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Aplikace matematiky, Vol. 34 (1989), No. 5, 364–374

Persistent URL: `http://dml.cz/dmlcz/104364`

34 (1989) APLIKACE MATEMATIKY No. 5, 364—374

ON MAXWELL EQUATIONS WITH THE PREISACH HYSTERESIS OPERATOR: THE ONE-DIMENSIONAL TIME-PERIODIC CASE

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(Received February 23, 1988)

Summary. Energy functionals for the Preisach hysteresis operator are used for proving the existence of weak periodic solutions of the one-dimensional systems of Maxwell equations with hysteresis for not too large right-hand sides. The upper bound for the speed of propagation of waves is independent of the hysteresis operator.

Keywords: Preisach hysteresis operator, Maxwell equations, periodic solutions.

AMS Classification: 35L60, 35B10, 78A25.

This research was motivated by Visintin's paper [7], where the author investigated systematically the possibilities of introducing hysteresis operators into partial differential equations. His results include existence theorems for parabolic equations with the Preisach hysteresis operator in a very general setting.

We try here to unify the approach of Preisach-Visintin with the Ishlinskiĭ model of hysteresis ([2], [3], [4]). In particular, we extend the notion of hysteresis energy functionals to the Preisach operator and derive the energy inequalities. For this purpose we must reduce the class of Preisach operators (the measure in the half-plane P generating the Preisach operator is assumed to be positive and absolutely continuous with a continuously differentiable density with respect to the Lebesgue measure). We preserve the notation from [3] – [6] in order to emphasize the correspondence between the two models for hysteresis.

The energy estimates are used in the second part for proving the existence of periodic solutions to the one-dimensional Maxwell equations in a ferromagnetic material with hysteresis of Preisach type. Let us note that the wave-propagation speed does not exceed the velocity of light. This property confirms the hyperbolicity of the system.

The local character of the existence theorem (the solutions are constructed only for small right-hand sides) is due to the fact that one of the energy inequalities is closely related to the convexity of hysteresis loops. The proof is based on the idea that the approximate solutions do not leave the region of convexity of the Preisach operator.

Let $u: [0, T] \to R^1$ be a continuous function and let $\varrho \in R^1$, $h > 0$ be given. We introduce the relay operator $z_{\varrho,h}$ in the following way: for $t \in [0, T]$ set $A_t = \{\tau \in [0, t];\ u(\tau) = \varrho \pm h\}$ and $t_M = \max A_t$ if $A_t \neq \emptyset$. We put

$$z_{\varrho,h}(u)(t) = z_{\varrho,h}(u)(0), \quad \text{if} \quad A_t = \emptyset, \tag{1.1}$$

$$z_{\varrho,h}(u)(t) = \begin{cases} +1, & \text{if} \quad u(t_M) = \varrho + h, \\ -1, & \text{if} \quad u(t_M) = \varrho - h, \end{cases} \tag{1.2}$$

$$z_{\varrho,h}(u)(0) = \begin{cases} J(u(0) - \varrho - h), & \varrho \geqq 0, \\ -J(-u(0) + \varrho - h), & \varrho < 0, \end{cases} \tag{1.3}$$

where $J(r) = 1$ for $r \geqq 0$, $J(r) = -1$ for $r < 0$. In Visintin's terminology [7], [8] we have $z_{\varrho,h}(u)(t) = f_{(\varrho-h,\varrho+h)}(u, \xi)(t)$, where the initial condition ξ corresponds to the "virginal state".

Following Krasnoselskiĭ and Pokrovskiĭ [2] (cf. also [3], [4]) we define the operator l_h. Let $u: [0, T] \to R^1$ be a continuous piecewise monotone function. We put

$$l_h(u)(0) = \begin{cases} 0, & \text{if} \quad |u(0)| \leqq h, \\ u(0) - h, & u(0) > h, \\ u(0) + h, & u(0) < -h, \end{cases} \tag{1.4}$$

$$l_h(u)(t) = \begin{cases} \max\{l_h(u)(t_0), u(t) - h\}, & t \in (t_0, t_1], \\ \quad \text{if } u \text{ is nondecreasing in } [t_0, t_1] \\ \min\{l_h(u)(t_0), u(t) + h\}, & t \in (t_0, t_1], \\ \quad \text{if } u \text{ is nonincreasing in } [t_0, t_1]. \end{cases} \tag{1.5}$$

We see that $l_h(u)$ is again a continuous piecewise monotone function. Moreover, for arbitrary continuous piecewise monotone functions u, v we have (cf. [2])

$$|l_h(u)(t) - l_h(v)(t)| \leqq \|u - v\|_{[0,t]}, \tag{1.6}$$

where

$$\|w\|_{[a,b]} = \max\{|w(s)|, s \in [a, b]\}.$$

The inequality (1.6) shows that l_h can be extended to a Lipschitz continuous operator in the space $C([0, T])$ of continuous functions with the norm $\|\cdot\|_{[0,T]}$.

