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variables. Arities of function and relation symbols are arbitrary cardinals. Infinitary conjunctions and quantifiers are admitted. This language is suitable for the study of concrete categories as the following results indicate.

Any concrete category is equivalent to a category of models of $L_{\infty,\infty}$. Any concrete category is equivalent to the category of all models of some theory of $L_{\infty^+,\infty}$ (class-indexed conjunctions are admitted). A theory of $L_{\infty,\infty}$ will be called normal if it has (up to the equivalence) only a set of n-ary atomic formulas for each cardinal n. A concrete category is strongly fibre-small (in the sense of Adámek, Herrlich and Strecker) iff it is equivalent to the category of all models of some normal theory of $L_{\infty,\infty}$. The Beck's theorem gives the characterization of categories of models of normal equational theories of $L_{\infty,\infty}$. Categories of models of normal universal Horn theories of $L_{\infty,\infty}$ was described in the foregoing author's paper (Arch. Math. (Brno) 4(1978), 219-226). Further, universal-uniquely existencial Horn theories correspond to the existence of a left adjoint to the underlying functor of the category of models and Horn theories to the existence of a stable left adjoint. The subsequent author's paper will also contain the model-theoretic treatment of Mac Neille completions and of (cartesian closed) initially complete categories.

SEQUENCE SOLUTIONS OF THE DIRICHLET PROBLEM

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Let X be a $\mathfrak{P}$-harmonic space with the countable base in the sense of Constantinescu and Cornea in which constants are harmonic functions. A sequence solution of the Dirichlet problem is the solution obtained as the limit of a properly defined sequence of functions. Suppose that $D \subset X$ is an open relatively compact set and $f \in C(\partial D)$ a boundary condition ($C(M)$ denotes the system ot continuous functions on M). A generalization of Lebesgue's procedure can be obtained in the following way: denote by ρ a metric compatible with the topology of X and by ε_x^t the Dirac measure ε_x swept-out on the complement of the open sphere with centre x and radius t. For an $r \in C(D)$, $0 < r(x) \leq \operatorname{dist}(x, X \setminus D)$, an increasing $g \in C([0,\infty[)$, $g(0) = 0$, $g > 0$ on $]0,\infty[$ and $G \in C(\bar{D})$ put

$$AG(x) = [g(r(x))]^{-1} \int_0^{r(x)} \varepsilon_x^t(G)\,dg(t), \quad x \in D.$$

Now choose an $F \in C(\bar{D})$, $F = f$ on ∂D and define $F_n = A^n F$, $n \in N$. Then $\{F_n\}$ is convergent (uniformly on compact subsets of D) to the solution which coincides with the PWB-solution H_f^D corresponding to f and D. Remark that in this context there is eventually more than one reasonable generalized solution. Some other related results will appear under the same title in Časopis Pěst. Mat.