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EQUATIONS OF MAGNETOHYDRODYNAMICS OF COMPRESSIBLE FLUID: PERIODIC SOLUTIONS

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1. INTRODUCTION

An initial boundary value problem for a system of equations of magnetohydrodynamics of incompressible, electrically conducting and viscous fluid was treated in [5]. The existence of time periodic solutions of a slightly more general system was dealt with in [4]. In [6] A. Valli proved the global existence and exponential stability of solutions to the initial-boundary value problem for the Navier-Stokes equations for the flow of compressible and barotropic fluid assuming that both the initial velocity and the external force are small and the initial density is not far from a constant. As a consequence it has been shown that small time periodic external forces give rise to periodic solutions of the problem in question.

The aim of this paper is to show that the methods of [6] can be applied when an initial-boundary value problem for a model of magnetohydrodynamics is studied. The model treated below consists of a standard system of equations, see [7], and [3] as far as the boundary conditions are concerned, in which the displacement current in the Maxwell equations and the Lorenz electric force in the momentum equation are allowed for. Unfortunately, Ohm's law is adopted in its simplest form neglecting both Hall's effect and the convective current.

By Ω we shall denote a region with a smooth boundary $\partial\Omega$ and homeomorphic to a ball. For $T > 0$ we set

$$Q_T = (0, T) \times \Omega\,, \quad \Sigma_T = (0, T) \times \partial\Omega\,.$$

In Q_T we shall take the following system of equations:

$$\varrho(v_t + (v\,.\,\nabla)\,v) = \eta\,\Delta v + (\zeta + \eta/3)\,\nabla\,\mathrm{div}\,v - \nabla p + qE + j \times B + \varrho b\,, \tag{1.1}$$

$$\varrho_t + \mathrm{div}\,(\varrho v) = 0\,, \tag{1.2}$$

$$B_t + \mathrm{rot}\,E = 0\,, \tag{1.3}$$

$$\operatorname{div} B = 0\,, \tag{1.4}$$

$$\varepsilon E_t + j - \mu^{-1} \operatorname{rot} B = 0\,, \tag{1.5}$$

$$q = \varepsilon \operatorname{div} E\,, \tag{1.6}$$

$$j = \varkappa(E + v \times B)\,. \tag{1.7}$$

Moreover, the functions v, ϱ, B, E are to satisfy the boundary conditions on Σ_T:

$$v = 0\,, \tag{1.8}$$

$$B_n = 0\,, \tag{1.9}$$

$$E_\tau = 0\,, \tag{1.10}$$

and the initial conditions on Ω:

$$v(0, \cdot) = v_0\,, \tag{1.11}$$

$$\varrho(0, \cdot) = \varrho_0\,, \tag{1.12}$$

$$B(0, \cdot) = B_0\,, \tag{1.13}$$

$$E(0, \cdot) = E_0\,. \tag{1.14}$$

In these equations we denote by

v	the velocity of the fluid,
ϱ	the density,
b	the given external mass force,
$p = p(\varrho)$	the pressure (the barotropic case),
B	the magnetic field,
E	the electric field,
j	the electric current,
q	the net charge.

The constants η, ζ, ε, μ and $\varkappa$ are supposed to be positive. The subscripts n and τ denote the normal and tangential components of a vector, i.e., if n denotes the unit outward normal to $\partial\Omega$ at a point $x \in \partial\Omega$ and "$\cdot$" the scalar product in R^3, then we set

$B_n = B \,.\, n$ – the normal component of B at x,
$E_\tau = E - (E \,.\, n)\, n$ – the tangential component of E at x.

As far as the notation is concerned we shall combine those of [6], [5] and [2]. The domain Ω remains fixed and therefore the symbol Ω in the notations of spaces will be suppressed.

We shall denote by H^k the space of real functions on Ω which along with their generalized derivatives up to order k belong to $L^2(\Omega)$. For $u \in H^k$ we set

$$\|u\|_k^2 = \Big(\sum_{|\alpha| \leqq k} \|D^\alpha u\|_0^2\Big)^{1/2}\,,$$

where

$$\|u\|_0^2 = \int_\Omega u^2(x)\,\mathrm{d}x\,.$$

The scalar product in $H^0 = L^2(\Omega)$ is

$$\langle u, v\rangle_0 = \int_\Omega u(x)\,v(x)\,\mathrm{d}x\,.$$

By H_0^1 we denote the closure of $C_0^\infty(\Omega)$ in H^1. For $0 < T < +\infty$, X a Banach space and j a non-negative integer, $C^j([0, T]; X)$ denotes the space of functions whose derivatives up to order j are continuous from $[0, T]$ into X. Similarly, $C_B^j(R^+, X)$ denotes the functions from R^+ into X whose derivatives up to order k are continuous and bounded on R^+.

The norms in $L^q(0, T; H^k)$ and $L^q(0, T, L^s(\Omega))$ will be denotet by $[\cdot]_{q,k,T}$ and $|||\cdot|||_{q,s,T}$, respectively.

In what follows we shall not make any difference in the notation of scalar and vector functions on Ω and Q_T.

A vector $v = (v_1, v_2, v_3)$ is called solenoidal if $\operatorname{div} v = 0$. Following [2], we denote

${}^\circ J$ the closure in $L^2(\Omega)$ of solenoidal vectors from $C_0^\infty(\Omega)$,

J the closure in $L^2(\Omega)$ of solenoidal vectors from $C^1(\Omega)$,

and for an integer l, $l \geqq 1$ we shall use the notations

$$J^l = \{u \in H^l;\ \operatorname{div} u = 0 \text{ in } \Omega\}\,,$$
$${}^\circ J_\tau^l = \{u \in J^l;\ u_\tau = 0 \text{ on } \partial\Omega\}\,,$$
$${}^\circ J_n^l = \{u \in J^l;\ u_n = 0 \text{ on } \partial\Omega\}\,,$$

and for $l = 0$

$${}^\circ J_\tau^0 = J\,,\quad {}^\circ J_n^0 = {}^\circ J\,.$$

Following the notation of [5] we set

$$\hat{J} = \{B = (B_1, B_2, B_3);\ B \in J^2,\ B_n = 0 \text{ and } (\operatorname{rot} B)_\tau = 0 \text{ on } \partial\Omega\}\,.$$

By Theorem 7.1 from [2], rot is a homeomorphism of ${}^\circ J_\tau^l$ onto ${}^\circ J_n^{l-1}$. The inverse mapping we denote by Z, i.e., if $B \in {}^\circ J_n^{l-1}$, we denote by ZB the function $w \in H^l$ satisfying $\operatorname{div} w = 0$, $\operatorname{rot} w = B$ in Ω and $w_\tau = 0$ on $\partial\Omega$.