This operator can be materialised by a simple device. Let us consider a cylinder of length $2h$ which is moved along its axis with help of a piston placed inside. If $u(t)$ denotes the position of the piston at the instant t, then $l_h(u)(t)$ corresponds to the position of the center of the cylinder at the same instant.

The following lemma establishes the relation between $z_{\varrho,h}$ and l_h.

(1.7) **Lemma.** *Let $h > 0$, $u \in C([0, T])$, $t \in [0, T]$ be given. Then we have $z_{\varrho,h}(u)(t) = -1$ for $\varrho > l_h(u)(t)$ and $z_{\varrho,h}(u)(t) = +1$ for $\varrho < l_h(u)(t)$.*

We postpone the proof of (1.7) and we recall the particular character of the hysteresis memory.

Let $u \in C([0, T])$, $t \in [0, T]$ be given. Put $\bar{t} = \max\{\tau \in [0, t];\ |u(\tau)| = \|u\|_{[0,t]}\}$. We denote $t_0 = \bar{t}$ if $u(\bar{t}) \leqq 0$ and $t_1 = \bar{t}$ if $u(\bar{t}) > 0$. Further, we put

$$(1.8) \qquad \begin{aligned} t_{2k} &= \max\{\tau \in [t_{2k-1}, t];\quad u(\tau) = \min\{u(\sigma);\ \sigma \in [t_{2k-1}, t]\}\},\\ t_{2k+1} &= \max\{\tau \in [t_{2k}, t];\quad u(\tau) = \max\{u(\sigma);\ \sigma \in [t_{2k}, t]\}\},\end{aligned}$$

until $t_n = t$.

The sequence $\{t_n\}$ is either finite or infinite. In the latter case we have

$$\lim_{n \to \infty} |u(t_{n+1}) - u(t_n)| = 0.$$

The following lemma is proved in [3], [4].

(1.9) **Lemma.** *Let* $u \in C([0, T])$, $t \in [0, T]$ *be given. Then we have* $l_h(u)(\bar{t}) = \max\{0, u(\bar{t}) - h\}$ *if* $\bar{t} = t_1$, $l_h(u)(\bar{t}) = \min\{0, u(\bar{t}) + h\}$ *if* $\bar{t} = t_0$, and

$$\begin{aligned} l_h(u)(t_{2k}) &= l_h(u)(t_{2k-1}) - \max\{0, u(t_{2k-1}) - u(t_{2k}) - 2h\},\\ l_h(u)(t_{2k+1}) &= l_h(u)(t_{2k}) + \max\{0, u(t_{2k+1}) - u(t_{2k}) - 2h\},\end{aligned}$$

where $\{t_n\}$ *is the sequence* (1.8).

This lemma shows that the hysteresis memory contains only the values of u at the points of the sequence (1.8). In particular, the hysteresis system "forgets" everything before $\bar{t}$.

Proof of (1.7). In the case $\|u\|_{[0,t]} \leqq h$ we have $l_h(u)(t) = 0$ and the assertion follows immediately from (1.1)–(1.3). For $\|u\|_{[0,t]} > h$ we construct the sequence (1.8) and prove (1.7) by induction over k. Let for instance $\bar{t} = t_1$ (the other case is analogous). Then $l_h(u)(\bar{t}) = u(\bar{t}) - h$. For $\varrho > u(\bar{t}) - h$ we have $\varrho > 0$ and $\varrho > u(0) - h$, hence $z_{\varrho,h}(u)(\bar{t}) = -1$ by (1.2) or (1.1), (1.3). For $\varrho < u(\bar{t}) - h$ we distinguish two cases: if $A_{\bar{t}} \neq \emptyset$, then $z_{\varrho,h}(u)(\bar{t}) = +1$ by (1.2), if $A_{\bar{t}} = \emptyset$, then $\varrho - h < \varrho + h < u(0)$ and we use (1.1), (1.3). Let us now assume that (1.7) is proved for $t = t_{2k}$ (the argument is the same for $t = t_{2k-1}$). If $u(t_{2k+1}) - u(t_{2k}) \leqq 2h$, then $l_h(u)(t) = l_h(u)(t_{2k})$ and $u(t) - h \leqq l_h(u)(t_{2k}) \leqq u(t) + h$ for $t \in [t_{2k}, t_{2k+1}]$, and (1.7) follows from (1.2). Let us assume $u(t_{2k+1}) - u(t_{2k}) > 2h$. Then we have $l_h(u)(t_{2k}) = u(t_{2k}) + h$, $l_h(u)(t_{2k+1}) = u(t_{2k+1}) - h$. For $\varrho > u(t_{2k+1}) - h$ or $\varrho < u(t_{2k}) - h$ the assertion is obvious. In the remaining case we have $u(t_{2k}) \leqq \varrho + h \leqq u(t_{2k+1})$, hence $z_{\varrho,h}(u)(t_{2k+1}) = +1$ by (1.2). Lemma (1.7) is proved.

