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BIFURCATION THEORY OF THE TIME-DEPENDENT VON KARMAN EQUATIONS

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1. FORMULATION OF THE PROBLEM

We shall deal with the existence and analysis of solutions of the nonlinear homogeneous Volterra integral equation

$$B^3(t) = c\, B(t) \int_0^t B^2(\tau)\, K(t - \tau)\, d\tau + e \int_0^t B(\tau)\, K(t - \tau)\, d\tau + (\lambda j - l)\, B(t)\,,$$

where $B(t)$ is an unknown function, $K(t - \tau) = \exp\left[-(1/\beta)(t - \tau)\right]$ is the kernel, c, e, j, l are positive constants and $\lambda > 0$ is a parameter.

This Volterra integral equation can be derived as the first approximation of the generalized time dependent von Karman equations for viscoelastic plates of a standard material [1]

(E1) $$K(1 + \alpha D_t)\, \Delta^2 w = (1 + \beta D_t)\, \{\lambda[w, F_0] + [w, F]\}\,,$$

(E2) $$(1 + \beta D_t)\, \Delta^2 F = -\tfrac{1}{2} h E(1 + \alpha D_t)\, [w, w]\,,$$

defined in a rectangular domain $\Omega = \{(x, y) \mid 0 \leqq x \leqq a,\ 0 \leqq y \leqq b\}$, where w is the transverse displacement of the plate, F is the stress function, Δ^2 is the biharmonic operator, $D_t = \partial/\partial t$, F_0 is the stress function corresponding to the given boundary loading, E is the modulus of elasticity, h the thickness of the plate, K the stiffness of the plate, $\alpha > 0$, $\beta > 0$ are viscous parameters such that $\alpha > \beta$, $\lambda > 0$ is the parameter of proportionality of the given boundary loading with respect to F_0 and

$$[f, g] = f_{xx} g_{yy} + f_{yy} g_{xx} - 2 f_{xy} g_{xy}\,.$$

We consider the following boundary conditions

(E3) $$w = w_{vv} = 0\,, \quad F = F_{ss} = 0 \quad \text{on} \quad \partial\Omega\,,$$

where $w_{\nu\nu}$, F_{ss} denote the second derivatives in the direction of an external normal and the tangent to $\partial\Omega$, respectively and homogeneous initial conditions

$$w\big|_{t=0^-}\ F\big|_{t=0^-} = 0 \quad \text{on} \quad \Omega\,.$$

We take F_0 in the form

$$F_0(x, y, t) = -y^2/2\,,$$

which corresponds to a constant pressure at the edges $x = 0$ and $x = a$.

The approximate solution of (E1), (E2) with boundary conditions (E3) can be sought in the form

$$F(x, y, t) = \sum_{\substack{m=1,2,\ldots \\ n=1,2,\ldots}} A_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\,,$$

$$w(x, y, t) = \sum_{\substack{m=1,2,\ldots \\ n=1,2,\ldots}} B_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}\,.$$

Restricting ourselves to the first terms, applying the Galerkin method and using the transformation

$$\int_0^t K(t-\tau) f(\tau)\,\mathrm{d}\tau = (1 + \beta D_t)^{-1} f(t)$$

for zero initial conditions we arrive at a system of two nonlinear Volterra integral equations, which after the elimination of $A(t)$ give

$$B^3(t) = \left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha} B(t) \int_0^t B^2(\tau)\, K(t-\tau)\,\mathrm{d}\tau + \tag{1}$$

$$+ P_1 \frac{1}{\alpha}\left(\frac{\alpha}{\beta} - 1\right) \int_0^t B(\tau)\, K(t-\tau)\,\mathrm{d}\tau + \left(P_2 \lambda \frac{\beta}{\alpha} - P_1\right) B(t)\,,$$

where

$$P_1 = 9K\pi^4(a^2 + b^2)^4/512hEa^4b^4\,,$$

$$P_2 = 9\pi^2(a^2 + b^2)^2/512hEa^2$$

are nonnegative constants.

