Sanjo Zlobec

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REGIONS OF STABILITY FOR ILL-POSED CONVEX PROGRAMS: AN ADDENDUM*)

SANJO ZLOBEC

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Summary. The marginal value formula in convex optimization holds in a more restrictive region of stability than that recently claimed in the literature. This is due to the fact that there are regions of stability where the Lagrangian multiplier function is discontinuous even for linear models.

1. INTRODUCTION

Consider the *convex mathematical model*

$$(\mathrm{P}, \theta) \qquad \operatorname*{Min}_{(x)} f^0(x, \theta)$$
$$\text{s.t.}$$
$$f^k(x, \theta) \leqq 0\,, \quad k \in P = \{1, \ldots, m\}$$

where $f^i: R^n \times R^p \to R$ are continuous functions and $f^i(\cdot, \theta): R^n \to R$ are convex for every $\theta \in R^p$, $i \in \{0\} \cup P$. The model is studied at some fixed $\theta = \theta^*$.

For every θ, we denote by

$F(\theta) = \{x \in R^n: f^k(x, \theta) \leqq 0,\ k \in P\}$ the *feasible set*;

$\tilde{x}(\theta)$ an *optimal solution*;

$\tilde{F}(\theta)$ the set of *all optimal solutions*;

$\tilde{f}(\theta) = f^0(\tilde{x}(\theta), \theta)$ the *optimal value*.

The model (P, θ) is considered as an *input-output system* with the *input* θ and the *output* $\{F(\theta), \tilde{F}(\theta), \tilde{f}(\theta)\}$.

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Assume that $\tilde{F}(\theta^*) \neq \emptyset$. Then at $\theta = \theta^*$ there are chunks of space R^p where continuity of the output is preserved. They are termed *regions of stability*. We recall (see e.g. [10], [11])

1.1. **Definition.** *Convex model* (P, θ) *is stable in a region* $S \subset R^p$ *at* $\theta = \theta^*$ *if, for some neighbourhood* $N(\theta^*)$ *of* θ^*, *both*

(i) $\theta \in N(\theta^*) \cap S \Rightarrow \tilde{F}(\theta) \neq \emptyset$ *and*
(ii) $\theta \in N(\theta^*) \cap S$ and $\theta \to \theta^* \Rightarrow \tilde{F}(\theta)$ is bounded and all its *limit* (*accumulation*) *points are in* $\tilde{F}(\theta^*)$. ∎

In order to formulate and construct specific regions of stability we denote, for a given θ,

$$P^=(\theta) = \{k \in P: x \in F(\theta) \Rightarrow f^k(x, \theta) = 0\}$$

$$P^<(\theta) = P \setminus P^=(\theta)$$

and

$$F^=(\theta) = \{x \in R^n: f^k(x, \theta) = 0,\ k \in P^=(\theta)\}.$$

We assume throughout this addendum that

$$\tilde{F}(\theta^*) \neq \emptyset \text{ and bounded}.$$

Then we recall (e.g. from [5], [10], [11]) that (P, θ) is stable in the following regions at $\theta = \theta^*$:

$$M(\theta^*) = \{\theta: F(\theta^*) \subset F(\theta)\};$$

$$V(\theta^*) = \{\theta: F^=(\theta^*) \subset F^=(\theta) \text{ and } f^k(x, \theta) \leqq 0 \quad \forall x \in F(\theta^*),\quad k \in P^=(\theta^*),\ k \notin P^=(\theta)\};$$

and in their (occasionally easier to construct) subsets:

$$Z_1(\theta^*) = \{\theta: F(\theta^*) \subset F(\theta) \subset F^=(\theta^*)\};$$

$$Z_2(\theta^*) = \{\theta: F^=(\theta^*) = F^=(\theta) \text{ and } f^k(x, \theta) \leqq 0,\quad \forall x \in F(\theta^*),\ k \in P^<(\theta) \setminus P^<(\theta^*)\};$$

$$V_1(\theta^*) = \{\theta: F^=(\theta^*) \subset F^=(\theta) \text{ and } f^k(x, \theta) \leqq 0,\quad \forall x \in F^=(\theta^*),\ k \in P^<(\theta) \setminus P^<(\theta^*)\};$$

$$W(\theta^*) = \{\theta: F^=(\theta^*) \subset F^=(\theta) \text{ and } P^=(\theta^*) = P^=(\theta)\};$$

$$Z(\theta^*) = \{\theta: F^=(\theta^*) = F^=(\theta) \text{ and } f^k(x, \theta) \leqq 0,\quad \forall x \in F^=(\theta^*),\ k \in P^<(\theta) \setminus P^<(\theta^*)\}.$$