We shall use an auxiliary operator V defined as follows: for $a \in H^k$, a scalar function, we set $Va = \operatorname{grad}\varphi$, where φ satisfies $\Delta\varphi = a$ in Ω and $\varphi = 0$ on $\partial\Omega$. Hence, for every positive integer k, V is a linear bounded operator from H^{k-1} into H^k. Moreover, $(Va)_\tau = 0$ on $\partial\Omega$. We start the investigation of the system (1.1)–(1.14) by reducing it to an equivalent system. If we denote

$$a(t) = \operatorname{div} E(t)\,,$$

then, by (1.3), we have rot $E = -B_t$ and, by the definition of V, rot $(E - Va) = -B_t$. Since div $(E - Va) = 0$ in Q and $(E - Va)_\tau = 0$ on $(0, T) \times \partial\Omega$ we immediately obtain

$$E = Va - ZB_t\,.$$

When j from (1.7) is inserted into (1.5) and div applied to the resulting relation, an equation for a is obtained. Further, (1.3), (1.5) and (1.7) yield a second order equation satisfied by B. Throughout the paper we shall suppose

$$0 < m \leqq \varrho_0(x) \leqq M \quad \text{on} \quad \Omega\,.$$

Setting

$$\bar{\varrho} = \int_\Omega \varrho_0(x)\,\mathrm{d}x/\mathrm{meas}\,(\Omega)$$

we denote

$$\sigma = \varrho - \bar{\varrho}$$

Thus the following system of equations for v, σ, B and a corresponds to (1.1)–(1.14):

(1.15) $$(\bar{\varrho} + \sigma)(v_t + (v\,.\,\nabla)\,v) = \eta\,\Delta v + (\zeta + \eta/3)\,\nabla\,\mathrm{div}\,v - \\ - \nabla p(\bar{\varrho} + \sigma) + \varepsilon a(Va - ZB_t) + \\ + \varkappa(Va - ZB_t + v \times B) \times B + (\bar{\varrho} + \sigma)\,b \quad \text{in} \quad Q_T\,,$$

(1.16) $$\sigma_t + v\,.\,\nabla\sigma + \sigma\,\mathrm{div}\,v + \bar{\varrho}\,\mathrm{div}\,v = 0 \quad \text{in} \quad Q_T\,,$$

(1.17) $$\varepsilon\mu B_{tt} + \varkappa\mu B_t + \mathrm{rot}\,\mathrm{rot}\,B = \varkappa\mu\,\mathrm{rot}\,(v \times B) \quad \text{in} \quad Q_T\,,$$

(1.18) $$\mathrm{div}\,B = 0 \quad \text{in} \quad Q_T\,,$$

(1.19) $$\varepsilon\varkappa^{-1}a_t + a = -\mathrm{div}\,(v \times B) \quad \text{in} \quad Q_T\,,$$

(1.20) $$v = 0 \quad \text{on} \quad \Sigma_T\,,$$

(1.21) $$B_n = 0 \quad \text{on} \quad \Sigma_T\,,$$

(1.22) $$\mathrm{rot}_\tau\,B = 0 \quad \text{on} \quad \Sigma_T\,,$$

(1.23) $$v(0, \cdot) = v_0 \quad \text{in} \quad \Omega\,,$$

(1.24) $$\sigma(0, \cdot) = \sigma_0 \quad \text{in} \quad \Omega\,,$$

(1.25) $$B(0, \cdot) = B_0 \quad \text{in} \quad \Omega\,,$$

(1.26) $$B_t(0, \cdot) = B_1 \quad \text{in} \quad \Omega\,,$$

(1.27) $$a(0, \cdot) = a_0 \quad \text{in} \quad \Omega\,,$$

where

$$\sigma_0 = \varrho_0 - \bar{\varrho}\,,$$

$$B_1 = -\operatorname{rot} E_0 \quad \text{and} \quad a_0 = \operatorname{div} E_0\,.$$

In (1.22) we have used $\operatorname{rot}_\tau B$ to denote $(\operatorname{rot} B)_\tau$. From now on we shall keep to this simplified notation.

We shall use results and methods of [6] when investigating the problem given by (1.15)–(1.27). It is easy to prove that from a solution of (1.15)–(1.27) one can get a solution of the original system (1.1)–(1.14).

2. LINEARIZED EQUATIONS

In this section we give some auxiliary assertions on existence of solutions of linearized problems. The first two are taken over from [6].

First we shall deal with the problem

$$\begin{aligned} &\tilde{\varrho} v_t + A v = F \quad \text{in} \quad Q_T\,, \\ &v = 0 \quad \text{on} \quad \Sigma_T\,, \\ &v(0) = v_0 \quad \text{in} \quad \Omega\,, \end{aligned} \tag{2.1}$$

where

$$A = -\eta\Delta - (\zeta + \eta/3)\operatorname{grad}\operatorname{div}$$

and $\tilde{\varrho}$, F and v_0 are given functions. By Lemma 2.2 from [6], we have