(1.10) **Definition.** *Let* μ *be a real function of two variables* $\varrho \in R^1$, $h \geqq 0$ *such that*

(i) μ, $\partial u/\partial\varrho$, $\partial^2\mu/\partial\varrho^2$ *are continuous in* $R^1 \times [0, \infty)$,

(ii) $\mu(\varrho, h) = -\mu(-\varrho, h)$ *for every* $\varrho, h \in R^1 \times [0, \infty)$,

(iii) $(\partial\mu/\partial\varrho)(\varrho, h) > 0$ *for every* $\varrho, h \in R^1 \times [0, \infty)$.

For $u \in C([0, T])$ *the value of the Preisach operator* Z *is defined by the formula*

$$Z(u)(t) = \lim_{K \to \infty} \frac{1}{2} \int_0^K \int_{-K}^K z_{\varrho,h}(u)(t) \frac{\partial\mu}{\partial\varrho}(\varrho, h) \, d\varrho \, dh .$$

Remarks.

(1.11) **Correctness.** The definition is meaningful, since by (1.7), (1.9) and (1.10) (ii) we have

$$Z(u)(t) = \int_0^\infty \mu(l_h(u)(t), h) \, dh ,$$

where $\mu(l_h(u)(t), h) = 0$ for $h \geqq \|u\|_{[0,t]}$.

(1.12) **Ishlinskii operator.** For $\mu(\varrho, h) = \varrho(\varphi^{-1})''(h)$, where φ is a given twice continuously differentiable concave function, $\varphi(0) = 0$, $+\infty > \varphi'(0+) > 0$, we derive from (1.11) and from (2.16) of [3] that $(\varphi'(0+))^{-1} I + Z = F^{-1}$, where I is the identity operator in $C([0, T])$ and F is the Ishlinskiĭ operator generated by φ.

From (1.11) we obtain further properties of the operator Z.

(1.13) **Lemma.** (i) *There exists a positive increasing continuous function* ψ *such that for* $u, v \in C([0, T])$ *we have*

$$|Z(u)(t) - Z(v)(t)| \leqq \psi(\max\{\|u\|_{[0,t]}, \|v\|_{[0,t]}\}) \, \|u - v\|_{[0,t]} .$$

In particular, Z *is a locally Lipschitz continuous operator in* $C([0, T])$.

(ii) *The operator* Z *is odd.*

Proof. Putting $\psi(V) = \int_0^V \max\{(\partial\mu/\partial\varrho)(\varrho, h), |\varrho| \leqq V\} \, dh$ and $V = \max\{\|u\|_{[0,t]}, \|v\|_{[0,t]}\}$ we obtain (1.13) (i) immediately from (1.11) and (1.6). The operators l_h are odd and μ is odd with respect to ϱ, hence Z is odd and Lemma (1.13) is proved.

(1.14) **Lemma.** *Let* $u \in C([0, T])$ *be absolutely continuous. Then* $Z(u)$ *is also absolutely continuous and the inequality*

$$0 \leqq (Z(u))'(t) \, u'(t) \leqq \psi(\|u\|_{[0,t]}) \, (u'(t))^2$$

holds almost everywhere.

Proof. Let us choose $t_2 > t_1 \geqq 0$ and put $v(t) = u(t)$ for $t \in [0, t_1]$, $v(t) = u(t_1)$ for $t \in (t_1, t_2]$. Lemma (1.13) (i) yields $|Z(u)(t_2) - Z(u)(t_1)| \leqq \psi(\|u\|_{[0,T]}) \, \|u(\cdot) - u(t_1)\|_{[t_1,t_2]}$, hence $Z(u)$ is absolutely continuous.