These subsets are needed to describe various properties of the convex model. Thus, a necessary condition and a sufficient condition for optimality of θ^* in *input opti-*

mization (see e.g. [6], [7], [8]) are stated over the sets $M(\theta^*)$ and $V(\theta^*)$. For bi--convex models these conditions are strengthened, but a necessary condition now holds over $V_1(\theta^*)$ while a sufficient condition holds over $Z_1(\theta^*) \cap Z_2(\theta^*)$ (see e.g. [8]). Continuity of the Lagrangian multiplier function is established in $V_1(\theta^*)$ while the function is generally discontinuous in $Z_1(\theta^*) \cap Z_2(\theta^*)$ (see [3]), etc. If Slater's condition holds, i.e. if

(1.1) "there exists $\hat{x}$ such that $f^k(\hat{x}, \theta^*) < 0\,, \quad k \in P$"

then $P^=(\theta^*) = \emptyset$, $F^=(\theta^*) = R^n$ and some of the above sets coincide. In particular, when (1.1) holds, one can specify

$$V(\theta^*) \equiv V_1(\theta^*) \equiv Z(\theta^*) \equiv Z_2(\theta^*) \equiv W(\theta^*) \equiv N(\theta^*)\,,$$

a neighbourhood of θ^*. Unfortunately, many real life situations (such as multiobjective decision making problems) are described by mathematical models (P, θ) for which Slater's condition does not or cannot hold. When studying such models one may have to use one or more regions of stability from the above variety. Recently, regions of stability $M(\theta^*)$ and $V(\theta^*)$ have been studied in abstract settings (e.g. in [4]).

In the next section we will show, by an example, that the marginal value formula does not generally hold on the region of stability $Z_2(\theta^*)$, contrary to the claim made in [5, Theorem 4.3]. Moreover, the formula does not work even on the smaller set $Z_1(\theta^*) \cap Z_2(\theta^*)$. However, we will prove that the formula does hold on the set $Z(\theta^*)$.

2. THE MARGINAL VALUE FORMULA

For some $\theta \in R^p$ consider the "reduced" Lagrangian

$$L^<(x, u; \theta) = f^0(x, \theta) + \sum_{k \in P^<(\theta)} u_k f^k(x, \theta)$$

where $P^<(\theta) = P \setminus P^=(\theta)$. It is well-known (see e.g. [6]) that $\tilde{x}(\theta) \in F^=(\theta)$ is an optimal solution of (P, θ) if, and only if, there exists $U(\theta) = (u_k(\theta))$, $u_k(\theta) \geqq 0$, $k \in P^<(\theta)$ such that

(2.1) $$L^<(\tilde{x}(\theta), u; \theta) \leqq L^<(\tilde{x}(\theta), U(\theta); \theta) \leqq L^<(x, U(\theta); \theta)$$

for every $x \in F^=(\theta)$ and every $u \in R_+^{q(\theta)}$. (Here $q(\theta)$ is the cardinality of the set $P^<(\theta)$ and $R_+^{q(\theta)}$ is the nonnegative orthant in $R^{q(\theta)}$.)

The marginal value formula will be formulated at an arbitrary but fixed $\theta = \theta^*$. Since for stable perturbations at θ^* we have $P^<(\theta^*) \subset P^<(\theta)$ (see e.g. [5] or [8, Theorem 3.1]) the Lagrangian multiplier function

(2.2) $$U_*(\theta) \equiv (u_k(\theta)), \quad k \in P^<(\theta^*)$$

appearing in (2.1) exists and is well-defined. In what follows we will consider the function $U_*(\theta)$ rather than $U(\theta) = (u_k(\theta))$, $k \in P^<(\theta)$. By concentrating only on the terms in $L^<$ belonging to the index set $P^<(\theta^*)$, we can further reduce the Lagrangian to

$$L_*^<(x, u; \theta) \equiv f^0(x, \theta) + \sum_{k \in P^<(\theta^*)} u_k f^k(x, \theta) .$$

Note that $L_*^<$ is used in (2.1) to characterize optimality of $\tilde{x}(\theta^*) \in F^=(\theta^*)$ for a fixed θ^*.