Lemma 2.1. *Let* $0 < m/2 \leqq \tilde{\varrho}(t, x) \leqq 2M$ *a.e. in* Q_T, $0 < m \leqq \tilde{\varrho}(0, x) \leqq M$ *a.e. in* Ω, $\nabla\tilde{\varrho} \in L^4(0, T;\ L^6(\Omega))$, $\tilde{\varrho}_t \in L^2(0, T;\ L^3(\Omega))$, $F \in L^2(0, T;\ H^1)$, $F_t \in L^2(0, T;\ H^{-1})$ *and* $v_0 \in H^2 \cap H^1_0$. *Then the solution* v *of* (2.1) *is such that* $v \in L^2(0, T;\ H^3) \cap C^0([0, T];\ H^2)$,

$$v_t \in L^2(0, T;\ H^1) \cap L^\infty(0, T;\ H^0)$$

and

$$\begin{aligned} &[v]^2_{\infty,2,T} + [v]^2_{2,3,T} + [v_t]^2_{\infty,0,T} + [v_t]^2_{2,1,T} \leqq \\ &\leqq c\{[F]^2_{2,1,T} + [F]^2_{\infty,0,T} + ([F_t]^2_{2,-1,T} + \|v_0\|^2_2 + \\ &+ \|F(0)\|^2_0)\,(1 + |||\nabla\tilde{\varrho}|||^4_{4,6,T} + |||\tilde{\varrho}_t|||^2_{2,3,T})\exp\,(c|||\tilde{\varrho}_t|||^2_{2,3,T})\}\,. \end{aligned} \tag{2.2}$$

Here H^{-1} denotes the dual of H^1_0. Further we shall need a solution to

$$\begin{aligned} &\sigma_t + \tilde{v}\,.\,\nabla\sigma + \sigma\operatorname{div}\tilde{v} + \bar{\varrho}\operatorname{div}\tilde{v} = 0 \quad \text{in} \quad Q_T\,, \\ &\sigma(0) = \sigma_0 \quad \text{in} \quad \Omega\,. \end{aligned} \tag{2.3}$$

By [6], Lemma 2.3, we have

Lemma 2.2. *Let $\tilde{v} \in L^1(0, T, H^3)$, $\tilde{v} \,.\, n = 0$ on Σ_T, and $\sigma_0 \in H^2$ with $\int_\Omega \sigma_0 \,\mathrm{d}x = 0$. Then there exists a unique solution σ of (2.3) such that $\sigma \in C^0([0, T]; H^2)$, $\int_\Omega \sigma(t, x) \,\mathrm{d}x = 0$ for each $t \in [0, T]$ and*

$$(2.4) \qquad [\sigma]_{\infty,2,T} \leqq c(\|\sigma_0\|_2 + 1) \exp\left(c[\tilde{v}]_{1,3,T}\right).$$

If, in addition, $\tilde{v} \in C^0([0, T], H^2)$, then $\sigma_t \in C^0([0, T]; H^1)$ and

$$(2.5) \qquad [\sigma_t]_{\infty,1,T} \leqq c[\tilde{v}]_{\infty,2,T}(\|\sigma_0\|_2 + 1) \exp\left(c[\tilde{v}]_{1,3,T}\right).$$

Further, we shall deal with the following system:

$$(2.6) \qquad \begin{aligned} &\varepsilon\mu B_{tt} + \varkappa\mu B_t + \operatorname{rot}\operatorname{rot} B = G \quad \text{in} \quad Q_T \\ &B_n = 0\,, \quad \operatorname{rot}_\tau B = 0 \quad \text{on} \quad \Sigma_T\,, \\ &B(0) = B_0\,, \quad B_t(0) = B_1 \quad \text{in} \quad \Omega\,. \end{aligned}$$

We give the following two existence results concerning (2.6).

Lemma 2.3. *Let $G \in L^\infty(0, T; {}^\circ J^1_n)$, $B_0 \in \hat{J}$ and $B_1 \in {}^\circ J^1_n$.*

(i) *Then there exists a unique $B \in L^\infty(0, T; \hat{J})$, $B_t \in L^\infty(0, T; {}^\circ J^1_n)$, $B_{tt} \in L^\infty(0, T; {}^\circ J)$ satisfying* (2.6).

(ii) *Moreover, B satisfies*

$$(2.7) \qquad \begin{aligned} &[B]^2_{\infty,2,T} + [B_t]^2_{\infty,1,T} \leqq c(\|B_0\|^2_2 + \|B_1\|^2_1 + T[G]^2_{\infty,1,T})\,, \\ &[B_{tt}]^2_{\infty,0,T} \leqq c\{\|B_0\|^2_2 + \|B_1\|^2_1 + T[G]^2_{\infty,1,T} + [G]^2_{\infty,0,T}\}\,. \end{aligned}$$

Lemma 2.4. *Let $G \in C([0, T]; {}^\circ J^1_n)$, $B_0 \in \hat{J}$, $B_1 \in {}^\circ J^1_n$.*

(i) *Then there exists a unique $B \in C([0, T]; \hat{J}) \cap C^1([0, T]; {}^\circ J^1_n) \cap C^2([0, T]; {}^\circ J)$ satisfying* (2.6).

(ii) *There exist positive constants d_1, d_2 such that, for every $0 \leqq \tau < t \leqq T$, the following inequality holds:*

$$(2.8) \qquad \begin{aligned} \psi(B)(t) - \psi(B)(\tau) + d_1 \int_\tau^t (\|\operatorname{rot}\operatorname{rot} B(\vartheta, \cdot)\|^2_0 + \|\operatorname{rot} B_t(\vartheta, \cdot)\|^2_0) \,\mathrm{d}\vartheta \leqq \\ \leqq d_2 \int_\tau^t (\|\operatorname{rot} G(\vartheta, \cdot)\|^2_0 + \|G(\vartheta, \cdot)\|^2_0) \,\mathrm{d}\vartheta\,, \end{aligned}$$

where ψ is defined by

$$(2.9) \qquad \begin{aligned} \psi(B)(t) = \|\operatorname{rot}\operatorname{rot} B(t)\|^2_0 + \varepsilon\mu \|\operatorname{rot} B_t(t, \cdot)\|^2_0 + \\ + \tfrac{1}{2}\varkappa\mu \langle \operatorname{rot} B(t), \operatorname{rot} B_t(t) \rangle_0 + \varkappa^2\mu(4\varepsilon)^{-1} \|\operatorname{rot} B(t)\|^2_0\,. \end{aligned}$$

Remark. It is easy to show that

$$c_1\,\psi(t) \leqq \|B_t(t,\cdot)\|_1^2 + \|B(t,\cdot)\|_2^2 \leqq c_2\,\psi(t)$$

with positive constants c_1 and c_2 which are independent of B.

