = \Omega(X^H)$. Using the exact homotopy sequence of the fibration $\Omega(X^H)\to P(X^H)\to X^H$ we can verify that $\Omega(X)$ satisfies, as an L-space of the L-isotropy type $\tilde{\mathcal{P}}\cap L$, the conditions of Theorem 2.3 (the role of the set $\mathcal{S}_i$ plays here a set $\mathcal{S}_{i+1} \cap L$).

By [8], $\Omega(X)$ has the L-homotopy type of an L-CW-complex. The G-Whitehead Theorem completes the proof.

q.e.d.

§ 3. Proof of Theorem 2.3. We assert that in order to prove Theorem 2.3 it suffices to construct the following sequence of G-CW-complexes X_n ($n = 0,1,\dots$) and equivariant mappings $f_n : X_n \to Y$, having the following properties:

i) X_n is a G-CW-complex of the G-isotropy type $\mathcal{S}_n$ and $f_n : X_n \to Y$ is an $\mathcal{S}_n$-n-G-equivalence,

ii) $(X_{n+1})^n_G = X_n$ and each X_n is a finite G-CW-complex,

iii) $f_{n+1} | X_n = f_n$.

Let us put $X = \bigcup_n X_n$ and let us define $f : X \to Y$ by $f(x) = f_n(x)$ for $x \in X_n$.

We have to prove that f is an $\widetilde{\mathcal{S}}$-G-weak homotopy equivalence. Suppose that $(H) \in \widetilde{\mathcal{S}}$ and let n be an integer. Let $L = \bigcap \{H' \mid (H') \in \mathcal{S}_{n+1}, H' \supset H\}$. Because X_{n+1} has the G-isotropy type $\mathcal{S}_{n+1}$, we have $(X_{n+1})^L = (X_{n+1})^H$.

By [7] the map $\mathrm{incl}_\# : \pi_n((X_{n+1})^H, *) \to \pi_n(X^H, *)$ is an isomorphism. The map $\mathrm{incl}_\# : \pi_n(Y^L, *) \to \pi_n(Y^H, *)$ is an isomorphism by ii) of Theorem 2.3. Hence the following diagram competes our proof.

$$\begin{array}{ccccc}
\pi_n((X_{n+1})^L, *) & = & \pi_n((X_{n+1})^H, *) & \xrightarrow[\mathrm{incl}_\#]{\approx} & \pi_n(X^H, *) \\
f^L_{n+1\#} \downarrow \approx & & f^H_{n+1\#} \downarrow & \swarrow f^H_\# & \\
\pi_n(Y^L, *) & \xrightarrow[\mathrm{incl}_\#]{\approx} & \pi_n(Y^H, *) & &
\end{array}$$

q.e.d.

We will need the following lemmas in our construction.

Lemma 3.1. Let G be a compact Lie group and K and H be its closed subgroups.

Then the space $(G/H)^K$ has the homotopy type of a finite CW-complex.

Proof: The group K acts locally smoothly on G/H [4,VI.2.4]. Hence, by [4,IV.3.3], the space $(G/H)^K$ is a compact topological manifold. The rest of proof follows from [6,page 744].

q.e.d.

Lemma 3.1, the G-cellular Approximation Theorem [7] and homotopical properties of attaching [10,chapter 2.3] allows us to prove the following lemma.

Lemma 3.2. Let G-CW-complex X have only finite number of G-cells. Then the space X^H has the homotopy type of a finite CW-complex for each closed subgroup H of the group G.

Lemma 3.3. Let Z be a connected CW-complex of finite type, let $\pi_1(Z,*)$ be finitely generated Abelian or finite groups and let $\pi_i(Z,*)$ be finitely generated groups for $i = 2,3,\ldots,k-1$.

Then the groups $\pi_k(Z,*)$ are finitely genrated $\pi_1(Z,*)$-modules.