We will need a guarantee that the "slopes" of certain components of $U_*(\theta)$ are finite as $\theta \to \theta^*$. This is formalized by requiring a rather weak condition called "Property $Z(\theta^*)$".

2.1. **Definition.** *Consider the convex model* (P, θ) *at some* $\theta = \theta^*$. *Assume that* $\tilde{F}(\theta^*) \neq \emptyset$ *and bounded. We say that the model* (P, θ) *satisfies Property* $Z(\theta^*)$ *if for every path*

(2.3) $\quad \theta \in Z(\theta^*)\,, \quad \theta \to \theta^* \quad$ *such that* $\quad \tilde{u}_k(\theta) \to \tilde{u}_k(\theta^*)\,, \quad k \in P^<(\theta^*)$

the following limits exist:

(2.4) $$\lim_{\substack{\theta \to \theta^* \\ \theta \in Z(\theta^*)}} \frac{\tilde{u}_k(\theta^*) - \tilde{u}_k(\theta)}{\|\theta^* - \theta\|}\,, \quad k \in P^<(\theta) \cap P(\tilde{x}(\theta^*), \theta^*) .$$

(*Here* $P(\tilde{x}(\theta^*), \theta^*) = \{k \in P: f^k(\tilde{x}(\theta^*), \theta^*) = 0\}$ *is the set of active constraints at* $\tilde{x}(\theta^*)$ *and* $\tilde{u}_k(\theta^*) \equiv 0$, $k \in P^<(\theta) \setminus P^<(\theta^*)$.)

The existence of at least one path with the property (2.3) is guaranteed by e.g. [3, Theorem 3.1]. Note that Property $Z(\theta^*)$ is somewhat stronger than "Property $U(\theta^*)$" used in [8] in that the latter requires that the limits (2.4) exist for *at least one* path (2.3).

Finally, for a neighbourhood $N(\theta^*)$ of θ^*, denote

$$B(\theta^*) = \left\{\frac{\theta - \theta^*}{\|\theta - \theta^*\|} : \theta \in Z(\theta^*) \cap N(\theta^*),\ \theta \neq \theta^*\right\},$$

and all its limit (accumulation) points, when $\theta \in Z(\theta^*)$, $\theta \to \theta^*$, by $B^0(\theta^*)$. Note that $B^0(\theta^*)$ is a subset of the unit sphere (possibly a singleton). When Slater's condition holds then $B(\theta^*)$ *is* the unit sphere.

In addition to convexity of $f^i(\cdot, \theta)$ for every θ, we also assume below $f^i(x, \cdot)$ is convex for every x, $i \in \{0\} \cup P$. Such models are called *bi-convex*. The marginal value formula follows.

2.2. **Theorem.** *Consider the bi-convex model* (P, θ) *at* $\theta = \theta^*$. *Let* $\tilde{F}(\theta^*) \neq \emptyset$ *and bounded. Suppose that the saddle point* $(\tilde{x}(\theta^*), \tilde{u}(\theta^*))$ *in* (2.1) *is unique for* $\theta = \theta^*$ *and that Property* $Z(\theta^*)$ *holds. Also suppose that the Lagrangian multiplier function* $U_*(\theta)$ *is unique for every* $\theta \in Z(\theta^*) \cap N(\theta^*)$, *where* $N(\theta^*)$ *is a neighbourhood of* θ^*. *If* $f^i(x, \cdot)$, $i \in \{0\} \cup P^<(\theta^*)$ *are differentiable in* $Z(\theta^*) \cap N(\theta^*)$, *and if* *the derivatives* $[f^i(x, \theta)]'_\theta$, $i \in \{0\} \cup P^<(\theta^*)$ *are continuous functions in* x *at* $\tilde{x}(\theta^*)$ *for* $\theta = \theta^*$, *then for every fixed path* $\theta \in Z(\theta^*)$, $\theta \to \theta^*$ *such that*

$$l = \lim_{\substack{\theta \in Z(\theta^*) \\ \theta \to \theta^*}} \frac{\theta - \theta^*}{\|\theta - \theta^*\|}$$

for some $l \in B^0(\theta^*)$, *we have*

$$\lim_{\substack{\theta \in Z(\theta^*) \\ \theta \to \theta^*}} \frac{\tilde{f}(\theta) - \tilde{f}(\theta^*)}{\|\theta - \theta^*\|} = ([L_*^< (\tilde{x}(\theta^*), \tilde{u}(\theta^*); \theta)]'_{\theta=\theta^*}, l)\,. \tag{2.5}$$