We shall sketch the proofs of these two lemmas using the method of [5]. Let $\{\alpha_k\}_{k=1}^{\infty}$ be the system of all functions from $\hat{J}$ satisfying

$$\operatorname{rot}\operatorname{rot}\alpha_k = \lambda_k\alpha_k \quad (\lambda_k > 0)\,,$$
$$\langle\alpha_k, \alpha_l\rangle_0 = \delta_l^k$$

with λ_k nondecreasing. $\{\alpha_k\}_{k=1}^{\infty}$ is a complete orthonormal system in $\mathring{J}$. For a function φ, let $\varphi_k = \langle\varphi, \alpha_k\rangle_0$. Then

(i) $\varphi \in {}^\circ J$ if and only if $\sum\limits_{k=1}^{\infty}\varphi_k^2 < +\infty$,

(ii) $\varphi \in {}^\circ J_n^1$ if and only if $\sum\limits_{k=1}^{\infty}\lambda_k\varphi_k^2 < +\infty$,

(iii) $\varphi \in \hat{J}$ if and only if $\sum\limits_{k=1}^{\infty}\lambda_k^2\varphi_k^2 < +\infty$.

Moreover,

$$\|\varphi\|_0^2 = \sum_{k=1}^{\infty}\varphi_k^2 \quad \text{for} \quad \varphi \in {}^\circ J\,,$$
$$\|\operatorname{rot}\varphi\|_0^2 = \sum_{k=1}^{\infty}\lambda_k\varphi_k^2 \quad \text{for} \quad \varphi \in {}^\circ J_n^1\,,$$
$$\|\operatorname{rot}\operatorname{rot}\varphi\|_0^2 = \sum_{k=1}^{\infty}\lambda_k^2\varphi_k^2 \quad \text{for} \quad \varphi \in \hat{J}\,.$$

Let us also recall that

$$c_1\|\varphi\|_1^2 \leqq \|\operatorname{rot}\varphi\|_0^2 \leqq c_2\|\varphi\|_1^2 \quad \text{for} \quad \varphi \in {}^\circ J_n^1\,, \tag{2.10}$$

and

$$c_1\|\varphi\|_2^2 \leqq \|\operatorname{rot}\operatorname{rot}\varphi\|_0^2 \leqq c_2\|\varphi\|_2^2 \quad \text{for} \quad \varphi \in \hat{J}\,. \tag{2.11}$$

If $a, b \in H^1$ and $a_\tau = 0$ on $\partial\Omega$, then

$$\int_\Omega \operatorname{rot} a\,.\,b = \int_\Omega a\,.\operatorname{rot} b\,.$$

Denoting $M_n = \operatorname{lin}\{\alpha_k\}_{k=1}^{n}$ and $B^n(t) = \sum\limits_{k=1}^{n} b_k(t)\,\alpha_k$, with b_k satisfying

$$\begin{gathered}\varepsilon\mu\,\ddot{b}_k(t) + \varkappa\mu\,\dot{b}_k(t) + \lambda_k\,b_k(t) = \langle G(t), \alpha_k\rangle_0\,,\\ b_k(0) = \langle B_0, \alpha_k\rangle_0\,, \quad \dot{b}_k(0) = \langle B_1, \alpha_k\rangle_0\,,\end{gathered} \tag{2.12}$$

we find that B^n satisfy

$$(2.13)\qquad \langle \varepsilon\mu\, B^n_{tt}(t) + \varkappa\mu\, B^n_t(t) + \operatorname{rot}\operatorname{rot} B^n(t), w\rangle_0 = \langle G(t), w\rangle_0\,, \quad w \in M_n\,.$$

Taking $w = \operatorname{rot}\operatorname{rot} B^n_t$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\varepsilon\mu\|\operatorname{rot} B^n_t(t)\|_0^2 + \|\operatorname{rot}\operatorname{rot} B^n(t)\|_0^2\} + \varkappa\mu\|\operatorname{rot} B^n_t(t)\|_0^2 \leqq (\varkappa\mu)^{-1}\|\operatorname{rot} G(t)\|_0^2\,.$$

Using this inequality and proceeding along the standard lines, we complete the proof of Lemma 2.3.

Substituting $w = \operatorname{rot}\operatorname{rot} B^n$ into (2.13), we get the inequality

$$\frac{\mathrm{d}}{\mathrm{d}t}\{2\varepsilon\mu\langle\operatorname{rot} B^n(t), \operatorname{rot} B^n_t(t)\rangle_0 + \varkappa\mu\|\operatorname{rot} B^n(t)\|_0^2\} -$$
$$- 2\varepsilon\mu\|\operatorname{rot} B^n_t(t)\|_0^2 + \|\operatorname{rot}\operatorname{rot} B^n(t)\|_0^2 \leqq \|G(t)\|_0^2\,,$$

which multiplied by $\varkappa/(4\varepsilon)$ and added to the preceding one yields

$$(2.14)\qquad \frac{\mathrm{d}}{\mathrm{d}t}[\psi(B^n)(t)] + \varkappa(4\varepsilon)^{-1}\|\operatorname{rot}\operatorname{rot} B^n(t)\|_0^2 + \tfrac{1}{2}\varkappa\mu\|\operatorname{rot} B^n_t(t)\|_0^2 \leqq$$
$$\leqq (\sigma\mu)^{-1}\|\operatorname{rot} G(t)\|_0^2 + (\sigma/4\varepsilon)\|G(t)\|_0^2\,.$$

If $G \in C([0, T]; {}^\circ J^1_n)$, then the series

$$G(t) = \sum_{k=1}^{\infty}\langle G(t), \alpha_k\rangle_0\,\alpha_k$$

converges in $C([0, T]; {}^\circ J^1_n)$. By direct computation, this implies that

$$\sum_{k=1}^{\infty} b_k(t)\,\alpha_k \quad \text{converges in} \quad C([0, T]; \hat{J})\,,$$

$$\sum_{k=1}^{\infty} \dot{b}_k(t)\,\alpha_k \quad \text{converges in} \quad C([0, T]; {}^\circ J^1_n)$$

and

$$\sum_{k=1}^{\infty} \ddot{b}_k(t)\,\alpha_k \quad \text{converges in} \quad C([0, T]; {}^\circ J)\,.$$

This gives part (i) of Lemma 2.4. Integrating (2.14) on $[\tau, t]$, we have (2.8) for $B = B^n$. Letting $n \to \infty$, we complete the proof.