Further, let $t \in (0, T)$ be arbitrarily chosen. If $u'(t) = 0$, then $(Z(u))'(t) = 0$ and (1.14) holds. Since the operator Z is odd, the cases $u'(t) > 0$ and $u'(t) < 0$ are symmetric. Let us suppose for instance $u'(t) > 0$. Then the sequence (1.8) is finite, $t = t_{2k+1}$ for some $k \geqq 0$. There are two possibilities:

a) $t = t_1 = \bar{t}$. Then we have $l_h(u)(t) = u(t) - h$, $(l_h(u))'(t) = u'(t)$ for $h < u(t)$ and $l_h(u)(t) = (l_h(u))'(t) = 0$ for $h > u(t)$.

Therefore,

$$(1.15) \qquad (Z(u))'(t) = \int_0^{u(t)} u'(t) \frac{\partial \mu}{\partial \varrho}(u(t) - h, h)\, \mathrm{d}h .$$

b) $t > \bar{t}$. Then we have by (1.9) $l_h(u)(t) = u(t) - h$, $(l_h(u))'(t) = u'(t)$ for $h < \frac{1}{2}(u(t) - u(t_{2k}))$ and $l_h(u)(t) = l_h(u)(t_{2k})$, $(l_h(u))'(t) = 0$ for $h > \frac{1}{2}(u(t) - u(t_{2k}))$. We obtain

$$(1.16) \qquad (Z(u))'(t) = \int_0^{1/2(u(t) - u(t_{2k}))} u'(t) \frac{\partial \mu}{\partial \varrho}(u(t) - h, h)\, \mathrm{d}h$$

and (1.14) follows from (1.15), (1.16). ∎

We now express the energies of a system with hysteresis. Put $\mu_\infty(h) = \lim_{\varrho \to +\infty} \mu(\varrho, h)$. Let $r(v, h)$ for $-\mu_\infty(h) < v < \mu_\infty(h)$ and $h \geqq 0$ be the partial inverse of μ, i.e. $\mu(r(v, h), h) = v$, $r(\mu(\varrho, h), h) = \varrho$. Let us denote

$$R(\xi, h) = \int_0^\xi r(v, h)\, \mathrm{d}v \quad \text{for} \quad |\xi| < \mu_\infty(h) .$$

We define the potential energies associated to the Preisach operator Z as

$$(1.17) \qquad P_1(u)(t) = \int_0^\infty R(\mu(l_h(u)(t), h), h)\, \mathrm{d}h \quad \text{for} \quad u \in C([0, T]) ,$$

$$(1.18) \qquad P_2(u)(t) = \tfrac{1}{2}(Z(u))'(t)\, u'(t) \quad \text{for} \quad u \in W^{1,1}(0, T) ,$$

where $W^{k,p}(0, T)$ denotes the usual Sobolev space.

We have the following energy inequalities.

(1.19) **Lemma.** *Let $u \in C([0, T])$ be absolutely continuous. Then the inequality*

$$(P_1(u))'(t) - (Z(u))'(t)\, u(t) \leqq 0$$

holds almost everywhere.

Proof. A straightforward computation yields

$$(P_1(u))'(t) - (Z(u))'(t)\, u(t) = \int_0^\infty (l_h(u))'(t)\,(l_h(u)(t) - u(t)) \,.$$

$$. \frac{\partial \mu}{\partial \varrho}(l_h(u)(t), h)\, \mathrm{d}h , \quad \text{and} \quad (l_h(u))'(t)\,(l_h(u)(t) - u(t)) \leqq 0$$

almost everywhere, thus (1.19) follows easily. ∎

The hypotheses (i)–(iii) of (1.10) imply that there exist $\gamma > 0$ and $U > 0$ such that

(1.20) $$\frac{1}{2}\frac{\partial\mu}{\partial\varrho}(\varrho-\xi,\xi)+\int_0^{\xi}\frac{\partial^2\mu}{\partial\varrho^2}(\varrho-h,h)\,\mathrm{d}h \geqq 2\gamma$$

for $|\varrho| \leqq U$, $0 \leqq \xi \leqq U$. This enables us to prove

(1.21) **Lemma.** *Let $u \in W^{2,1}(0, T)$ be given such that $\|u\|_{[0,T]} \leqq U$. Then for arbitrary t, s, $0 \leqq s < t \leqq T$ we have*

$$\textstyle\int_s^t (Z(u))'(\sigma)\,u''(\sigma)\,\mathrm{d}\sigma \leqq P_2(u)(t) - P_2(u)(s) - \gamma\int_s^t |u'(\sigma)|^3\,\mathrm{d}\sigma\,.$$