Proof: Let the group $\pi_1(Z,*)$ be finitely generated Abelian. Let us take the universal covering space $\tilde{Z}$ of the space Z. $H_k(\tilde{Z})$ is finitely generated $\pi_1(Z,*)$-module, because Z has finitely many cells in each dimension and $\mathbb{Z}(\pi_1(Z,*))$ is a Noetherian ring. Let us denote $\mathcal{C}$ the class of finitely generated Abelian groups. Making use of the Generalized Hurewicz Theorem modulo $\mathcal{C}$ (see [11]), we obtain, that $\pi_k(\tilde{Z},\tilde{*}) = \pi_k(Z,*)$ is $\mathcal{C}$-isomorphic to $H_k(\tilde{Z})$. Hence $\pi_k(Z,*)$ is finitely generated $\pi_1(Z,*)$-module.

If the group $\pi_1(Z,*)$ is finite, the universal covering space $\tilde{Z}$ contains only finitely many cells in each dimension, so $H_k(\tilde{Z})$ is a finitely generated Abelian group.

The rest of the proof follows from the Generalized Hurewicz Theorem modulo $\mathcal{C}$ in the case.

q.e.d.

3.4. Let $\mathcal{S}$ be a set of G-isotropy types and let M be a set of closed subgroups of the group G containing one group from each class in $\mathcal{S}$. Let, for $H \in M$, $\{y_i^H\}_{i \in I^H}$ be a set containing one point from each component of the space Y^H. Let $f: X \to Y$ be an equivariant map, which, for any j, generates the isomorphism $f_\#^H : \pi_j(X^H, *) \to \pi_j(Y^H, f(*))$ for each $H \in M$ and $* \in \{y_i^H\}$.

Then the map f induces the isomorphism for each H with $(H) \in \mathcal{S}$ and $* \in Y^H$.

3.5. Let us consider the situation, described in Theorem 2.3. Let, for $n = 0,1,\ldots, M_n$ be a set of closed subgroups of the group G, containing one point from each class in $\mathcal{S}_n$. We can suppose that $M_0 \subset M_1 \subset \ldots \subset \bigcup_n M_n = M$. Let $\{y_i^H\}_{i \in I^H}$ be, for $H \in M$, a set containing one point from each component of the space Y^H. By ii) of 2.3 we can suppose that $\{y_i^H\}_{i \in I^H} \subset \{y_i^{H'}\}_{i \in I^{H'}}$ for suitable $H' \in M_0$. So we can suppose, by i) of 2.3, that the system $\{y_i^H\}_{i \in I^H}$ is finite for each $H \in M$. Now, we are able to construct our sequence.

3.6. Let us put $X_0 = \dot{\bigcup}_{\substack{H \in M_0 \\ i \in I^H}} ((G/H) \times \{y_i^H\})$,

$x_i^H = (eH, y_i^H)$, where e denotes the unit of the group G, and let us define $f_0: X_0 \to Y$ by $f_0(gx_i^H) = gy_i^H$.

3.7. Let us consider relations of the following kind: Let $a = (gP, y_i^P)$ and $b = (hQ, y_j^Q)$, $P, Q \in M_0$, be points of the space X_0^H for any $H \in M_1$ such that $f_0(a)$ and $f_0(b)$ belongs to the same component of the space Y^H.

Let us define $\varphi: ((G/H) \times \partial I) \to X_0$ by $\varphi(kH, 0) = (kgP, y_i^P)$ and $\varphi(kH, 1) = (kHQ, y_j^Q)$. The space $((G/H) \times I) \cup_\varphi X_0$ is then well defined.

We can deduce by finitness of the systems M_0 and M_1 that we obtain, attaching finitely many those relations, the space X_0' such, that the clear extension $f_0': X_0' \longrightarrow Y$ induces the isomorphism of the components of the spaces $(X_0')^H$ and Y^H for $H \in M_1$.