The last lemma in this section deals with the equation obtained by linearizing (1.19). We are looking for a function a satisfying

$$(2.15)\qquad \varepsilon\varkappa^{-1}a_t + a = h\,, \quad 0 \leqq t \leqq T, \quad a(0) = a_0\,.$$

The proof of the following lemma is straightforward.

Lemma 2.5. *Let* $h \in L^\infty(0, T; H^k)$ *and* $a_0 \in H^k$.

(i) *There is a unique* $a \in C(0, T; H^k)$ *with* $a_t \in L^\infty(0, T; H^k)$ *satisfying* (2.14). *Moreover,*

$$(2.16) \qquad \begin{aligned} [a]^2_{\infty,k,T} &\leqq c(\|a_0\|^2_k + T[h]^2_{\infty,k,T}), \\ [a_t]^2_{\infty,k,T} &\leqq c(\|a_0\|^2_k + [h]^2_{\infty,k,T}). \end{aligned}$$

(ii) *If* $h \in C([0, T]; H^k)$, *then* $a \in C^1([0, T]; H^k)$ *and for every* τ, t, $0 \leqq \tau < t \leqq T$, *we have*

$$(2.17) \qquad \|a(t)\|^2_k - \|a(\tau)\|^2_k + d_3 \int_\tau^t \|a(\vartheta)\|^2_k \, \mathrm{d}\vartheta \leqq d_4 \int_\tau^t \|h(\vartheta)\|^2_k \, \mathrm{d}\vartheta$$

with positive constants d_3 *and* d_4 *independent of* h.

3. LOCAL EXISTENCE

We set

$$\begin{aligned} X_T &= \{v;\ v \in L^\infty(0, T; H^2) \cap L^2(0, T; H^3), \\ &\qquad v_t \in L^\infty(0, T; H^0) \cap L^2(0, T; H^1)\}, \\ Y_T &= \{\sigma;\ \sigma \in L^\infty(0, T; H^2),\ \sigma_t \in L^\infty(0, T, H^1)\}, \\ \mathscr{B}_T &= \{B;\ B \in L^\infty(0, T; \hat{J}),\ B_t \in L^\infty(0, T; {}^\circ J^1_n),\ B_{tt} \in L^\infty(0, T; {}^\circ J)\}, \\ \mathscr{A}_T &= \{a;\ a \in L^\infty(0, T; H^1),\ a_t \in L^\infty(0, T; H^1)\}. \end{aligned}$$

The space X_T is equipped with the norm

$$\|v\|_{X_T} = \max\{[v]_{\infty,2,T}, [v]_{2,3,T}, [v_t]_{\infty,0,T}, [v_t]_{2,1,T}\}.$$

The norms in Y_T, $\mathscr{B}_T$ and $\mathscr{A}_T$ are defined in an obvious manner ensuring the completeness of the spaces involved.

Let T, K_1, K_2 be positive constants. Following [6] we introduce

$$\begin{aligned} R_T = \{&(v, \sigma, B, a);\ v \in X_T,\ \sigma \in Y_T,\ B \in \mathscr{B}_T,\ a \in \mathscr{A}_T, \\ &\|v\|_{X_T} \leqq K_1,\quad v(0) = v_0, \\ &[\sigma]_{\infty,2,T} \leqq K_1,\quad [\sigma_t]_{\infty,1,T} \leqq K_2,\quad \sigma(0) = \varrho_0 - \bar{\varrho}, \\ &0 < \tfrac{1}{2}m \leqq \bar{\varrho} + \sigma(t, x) \leqq 2M \quad \text{a.e. in } Q_T, \\ &[B]_{\infty,2,T} + [B_t]_{\infty,1,T} \leqq K_1,\quad [B_{tt}]_{\infty,0,T} \leqq K_2 \\ &B(0) = B_0,\quad B_t(0) = B_1, \\ &[a]_{\infty,1,T} \leqq K_1,\quad [a_t]_{\infty,1,T} \leqq K_2,\quad a(0) = a_0\}. \end{aligned}$$

For any $(\tilde{v}, \tilde{\sigma}, \tilde{B}, \tilde{a}) \in R_T$ we denote by (v, σ, B, a) the functions given by

(i) v is a solution of (2.1) with

$$\begin{aligned} \tilde{\varrho} &= \bar{\varrho} + \tilde{\sigma}\,, \\ F &= \tilde{F} + \tilde{\varrho} b\,, \\ \tilde{F} &= -\tilde{\varrho}(\tilde{v} \,.\, \nabla)\, \tilde{v} - p'(\tilde{\varrho})\, \nabla \tilde{\sigma} + \varepsilon\, \tilde{a}(V\tilde{a} - Z\tilde{B}_t) + \\ &\quad + \varkappa(V\tilde{a} - Z\tilde{B}_t + \tilde{v} \times \tilde{B}) \times \tilde{B}\,, \end{aligned}$$

(ii) σ is a solution of (2.3) with $\sigma_0 = \varrho_0 - \bar{\varrho}$,

(iii) B is a solution of (2.6) with $G = \varkappa\mu \operatorname{rot}(\tilde{v} \times \tilde{B})$,

(iv) a is a solution of (2.15) with $h = -\operatorname{div}(\tilde{v} \times \tilde{B})$.