Proof. Repeating the proof of of (1.17) of [4] we see from (1.8), (1.9) that it suffices to assume that u is increasing and that there is at most one change of memory level in $[s, t]$, i.e. four cases are possible:

(a) $l_h(u)(\sigma) = l_h(u)(t_{2k}) + \max\{0, u(\sigma) - u(t_{2k}) - 2h\}\,,\quad \sigma \in [s, t]\,,$

(b) $l_h(u)(\sigma) = \max\{0, u(\sigma) - h\}\,,\quad \sigma \in [s, t]\,,$

(c) $$l_h(u)(\sigma) = \begin{cases} l_h(u)(t_{2k+2}) + \max\{0, u(\sigma) - u(t_{2k+2}) - 2h\}\,, & \sigma \in [s, \tau]\,,\\ l_h(u)(t_{2k}) + \max\{0, u(\sigma) - u(t_{2k}) - 2h\}\,, & \sigma \in (\tau, t]\,,\end{cases}$$
$u(t_{2k}) < u(t_{2k+2})\,,$

(d) $$l_h(u)(\sigma) = \begin{cases} l_h(u)(t_{2k}) + \max\{0, u(\sigma) - u(t_{2k}) - 2h\}\,, & \sigma \in [s, \tau]\,,\\ \max\{0, u(\sigma) - h\}\,, & \sigma \in (\tau, t]\,, \quad k = 0 \text{ or } 1\,.\end{cases}$$

Let us prove the lemma in the case (c) (the others are analogous). By (1.16) we have

$$\int_s^{\tau}(Z(u))'(\sigma)\,u''(\sigma)\,\mathrm{d}\sigma = \frac{1}{2}(u'(\tau))^2\int_0^{1/2(u(\tau)-u(t_{2k+2}))}\frac{\partial\mu}{\partial\varrho}(u(\tau)-h,h)\,\mathrm{d}h -$$
$$- P_2(u)(s) - \frac{1}{2}\int_s^{\tau}|u'(\sigma)|^3\cdot\Big[\frac{1}{2}\frac{\partial\mu}{\partial\varrho}\Big(\frac{1}{2}(u(\sigma)+u(t_{2k+2})),\frac{1}{2}(u(\sigma)-u(t_{2k+2}))\Big) +$$
$$+\int_0^{1/2(u(\sigma)-u(t_{2k+2}))}\frac{\partial^2\mu}{\partial\varrho^2}(u(\sigma)-h,h)\,\mathrm{d}h\Big]\,\mathrm{d}\sigma$$

and similarly for $\int_\tau^t$. Since we have

$$\frac{1}{2}(u'(\tau))^2\Big[\int_0^{1/2(u(\tau)-u(t_{2k+2}))}\frac{\partial\mu}{\partial\varrho}(u(\tau)-h,h)\,\mathrm{d}h -$$
$$-\int_0^{1/2(u(\tau)-u(t_{2k}))}\frac{\partial\mu}{\partial\varrho}(u(\tau)-h,h)\,\mathrm{d}h\Big] \leqq 0\,,$$

we use (1.20) and Lemma is proved. ■

Let us pass to periodic functions. We denote by C_ω the space of continuous ω-periodic functions with the norm $\|u\| = \max\{|u(t)|,\ t \in R^1\}$.

Lemma (1.9) implies that $l_h(u)$ is ω-periodic for $t \geqq \omega$ if $u \in C_\omega$. Therefore, Z can be considered as a continuous operator in C_ω.

In the sequel, we deal with functions $u(x, t)$, $x \in [0, \pi/2]$, $t \in R^1$, such that $u(x, t + \omega) = u(x, t)$. For the corresponding L^p-spaces, $1 \leqq p < \infty$, we use an obvious notation $L^p_\omega(0, \pi/2)$, with the norm

$$\|u\|_p = \left(\int_0^{\pi/2} \int_0^\omega |u(x, t)|^p \, dt \, dx\right)^{1/p}.$$

The space of continuous ω-periodic functions is denoted by $C_\omega([0, \pi/2])$, with the norm $\|u\|_\infty = \max \{|u(x, t)|, x \in [0, \pi/2], t \in R^1\}$.