Proof. The proof has two parts: first we prove the result for the case when the objective function $f^0(\tilde{x}(\theta^*), \cdot)$ is strictly convex and then we use Tihonov's regularization (e.g. [2], [5]) to prove it for a general convex function. For the strictly convex case, it is easy to show (see e.g. the proof of [8, Lemma 4.4] that

$$([L_*^<(\tilde{x}(\theta^*), \tilde{u}(\theta^*); \theta)]'_\theta,\ \theta - \theta^*) > \tilde{f}(\theta) - \tilde{f}(\theta^*) + \varepsilon(\theta) \tag{2.6}$$

for every $\theta \in V_1(\theta^*)$, $\theta \neq \theta^*$ and sufficiently close to θ^*, where

$$\varepsilon(\theta) \equiv L^<(\tilde{x}(\theta^*), \tilde{u}(\theta^*); \theta) - L^<(\tilde{x}(\theta^*), \tilde{u}(\theta); \theta)\,.$$

(Here we have used the fact that, under the assumptions of the theorem, $U_*(\theta)$ is a continuous function, see [3, Corollary 3.2].)

Since $Z(\theta^*) \subset V_1(\theta^*)$, the formula (2.6) also holds for $\theta \in Z(\theta^*)$, $\theta \neq \theta^*$ and close to θ^*. On the other hand, using Lemma 4.2 from [5], we have

$$\tilde{f}(\theta) - \tilde{f}(\theta^*) \geqq f^0(\tilde{x}(\theta), \theta) + \sum_{k \in P^<(\theta)} u_k f^k(\tilde{x}(\theta), \theta) - L_*^<(x, \tilde{u}(\theta^*); \theta^*)$$

for every $x \in F^=(\theta^*)$ and every $u \in R_+^{q(\theta)}$.

Now specify $u = (u_k)$ as follows:

$$u_k = \begin{cases} \tilde{u}_k(\theta^*) & \text{if} \quad k \in P^<(\theta^*) \\ 0 & \text{if} \quad k \in P^<(\theta) \setminus P^<(\theta^*) \end{cases}$$

and $x = \tilde{x}(\theta)$. (The former is possible since, as noted earlier, $P^<(\theta^*) \subset P^<(\theta)$; the latter is possible since $\tilde{x}(\theta) \in F(\theta) \subset F^=(\theta) = F^=(\theta^*)$.) This gives

$$\begin{aligned} \tilde{f}(\theta) - \tilde{f}(\theta^*) &\geqq L_*^<(\tilde{x}(\theta), \tilde{u}(\theta^*); \theta) - L_*^<(\tilde{x}(\theta), \tilde{u}(\theta^*); \theta^*) > \\ &> ([L_*^<(\tilde{x}(\theta), \tilde{u}(\theta^*); \theta)]'_{\theta=\theta^*},\ \theta - \theta^*) \end{aligned} \tag{2.7}$$

by strict convexity of $L_*^< = L_*^<(\theta)$ and the gradient inequality. The difference $\tilde{f}(\theta) - \tilde{f}(\theta^*)$ is thus bounded by (2.6) and (2.7). Now divide by $\|\theta - \theta^*\| > 0$ and set $\theta \to \theta^*$ such that

$$\lim_{\substack{\theta \in Z(\theta^*) \\ \theta \to \theta^*}} \frac{\theta - \theta^*}{\|\theta - \theta^*\|} = l \in B^0(\theta^*)\,.$$

First we note that

$$\lim_{\substack{\theta \in Z(\theta^*) \\ \theta \to \theta^*}} \frac{\varepsilon(\theta)}{\|\theta - \theta^*\|} = 0$$

by the Property $Z(\theta^*)$, see e.g. [8]. Therefore the limits on the bounds coincide, i.e.