Using

$$\begin{gathered} |||\nabla \tilde{\varrho}|||_{4,6,T}^4 + |||\tilde{\varrho}_t|||_{2,3,T}^2 \leqq c(K_1, K_2)\, T\,, \\ [F]_{\infty,0,T}^2 \leqq c\{\|\tilde{F}(0)\|_0^2 + T[\tilde{F}_t]_{2,0,T}^2 + [b]_{\infty,0,T}^2\} \end{gathered}$$

and Lemma 2.1, we get

$$\begin{aligned} \|v\|_{X_T}^2 \leqq c\{&[\tilde{F}]_{2,1,T}^2 + [\tilde{F}_t]_{2,-1,T}^2 + T[\tilde{F}_t]_{2,0,T}^2 + \\ &+ \|v_0\|_2^2 + \|\tilde{F}(0)\|_0^2 + [b]_{\infty,0,T}^2 + [\tilde{\varrho} b]_{2,1,T}^2 + \\ &+ [(\tilde{\varrho} b)_t]_{2,-1,T}^2\}\, (1 + Tc(K_1, K_2)) \exp\,(Tc(K_1, K_2))\,. \end{aligned}$$

It is not difficult to show that

$$\|\tilde{F}(0)\|_0^2 \leqq P(\|v_0\|_2, \|\sigma_0\|_1, \|B_0\|_1, \|B_1\|_0, \|a_0\|_1)\,,$$

where P is a polynomial, and

$$[\tilde{F}]_{2,1,T}^2 \leqq c(K_1, K_2)\, T\,,$$

$$[\tilde{F}_t]_{2,-1,T}^2 \leqq c(K_1, K_2)\, T\,, \tag{3.1}$$

$$[\tilde{F}_t]_{2,0,T}^2 \leqq c(K_1, K_2)\,. \tag{3.2}$$

This is analogous to [6] since F appearing there is extended here only by a part $\bar{F}$ which comes from the electrodynamical forces, i.e.,

$$\bar{F} = \varepsilon a(Va - ZB_t) + \varkappa(Va - ZB_t + v \times B) \times B\,.$$

But for $\bar{F}$ we even have

$$\|\bar{F}_t(t)\|_0^2 \leqq c(K_1, K_2) \quad \text{for all} \quad t\,,$$

which implies (3.1) and (3.2). Thus the function v satisfies

$$\begin{aligned} \|v\|_{X_T}^2 \leqq \{&c(K_1, K_2)\, T + c(K_1, K_2)\,([b]_{2,1,T}^2 + [b_t]_{2,-1,T}^2) + \\ &+ cP(\|v_0\|_2, \|\sigma_0\|_1, \|B_0\|_1, \|B_1\|_0, \|a_0\|_1) + \\ &+ c[b]_{\infty,0,T}^2\}\, (1 + c(K_1, K_2)\, T) \exp\,(c(K_1, K_2)\, T)\,. \end{aligned}$$

The function σ defined in (ii) is estimated as in [6]. The function B defined in (iii) is estimated with the help of (2.7). As $G = \varkappa\mu \operatorname{rot}(\tilde{v} \times \tilde{B})$, we have

$$\|G(t)\|_1^2 \leqq c(K_1) \quad \text{for a.e. } t\,, \quad 0 < t < T\,.$$

Using Lemma 2.3 we obtain the estimates

$$[B]^2_{\infty,2,T} + [B_t]^2_{\infty,1,T} \leqq c(K_1)\,T + c(\|B_0\|_2^2 + \|B_1\|_1^2)$$

and

$$[B_{tt}]^2_{\infty,0,T} \leqq c(K_1)\,(1 + T) + c(\|B_0\|_2^2 + \|B_1\|_1^2)\,.$$

Further, by Lemma 2.5 with $k = 1$, we get for a defined in (iv)

$$\begin{aligned} [a]^2_{\infty,1,T} &\leqq c(K_1)\,T + c\|a_0\|_1\,, \\ [a_t]^2_{\infty,1,T} &\leqq c(K_1)\,(1 + T) + c\|a_0\|_1\,. \end{aligned}$$

The estimates of v, σ, B and a show that there are positive K_1, K_2 and T such that $R_T \neq \emptyset$ and $(v, \sigma, B, a) \in R_T$ for any $(\tilde{v}, \tilde{\sigma}, \tilde{B}, \tilde{a}) \in R_T$. The correspondence $(\tilde{v}, \tilde{\sigma}, \tilde{B}, \tilde{a}) \to (v, \sigma, B, a)$ thus defined will be denoted by Φ.

Further we introduce the space $\mathscr{X}$ by

$$\begin{aligned} \mathscr{X} = \{(v, \sigma, B, a);\ & v \in L^\infty(0, T; H^1),\ \sigma \in L^\infty(0, T; H^1)\,, \\ & B \in L^\infty(0, T; H^1),\ B_t \in L^\infty(0, T, H^0),\ a \in L^\infty(0, T; H^0)\}\,, \end{aligned}$$

with a norm defined as the maximum of the corresponding norms of v, σ, B and a. The mapping Φ maps R_T into itself and, as it is not difficult to show, it is continuous in the norm of $\mathscr{X}$. By Schauder's theorem, there is a fixed point (v, σ, B, a) of Φ, i.e. a local solution of (1.15)–(1.27). This solution is unique, as one can show proceeding along the lines of [6].

The first component v of the solution satisfies

$$v \in C^0([0, T]; H^2) \quad \text{and} \quad \|v\|_{\infty,2,T} \leqq K_1\,.$$

We shall look for a solution $\bar{B} \in C^0([0, T]; \hat{J}) \cap C^1([0, T]; {}^\circ J^1_n) \cap C^2([0, T]; {}^\circ J)$ of

$$\begin{aligned} &\varepsilon\mu\bar{B}_{tt} + \varkappa\mu\bar{B}_t + \operatorname{rot}\operatorname{rot}\bar{B} = \varkappa\mu \operatorname{rot}(v \times \bar{B}) \quad \text{in} \quad Q_T\,, \\ &\operatorname{div}\bar{B} = 0 \quad \text{in} \quad Q_T\,, \\ &\bar{B}_n = \operatorname{rot}_\tau \bar{B} = 0 \quad \text{on} \quad \Sigma_T\,, \\ &\bar{B}(0) = B_0\,, \quad \bar{B}_t(0) = B_1 \quad \text{in} \quad \Omega\,. \end{aligned} \tag{3.3}$$