For $u \in C_\omega([0, \pi/2])$ we can define the value of the Preisach operator

$$(1.22) \qquad Z(u)(x, t) = Z(u(x, \cdot))(t), \quad x \in [0, \pi/2], \quad t \geqq 0.$$

Lemma (1.13) (i) shows that Z is again a locally Lipschitz continuous operator in $C_\omega([0, \pi/2])$.

2. MAXWELL EQUATIONS

The one-dimensional Maxwell equations in a ferromagnetic material can be written in the form ([1])

$$(2.1) \qquad \begin{aligned} \varepsilon_0 \varepsilon E_t + H_x + \sigma(E) &= g \\ \mu_0(H + J)_t + E_x &= 0, \end{aligned}$$

where E, H are the intensities of the electric and magnetic field, respectively, J is the magnetization, σ is a given function representing the conductivity, $\varepsilon_0 > 0$, $\mu_0 > 0$, $\varepsilon > 1$ are given constants, $\varepsilon_0 \mu_0 = c^{-2}$, c is the velocity of light and g is the given density of the imposed electric current. The Preisach model for the ferromagnetism consists in putting

$$(2.2) \qquad J = Z(H),$$

where Z is the operator (1.10), (1.22).

(2.3) **Proposition.** *Let us suppose $\sigma(E) \, . \, E \geqq 0$ for every $E \in R^1$. Then the speed of propagation of electromagnetic waves governed by (2.1), (2.2) does not exceed $c/\sqrt{\varepsilon}$.*

Proof. The argument is the same as in [6]. We assume that E, H are solutions of (2.1), (2.2) with $g \equiv 0$ such that $E(x, 0) = H(x, 0) = 0$ for $x \in [a, b]$. We integrate the function

$$\left[\mu_0\left(\tfrac{1}{2}H^2 + P_1(H)\right) + \frac{\varepsilon_0 \varepsilon}{2} E^2\right]_t + (EH)_x$$

over a trapezoidal domain bounded by the straight lines $t = 0$, $t = \bar{t}$, $x = a + \lambda t$, $x = b - \lambda t$, where $\lambda = c/\sqrt{\varepsilon}$, $0 < \bar{t} < (b - a)/2$. Lemma (1.19) and the Green theorem yield $H(x, \bar{t}) = E(x, \bar{t}) = 0$ for $x \in (a + \lambda \bar{t}, b - \lambda \bar{t})$, i.e. λ is an upper bound for the wave-propagation speed. ∎

The values of the constants in (2.1) are irrelevant for the existence theorem. For this reason we now consider the system in the form

$$(2.4) \qquad \begin{aligned} E_t + H_x + \sigma(E) &= g \\ H_t + (Z(H))_t + E_x &= 0 . \end{aligned}$$

(2.5) **Theorem.** *Let σ be a continuously differentiable function, $\sigma' \geqq 0$, $\sigma(0) = 0$, and let Z be the Preisach operator (1.10), (1.22). Then there exists $\delta > 0$ such that every $g \in L^2_\omega(0, \pi/2)$, $g_t \in L^2_\omega(0, \pi/2)$, $\|g\|_2 + \|g_t\|_2 < \delta$ there exist $E, H \in C_\omega([0, \pi/2])$, $E_t, H_x \in L^2_\omega(0, \pi/2)$, $E_x, H_t \in L^3_\omega(0, \pi/2)$, $E(0, t) = H(\pi/2, t) = 0$, such that the system (2.4) is satisfied almost everywhere in $(0, \pi/2) \times (\omega, +\infty)$.*

Remarks

(2.6) **Uniqueness.** If the operator Z is not of the form (1.12), where it is possible to use the monotonicity of the Ishlinskiĭ operator (cf. [5]), the answer is not known.

(2.7) **Boundary conditions.** The situation is more complicated here than in the "Ishlinskiĭ" case. For instance, with the boundary conditions $E(0, t) = E(\pi/2, t) = 0$, even the problem of existence of solutions seems to be open.