$$\lim_{\substack{\theta \in Z(\theta^*) \\ \theta \to \theta^*}} \left([L_*^<(\tilde{x}(\theta^*), \tilde{u}(\theta^*); \theta]'_\theta , \frac{\theta - \theta^*}{\|\theta - \theta^*\|} \right) =$$

$$= \lim_{\substack{\theta \in Z(\theta^*) \\ \theta \to \theta^*}} \left([L_*^< (\tilde{x}(\theta), \tilde{u}(\theta^*); \theta)]'_{\theta=\theta^*} , \frac{\theta - \theta^*}{\|\theta - \theta^*\|} \right) =$$

$$= ([L_*^<(\tilde{x}(\theta^*), \tilde{u}(\theta^*); \theta)]'_{\theta=\theta^*}, l)$$

by the continuous differentiability of a differentiable convex function and uniqueness of $\tilde{x}(\theta^*)$. This proves (2.5).

In order to prove the formula for a convex objective function, we consider the "regularized" problem

$$\text{(TP, } \theta) \qquad \begin{aligned} &\min_{(x)} F^0(x, \theta, \varepsilon) \equiv f^0(x, \theta) + \varepsilon\|\theta\|^2 \\ &\text{s.t.} \\ &f^k(x, \theta) \leqq 0 , \quad k \in P \end{aligned}$$

where $\varepsilon > 0$. Now $F^0(x, \cdot, \varepsilon)$ is strictly convex and therefore

$$\lim_{\substack{\theta \in Z(\theta^*) \\ \theta \to \theta^*}} \frac{\tilde{F}(\theta, \varepsilon) - \tilde{F}(\theta^*, \varepsilon)}{\|\theta - \theta^*\|} = ([L_*^<(\tilde{x}(\theta^*, \varepsilon), \tilde{u}(\theta^*, \varepsilon); \theta; \varepsilon)]'_{\theta=\theta^*}, l) \tag{2.8}$$

with l as before. Here

$$L_*^<(\tilde{x}(\theta^*, \varepsilon), \tilde{u}(\theta^*, \varepsilon); \theta; \varepsilon) =$$

$$= f^0(\tilde{x}(\theta^*, \varepsilon), \theta) + \varepsilon\|\theta\|^2 + \sum_{k \in P^<(\theta^*)} \tilde{u}_k(\theta^*, \varepsilon) f^k(\tilde{x}(\theta^*, \varepsilon), \theta) ,$$

$\tilde{F}(\theta, \varepsilon)$ is the optimal value of (TP, θ) and $(\tilde{x}(\theta^*, \varepsilon), \tilde{u}(\theta^*, \varepsilon))$ is a saddle point. Since the feasible set of (TP, θ) does not depend on ε, under the assumptions of the theorem $\varepsilon \to 0$ implies $(\tilde{x}(\theta^*, \varepsilon), \tilde{u}(\theta^*, \varepsilon)) \to (\tilde{x}(\theta^*), \tilde{u}(\theta^*))$. Therefore (2.8) gives (2.5). ∎

The example below (adjusted from [3]) shows that the marginal value formula does not generally work on the region of stability $Z_1(\theta^*) \cap Z_2(\theta^*)$ even for bi-linear models.

2.3. Example. Consider the model

$$\min f^0 = x$$

s.t.

$$f^1 = -\theta x \leqq 0$$

$$f^2 = -\theta - x \leqq 0$$

around $\theta^* = 0$.

Here

$$F(\theta) = \begin{cases} [0, \infty) & \text{if} \quad \theta \geqq 0 \\ \emptyset & \text{if} \quad \theta < 0 . \end{cases}$$

For $\theta \geqq 0$ we have the following situation:

$$P^{<}(\theta) = \begin{cases} \{2\} & \text{if} \quad \theta = 0 \\ \{1, 2\} & \text{if} \quad \theta > 0 \end{cases}$$

while $F^{=}(\theta) = (-\infty, \infty)$ for every $\theta \geqq 0$. Further,

$$Z_1(\theta^*) = Z_2(\theta^*) = Z_1(\theta^*) \cap Z_2(\theta^*) = [0, \infty) .$$