We put

$$\mathscr{Y}_T = C^0([0, T]; \hat{J}) \cap C^1([0, T]; {}^\circ J^1_n)$$

and by Ψ we denote the operator assigning, according to Lemma 2.4 (i), to $G \in C([0, T]; {}^\circ J^1_n)$ the function B satisfying (2.6). Since

$$\|\varkappa\mu \operatorname{rot}(v(t) \times \bar{B}(t))\|^2_{H^1} \leqq c\|v(t)\|_2^2\,\|\bar{B}(t)\|_2^2\,, \quad 0 \leqq t \leqq T\,,$$

the mapping $\Psi(\varkappa\mu \operatorname{rot}(v \times \bar{B}))$ is a mapping of $\mathscr{Y}_T$ into itself, and moreover, its fixed point is a solution to (3.3). By (2.7) we find that for T sufficiently small, $\bar{B} \to \Psi(\varkappa\mu \operatorname{rot}(v \times \bar{B}))$ is a contraction on $\mathscr{Y}_T$. The fixed point of this contraction is a solution of (3.3). Obviously, also $\bar{B}_{tt} \in C^0([0, T]; {}^\circ J)$. But B and $\bar{B}$ coincide, therefore

$$B \in C^0([0, T]; \hat{J}) \cap C^1([0, T]; {}^\circ J^1_n) \cap C^2([0, T]; {}^\circ J)$$

and B satisfies the estimate (2.8) with $G = \varkappa\mu \operatorname{rot}(v \times B)$. Similarly, $a \in C^1([0, T]; H^1)$ and satisfies (2.17) with $h = -\operatorname{div}(v \times B)$. Hence, the following theorem is proved.

Theorem 3.1. *Let* $b \in L^2_{loc}(R^+; H^1)$, $b_t \in L^2_{loc}(R^+; H^{-1})$, $p \in C^3$ *with* $p' > 0$. *Further let* $v_0 \in H^2 \cap H^1_0$, $\varrho_0 \in H^2$, $0 < m \leqq \varrho_0(x) \leqq M$ *in* Ω, $B_0 \in \hat{J}$, $E_0 \in H^2$, $(E_0)_\tau = 0$ *on* $\partial\Omega$. *Then there is (a sufficiently small)* $T > 0$ *and functions*

$$\begin{aligned}
&v \in L^2(0, T; H^3) \cap C^0([0, T]; H^2) \quad \textit{with} \\
&v_t \in L^2(0, T; H^1) \cap C^0([0, T]; H^0), \\
&\varrho \in C^0([0, T]; H^2) \quad \textit{with} \quad \varrho_t \in C^0([0, T]; H^1) \quad \textit{and} \\
&\qquad \varrho(t, x) > 0 \quad \textit{on} \quad Q_T, \\
&B \in C^0([0, T]; \hat{J}) \cap C^1([0, T]; {}^\circ J^1_n) \cap C^2([0, T]; {}^\circ J), \\
&E \in C^1([0, T]; H^2)
\end{aligned}$$

such that (v, ϱ, B, E) *satisfy* (1.1)–(1.14).

4. GLOBAL AND PERIODIC SOLUTIONS

Let φ be defined by (4.47) of [6], i.e.,

$$\varphi(t) =]|v(t)|[^2_1 +]|\sigma(t)|[^2_2 + c_4\|v_t(t)\|^2_0 + \bar{c}_4 \frac{p_1}{\bar{\varrho}} \|\sigma_t(t)\|^2_0 + \bar{c}_5[v(t)]^2_2,$$

where $[\cdot]_2$ is the sum of L^2-norms of interior and tangential derivatives of orders less or equal to 2 and $]|\cdot|[_k$ is a norm equivalent to $\|\cdot\|_k$. Denoting

$$\Phi = \|v\|^2_3 + \|\sigma\|^2_2 + \|v_t\|^2_1 + \|\sigma_t\|^2_1,$$

we have, by integrating (4.48) in [6] over (τ, t), $0 \leqq \tau < t \leqq T$,

$$\varphi(t) - \varphi(\tau) + \int_\tau^t \Phi(\vartheta)\, \mathrm{d}\vartheta \leqq \tag{4.1}$$

$$\leqq c \int_\tau^t \{\Phi(\vartheta)\, (\varphi(\vartheta) + \varphi^2(\vartheta)) + \|f^1(\vartheta)\|^2_1 + \|f^1_t(\vartheta)\|^2_{-1} + \beta(\vartheta)\}\, \mathrm{d}\vartheta,$$

where

$$f^1 = \frac{1}{\bar{\varrho} + \sigma} \{\varepsilon\, a(Va - ZB_t) + \varkappa(Va - ZB_t + v \times B) \times B\}$$

and

$$\beta(t) = \|b(t)\|_1^2 + \|b_t(t)\|_{-1}^2 .$$

In what follows we drop B from $\psi(B)(t)$ defined in (2.9), writing $\psi(t) = \psi(B)(t)$. Using this ψ we set

$$\chi(t) = \varphi(t) + \psi(t) + \|a(t)\|_1^2 .$$

Further we denote

$$\Psi(t) = d_1(\|\operatorname{rot}\operatorname{rot} B(t)\|_0^2 + \|\operatorname{rot} B_t(t)\|_0^2)$$

and

$$\Pi(t) = \Phi(t) + \Psi(t) + \|a(t)\|_1^2 .$$

Using (4.1), estimates (2.8) and (2.16) we get, for $0 \leqq \tau < t \leqq T$,

$$\begin{aligned}(4.2)\qquad \chi(t) - \chi(\tau) + \int_\tau^t \Pi(\vartheta)\,\mathrm{d}\vartheta &\leqq c \int_\tau^t \Phi(\vartheta)\,(\varphi(\vartheta) + \varphi^2(\vartheta))\,\mathrm{d}\vartheta + \\ &+ c\int_\tau^t (\|f^1\|_1^2 + \|f_t^1\|_{-1}^2)\,\mathrm{d}\vartheta + c\int_\tau^t (\|\operatorname{rot} G\|_0^2 + \|G\|_0^2)\,\mathrm{d}\vartheta + \\ &+ c\int_\tau^t \|h(\vartheta)\|_1^2\,\mathrm{d}\vartheta + c\int_\tau^t \beta(\vartheta)\,\mathrm{d}\vartheta ,\end{aligned}$$

where

$$\begin{aligned} G &= \varkappa\mu \operatorname{rot}(v \times B) , \\ h &= \operatorname{div}(v \times B) . \end{aligned}$$