Proof of (2.5). The idea of the proof is the same as in [3]. We apply the Galerkin method. Let us denote $w_j(t) = \sin(2\pi j t/\omega)$ for $j > 0$, $w_j(t) = \cos(2\pi j t/\omega)$ for $j \leqq 0$. For a fixed integer $n > 0$ we look for functions

$$E^{(n)}(x, t) = \sum_{j=-n}^{n} \sum_{k=0}^{n} E_{jk}\, w_j(t) \sin(2k + 1)\, x ,$$

$$H^{(n)}(x, t) = \sum_{j=-n}^{n} \sum_{k=0}^{n} H_{jk}\, w_j(t) \cos(2k + 1)\, x ,$$

where E_{jk}, H_{jk} are real numbers satisfying the system

$$(2.8) \qquad \begin{gathered} \int_\omega^{2\omega} \int_0^{\pi/2} (E_t^{(n)} + H_x^{(n)} + \sigma(E^{(n)}) - g)\, w_j(t) \sin(2k + 1)\, x \,\mathrm{d}x \,\mathrm{d}t = 0 , \\ \int_\omega^{2\omega} \int_0^{\pi/2} [(H_t^{(n)} + E_x^{(n)})\, w_j(t) - Z(H^{(n)})\, w_j'(t)] \cos(2k + 1)\, x \,\mathrm{d}x \,\mathrm{d}t = 0 , \\ j = -n, \ldots, n , \quad k = 0, \ldots, n . \end{gathered}$$

We see immediately that every solution of (2.8) fulfils

$$\int_0^{\pi/2} \int_\omega^{2\omega} (Z(H^{(n)}))_t\, H_{tt}^{(n)} - \sigma'(E^{(n)})\, (E_t^{(n)})^2 + g_t E_t^{(n)}) \,\mathrm{d}x \,\mathrm{d}t = 0 .$$

Let us suppose

$$(2.9) \qquad \|H^{(n)}\|_\infty \leqq U , \quad \|g\|_2 + \|g_t\|_2 < \delta .$$

By Lemma (1.21) we obtain

$$\|H_t^{(n)}\|_3^3 \leqq \delta/\gamma \|E_t^{(n)}\|_2 .$$

Moreover, (2.8) implies

$$\|E_t^{(n)}\|_2^2 = \int_0^{\pi/2} \int_\omega^{2\omega} ((H_t^{(n)})^2 + Z(H^{(n)})_t H_t^{(n)} + gE_t^{(n)})\,dt\,dx\,.$$

Hence (1.14) yields

$$\|E_t^{(n)}\|_2^2 \leqq [(1 + \psi(U))\,\delta/\gamma]^{2/3}\,\|E_t^{(n)}\|_2^{2/3} + \delta\|E_t^{(n)}\|_2\,.$$

In particular we have

$$\|H_t^{(n)}\|_3^3 + \|E_t^{(n)}\|_2^2 \leqq \text{const.}\,\delta^2\,. \tag{2.10}$$

Let $\varphi \in L^2_\omega(0, \pi/2)$ be an arbitrary function and let us denote by φ_n the projection of φ onto the subspace generated by $\{w_j(t)\cos(2k+1)x,\ j = -n, \ldots, n,\ k = 0, \ldots, n\}$.

Using (2.8), (2.10), (1.14) we obtain

$$\left|\int_0^{\pi/2} \int_\omega^{2\omega} E_x^{(n)}\varphi\,dt\,dx\right| = \left|\int_0^{\pi/2} \int_\omega^{2\omega} (H_t^{(n)} + Z(H^{(n)})_t)\,\varphi_n\,dt\,dx\right| \leqq$$
$$\leqq \text{const.}\,\delta^{2/3}\|\varphi_n\|_{3/2}\,.$$

Since $\|\varphi - \varphi_n\|_2 \to 0$ as $n \to \infty$, we have also $\|\varphi - \varphi_n\|_{3/2} \to 0$ as $n \to \infty$, hence the inequality

$$\left|\int_0^{\pi/2} \int_\omega^{2\omega} E_x^{(n)}\varphi\,dt\,dx\right| \leqq \text{const.}\,\delta^{2/3}\|\varphi\|_{3/2}$$

holds for every $\varphi \in L^2_\omega(0, \pi/2)$, and consequently for every $\varphi \in L^{3/2}_\omega(0, \pi/2)$. In other words, we have

$$\|E_x^{(n)}\|_3^3 \leqq \text{const.}\,\delta^2\,, \quad \|E^{(n)}\|_3^3 \leqq \text{const.}\,\delta^2\,. \tag{2.11}$$

The space $\{u \in L^3_\omega(0, \pi/2);\ u_t \in L^2_\omega(0, \pi/2),\ u_x \in L^3_\omega(0, \pi/2)\}$ is (compactly) embedded into $C_\omega([0, \pi/2])$. More precisely, we have $|u(x, t) - u(y, s)| \leqq \text{const.}\,(|x - y|^{1/5} + |t - s|^{1/7})$ for $x, y \in [0, \pi/2]$, $t, s \in R^1$.