In order to apply the marginal value formula (2.5) for $\theta > 0$, $\theta \to \theta^*$, we find that

$$L^{<}_{*}(x, u; \theta) = f^0(x, \theta) + \sum_{k \in P^{<}(\theta^*)} u_k f^k(x, \theta) = x + u_2(-\theta - x) .$$

Since $\tilde{x}(\theta^*) = 0$ and $\tilde{u}_2(\theta^*) = 1$, this gives

$$L^{<}_{*}(\tilde{x}(\theta^*), \tilde{u}(\theta^*); \theta) = -\theta$$

and

$$[L^{<}_{*}(\tilde{x}(\theta^*), \tilde{u}(\theta^*); \theta)]'_{\theta = \theta^*} = -1 .$$

On the other hand, since

$$\frac{\theta - \theta^*}{|\theta - \theta^*|} = 1$$

for every $\theta > 0$, we find that $l = 1$. Therefore the right-hand side in the marginal value formula (2.5) is

$$([L^{<}_{*}(\tilde{x}(\theta^*), \tilde{u}(\theta^*); \theta)]'_{\theta = \theta^*}, l) = -1 .$$

But $\tilde{x}(\theta) = 0$ and $\tilde{f}(\theta) = 0$ for every $\theta \geqq 0$. This implies

$$\lim_{\substack{\theta > 0 \\ \theta \to 0}} \frac{\tilde{f}(\theta) - \tilde{f}(\theta^*)}{|\theta - \theta^*|} = 0 .$$

We have obtained a contradiction.

Conclusion. The marginal value formula does not generally work for the set $Z_1(\theta^*) \cap Z_2(\theta^*)$. ■

Comment. The failure of the marginal value formula on the set $Z_2(\theta^*)$ is caused here by discontinuity of the Lagrange multiplier function. Indeed, in the above example we find that

$$\tilde{u}_2(\theta) = \begin{cases} 1 & \text{if} \quad \theta = 0 \\ 0 & \text{if} \quad \theta > 0 \end{cases} .$$

Without the continuity one cannot arrive at the relation (2.6). The uniqueness assumptions in Theorem 2.2 can be omitted; the marginal value formula then assumes a minimax form (see [1] and [5]). ∎

The formula (2.5) is an important tool in input optimization. If the inner product in (2.5) is negative for some path $\theta \in Z(\theta^*)$; $\theta \to \theta^*$ then, locally along this path, $\tilde{f}(\theta) < \tilde{f}(\theta^*)$. This generates a new $\theta = \theta_{NEW}$ that "improves" the model from (P, θ^*) to (P, θ_{NEW}). The paths are chosen in computable subsets of the stable region $Z(\theta^*)$. This leads to "optimal realizations" of mathematical models (see [8], [9]). When Slater's condition is satisfied ,then $B^0(\theta^*)$ is the unit sphere, $Z(\theta^*) = N(\theta^*)$, Property $Z(\theta^*)$ holds for differentiable functions, and the marginal value formula holds in a neighbourhood of θ^* (see [1]).

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Souhrn

OBLASTI STABILITY PRO NEKOREKTNĚ FORMULOVANÉ PROBLÉMY KONVEXNÍHO PROGRAMOVÁNÍ: DODATEK

SANJO ZLOBEC

Formule pro marginální hodnotu v konvexní optimalizaci platí v užší oblasti než bylo uvedeno původně v literatuře. To plyne ze skutečnosti, že existují oblasti stability, ve kterých Lagrangeův multiplikátor je nespojitou funcí i pro lineární modely.

Резюме

ОБЛАСТИ УСТОЙЧИВОСТИ ДЛЯ НЕКОРРЕКТНЫХ ВЫПУКЛЫХ ПРОГРАММ: ДОБАВЛЕНИЕ

Sanjo Zlobec

Формула для маргинального значения в выпуклой оптимизации верна в более ограниченной области устойчивости, чем недавно утверждалось в литературе. Причиной тому является существование областей устойчивости, в которых мультипликатор Лагранжа является разрывной функцией даже для линейных моделей.

Author's address: Prof. *Sanjo Zlobec*, McGill University, Department of Mathematics and Statistics, Burnside Hall, 805 Sherbrooke Street West, Montreal, Quebec, Canada H3A 2K6.