Given positive α_1 and α_2 we set

$$X(t) = \tfrac{1}{2}\,\Pi(t) + \alpha_1\|B_{tt}\|_0^2 + \alpha_2\|a_t\|_1^2 ,$$

i.e.

$$\begin{aligned} X(t) = \|v\|_3^2 + \|v_t\|_1^2 + \|\sigma\|_2^2 + \|\sigma_t\|_1^2 + d_1\|\operatorname{rot}\operatorname{rot} B\|_0^2 + \\ + d_2\|\operatorname{rot} B_t\|_0^2 + \alpha_1\|B_{tt}\|_0^2 + \|a\|_1^2 + \alpha_2\|a_t\|_1^2 . \end{aligned}$$

A direct computation shows that

$$\|f^1\|_1^2 + \|f_t\|_{-1}^2 + \|\operatorname{rot} G\|_0^2 + \|G\|_0^2 + \|h\|_1^2 \leqq c\,X(\chi + \chi^3) .$$

Using this inequality and estimating $\|B_{tt}\|_0^2$ and $\|a_t\|_1^2$ with the help of (1.17) and (1.19), we see that α_1 and α_2 can be chosen in such a way that the following inequality holds $0 \leqq \tau < t \leqq T$:

$$(4.3)\quad \chi(t) - \chi(\tau) + \int_\tau^t X(\vartheta)\,\mathrm{d}\vartheta \leqq c\int_\tau^t X(\vartheta)\,(\chi(\vartheta) + \chi^3(\vartheta))\,\mathrm{d}\vartheta + \tilde{c}\int_\tau^t \beta(\vartheta)\,\mathrm{d}\vartheta .$$

As the function $\chi(t)$ is continuous on $[0, T]$ and

$$\chi(t) \leqq \bar{c}\,X(t) \quad \text{for a.e.} \quad t \in (0, T) ,$$

one easily proves that there are two constants δ and A such that the following implication holds: If $\chi(0) \leqq A$ and $\beta(t) \leqq \delta$ for a.e. $t \in (0, T)$, then $\chi(t) \leqq A$ for all $t \in [0, T]$. This proves the following theorem on global existence:

Theorem 4.1. *Let the assumptions of Theorem* 3.1 *be satisfied. Moreover, let* $\|v_0\|_2 + \|\varrho_0 - \bar{\varrho}\|_2 + \|B_0\|_2 + \|E_0\|_2$ *and* $[b]_{\infty,1,\infty} + [b_t]_{\infty,-1,\infty}$ *be sufficiently small. Then there exist unique*

$$\begin{aligned} &v \in L^2_{loc}(R^+; H^3) \cap C^0_B(R^+; H^2) \quad \textit{with} \quad v_t \in L^2_{loc}(R^+; H^1) \cap C^0_B(R^+; H^0)\,, \\ &\varrho \in C^0_B(R^+; H^2) \quad \textit{with} \quad \varrho_t \in C^0_B(R^+; H^1)\,, \\ &B \in C^0_B(R^+; \hat{J}) \cap C^1_B(R^+; {}^\circ J^1_n) \cap C^2_B(R^+; {}^\circ J)\,, \\ &E \in C^1_B(R^+; H^2) \end{aligned}$$

such that (v, ϱ, B, E) *satisfy* (1.1)–(1.14) *on* R^+.

If $(v_i, \sigma_i, B_i, a_i)$, $i = 1, 2$ are solutions of (1.15)–(1.22) with the initial conditions $(v_{i0}, \sigma_{i0}, B_{i0}, B_{i1}, a_{i0})$, we denote

$$\begin{aligned} w &= v_1 - v_2\,, & w_0 &= v_{10} - v_{20}\,. \\ \eta &= \sigma_1 - \sigma_2\,, & \eta_0 &= \sigma_{10} - \sigma_{20}\,, \\ D &= B_1 - B_2\,, & D_0 &= B_{10} - B_{20}\,, \quad D_1 = B_{11} - B_{21}\,, \\ d &= a_1 - a_2\,, & d_0 &= a_{10} - a_{20}\,. \end{aligned}$$

Proceeding as in [6] we find that there are positive A, ε, δ such that

$$\begin{aligned} &\|w(t)\|_0 + \|\eta(t)\|_0 + \|D(t)\|_1 + \|D_t(t)\|_0 + \|d(t)\|_0 \leqq \\ &\leqq Ae^{-\varepsilon t}(\|w_0\|_0 + \|\eta_0\|_0 + \|B_0\|_1 + \|B_1\|_0 + \|a\|_0)\,, \quad t \in R^+\,, \quad \text{provided} \\ &\|v_{i0}\|_2 + \|\sigma_{i0}\|_2 + \|B_{i1}\|_1 + \|a_i\|_1 < \delta \quad \text{for} \quad i = 1, 2\,. \end{aligned} \tag{4.4}$$

Using (4.4) we can follow the approach of [6] and prove the existence of periodic solutions.

Theorem 4.2. *Let* $b \in L^\infty(R^+; H^1)$, $b_t \in L(R^+; H^{-1})$ *be T-periodic in t and* $p \in C^3$, $p_\varrho > 0$.

If $[b]_{\infty,1,\infty} + [b_t]_{\infty,-1,\infty}$ *is sufficiently small, then there exists a T-periodic solution of* (1.1)–(1.14).

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Souhrn

PERIODICKÁ ŘEŠENÍ ROVNIC MAGNETOHYDRODYNAMIKY STLAČITELNÝCH TEKUTIN

MILAN ŠTĚDRÝ, OTTO VEJVODA

Je dokázána globální existence a exponenciální stabilita řešení daného systému rovnic v případě, že počáteční rychlosti a vnější síly jsou malé a počáteční hustota se příliš neliší od konstantní. Jsou-li kromě toho vnější síly periodické, existuje řešení periodické se stejnou periodou. Systém uvažovaných rovnic se trochu liší od obvykle uvažovaného systému; například posuvný proud není zanedbán.

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