This gives

$$\|E^{(n)}\|_\infty \leqq \text{const.}\,\delta^{2/3}\,, \quad \|\sigma(E^{(n)})\|_\infty \leqq \sigma(\text{const.}\,\delta^{2/3})\,, \tag{2.12}$$

and from (2.8), (2.10), (2.12) we get

$$\|H_x^{(n)}\|_2 \leqq \alpha(\delta)\,, \tag{2.13}$$

where α is a continuous function, $\alpha(0) = 0$. An analogous embedding as above yields

$$\|H^{(n)}\|_\infty \leqq \text{const.}\,\alpha(\delta)\,. \tag{2.14}$$

The system (2.8) is a (nonlinear) vector equation of the form

$$\Phi(V) = G\,, \tag{2.15}$$

where $V = \{E_{jk}, H_{jk},\ j = -n, \ldots, n,\ k = 0, \ldots, n\}$ and G is the right-hand side vector. We equip the space $R^{2(n+1)(2n+1)}$ of vectors V with the norm

$$|V| = \|E^{(n)}\|_\infty + \|H^{(n)}\|_\infty$$

and denote by $B_U^{(n)}$ the ball $\{V \in R^{2(n+1)(2n+1)};\ |V| \leqq U\}$. Let us consider the system

$$\Phi_\varepsilon(V) = \varepsilon G\,, \quad \varepsilon \in [0, 1] \tag{2.16}$$

analogous to (2.8), where $\sigma(E^{(n)})$, g are replaced by $\varepsilon\,\sigma(E^{(n)})$, εg, respectively. Indeed, the estimates (2.10)–(1.14) remain valid for the solutions of (2.16) independently of ε, provided that (2.9) holds. Consequently, for $\delta > 0$ sufficiently small the system (2.16) has no solution on the boundary of $B_U^{(n)}$ for $\varepsilon \in [0, 1]$ and the topological degree $\mathrm{d}(\Phi_\varepsilon(\cdot) - \varepsilon G, B_U^{(n)}, 0)$ is independent of ε. The mapping Φ_0 is odd, hence $\mathrm{d}(\Phi(\cdot) - G, B_U^{(n)}, 0) = \mathrm{d}(\Phi_0, B_U^{(n)}, 0) \neq 0$. Thus we have proved that for every $n \geqq 1$ the system (2.8) has at least one solution in the interior of $B_U^{(n)}$ such that (2.10)–(2.14) hold. Notice that δ may be chosen indepedently of n.

Using once more the embedding theorems quoted above we conclude that there exists a subsequence $\{E^{(m)}, H^{(m)}\}$ of $\{E^{(n)}, H^{(n)}\}$ and functions $E, H \in C_\omega([0, \pi/2])$, $E_t, H_x \in L^2_\omega(0, \pi/2)$, $E_x, H_t \in L^3_\omega(0, \pi/2)$ such that $E_t^{(m)} \to E_t$, $H_x^{(m)} \to H_x$ in $L^2_\omega(0, \pi/2)$ weak, $E_x^{(m)} \to E_x$, $H_t^{(m)} \to H_t$ in $L^3_\omega(0, \pi/2)$ weak, $E^{(n)} \to E$, $H^{(n)} \to H$ uniformly. A standard limit procedure in (2.8) completes the proof.

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Souhrn

O MAXWELLOVÝCH ROVNICÍCH S PREISACHOVÝM HYSTEREZNÍM OPERÁTOREM: JEDNOROZMĚRNÝ ČASOVĚ PERIODICKÝ PŘÍPAD

PAVEL KREJČÍ

Pomocí funkcionálů energie pro Preisachův hysterezní operátor je dokázána existence slabých periodických řešení jednorozměrné soustavy Maxwellových rovnic s hysterezí pro nepříliš velké pravé strany. Horní odhad pro rychlost šíření vln nezávisí na hysterezním operátoru.

Резюме

ОБ УРАВНЕНИЯХ МАКСВЕЛЛА С ГИСТЕРЕЗИСНЫМ ОПЕРАТОРОМ ПРЕЙСАХА: ОДНОМЕРНЫЙ ПЕРИОДИЧЕСКИЙ ПО ВРЕМЕНИ СЛУЧАЙ

PAVEL KREJČÍ

С помощью функционалов энергии для гистерезисного оператора Прейсаха доказывается существование слабых периодических решений одномерной системы уравнений Максвелла с гистерезисом для не слишком больших правых частей. Оценка сверху для скорости распространения волн не зависит от гистерезисного оператора.

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