Let, for $H \in M_1$, be J_1^H the finite system of generators of $\pi_1(Y^H, y_1^H)$. Elements of the system are represented by mappings $\alpha: (S^1_\alpha, s_0^\alpha) \longrightarrow (Y^H, y_1^H)$. Let us put $O_1^H = \dot{\bigcup}_{\alpha \in J_1^H}((G/H) \times S^1_\alpha)$ and let us define

$$\psi_1^H: \dot{\bigcup}_{\alpha \in J_1^H}((G/H) \times \{s_0^\alpha\}) \longrightarrow X_0' \text{ by } \psi_1^H(gH, s_0^\alpha) = gx_1^H .$$

Let X_1 be the space obtained from X_0' by attaching the spaces O_1^H by those mappings. The extension $f_1: X_1 \longrightarrow Y$ is clear. We can verify, that the object satisfies our conditions.

3.8. Let us suppose that we have already constructed the sequence of spaces and mappings $f_i: X_i \longrightarrow Y$ for $i \leq k$.

We assert that, for $H \in M_{k+1}$, $\mathrm{Ker}(f_{k*}: \pi_k(X_k^H, *) \longrightarrow \pi_k(Y^H, *))$ is finitely generated $\pi_1(X_k^H, *)$-module for $k \geq 2$ and finitely generated subgroup for $k = 1$.

The case $k = 1$ is clear. Indeed, the groups $\pi_1(X_1^H, *)$ are finitely generated, because the space X_1^H has the homotopy type of a CW-complex of a finite type.

If the group $\pi_1(Y^H, *)$ is finite, the kernel is, as a subgroup of a finitely generated group with finite index, finitely generated, too (see [5], chapter VII, 2.1).

If $\pi_1(Y^H, *)$ is finitely generated Abelian group, we can suppose that $\pi_1(X_1^H, *)$ is an Abelian group, too. In the opposite case we attach finitely many G-2-cells to kill the commutators of the set of generators of the group $\pi_1(X_1^H, *)$. Hence Ker f_* is, as a subgroup of the finitely generated Abelian group, also finitely generated. By Lemma 3.2 the space X_k^H has the homotopy type of a finite CW-complex for $k \geq 2$. By Lemma 3.3 the groups $\pi_k(X_k^H, *)$ are finitely generated $\pi_1(X_k^H, *)$-modules. Because $Z(\pi_1(X_k^H, *))$ is a Noetherian ring, we obtain our assertion.

3.9. Let, for $H \in M_{k+1}$, C_1^H denotes the system of generators of $\mathrm{Ker}(f_{k\#}: \pi_k(X_k^H, x_1^H) \longrightarrow \pi_k(Y^H, y_1^H))$ as a $Z(\pi_1(X_k^H, x_1^H))$-module for $k \geq 2$ and as a normal subgroup for $k = 1$. By 3.8 the systems C_1^H are finite. Elements of C_1^H are represented by mappings $\beta : (S_\beta^k, s_0^\beta) \longrightarrow (X_k^H, x_1^H)$. Let us put $R_1^H = \dot{\bigcup}_{\beta \in C_1^H}((G/H) \times E_\beta^{k+1})$ and let us define

$$\varphi_1^H : \partial R_1^H \longrightarrow X_k \text{ by } \varphi_1^H(gH, s_\beta) = g(\beta(s_\beta)).$$

Let X_k' denote the space, obtained by attaching the spaces R_1^H by those mappings. The definition of the extension $f_k' : X_k' \longrightarrow Y$ is clear.

In the following commutative diagram

$$\begin{array}{ccc} \pi_k((X_k')^H, x_1^H) & \xleftarrow{\ \mathrm{incl}_\#\ } & \pi_k(X_k^H, x_1^H) \\ & f'^H_{k\#} \searrow \quad \swarrow f^H_{k\#} & \\ & \pi_k(Y^H, y_1^H) & \end{array}$$

the mapping $\mathrm{incl}_\#$ is an epimorphism (see Theorem 4.3 in [7]). This fact implies, similarly as in the chapter 5.2 of [12], that

$$\pi_k((X_k')^H, x_1^H) \cong \pi_k(Y^H, y_1^H) \quad \text{for } H \in M_{k+1}$$

The space X_{k+1} and the map f_{k+1} is obtained from X_k' and f_k' similarly, as in the end of 3.7. This completes our construction.

q.e.d.

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