Definition. *Function $B(t)$ is called a solution of* (1) *if*:

a) *$B(t)$ is absolutely continuous on* $\langle 0, \infty)$;

b) $B(t) \in C^\infty((0, \infty))$;

c) *$B(t)$ fulfils* (1) *on* $\langle 0, \infty)$.

Definition. *A point $\lambda = \lambda_{cr}$ is a critical point of* (1) *if*:

a) *for $\lambda < \lambda_{cr}$ the integral equation* (1) *has only the trivial solution*;

b) *for $\lambda = \lambda_{cr}$, in addition to the trivial solution there exist at least two symmetric nontrivial solutions of* (1) *which are bifurcating from the zero value* (Fig. 1);

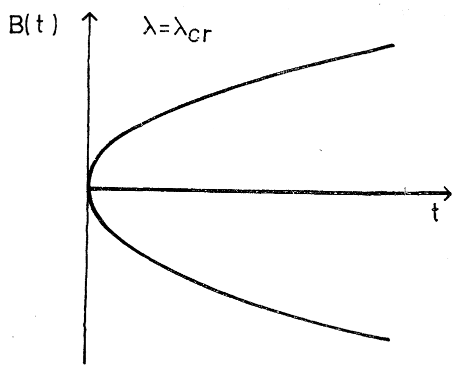

Fig. 1.

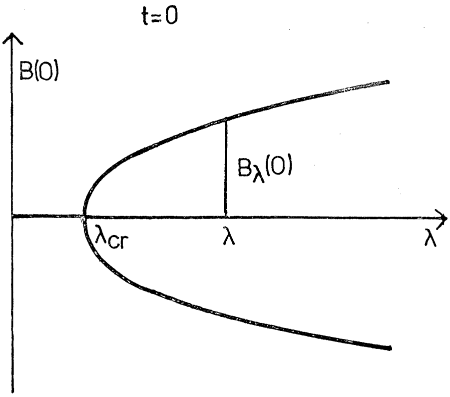

Fig. 2.

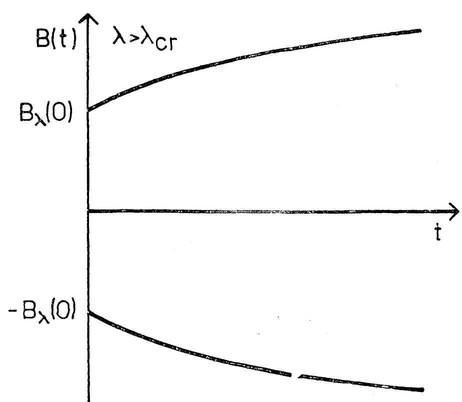

Fig. 3.

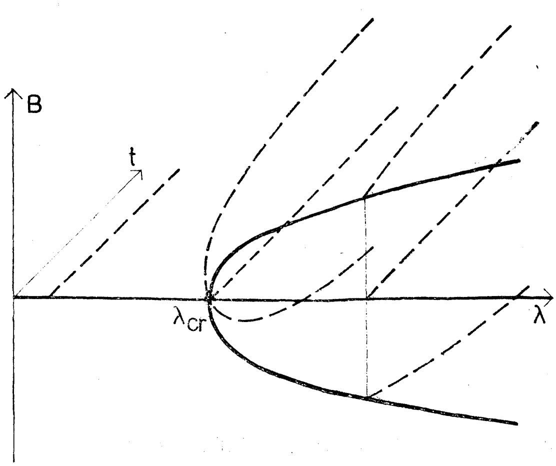

Fig. 4.

c) *for* $\lambda > \lambda_{cr}$, *in addition to the trivial solution there exist exactly two symmetric solutions of* (1) *starting with a jump* $B_\lambda(0^+)$ *at* $t = 0^+$ (Fig. 2). *They are functions of time* (Fig. 3).

To understand the situation see Fig. 4.

The main goal of this paper is to prove the existence of a critical point for the problem (1).

2. PRELIMINARY ANALYSIS

Lemma 1. *Let* $B(t) \in L_\infty((0, \infty))$, *let* $B(t)$ *fulfils* (1) *and let* $B(t)$ *fulfils one of the conditions*

a) $B(t) \geqq g(t)$ a.e.;

b) $B(t) \leqq -g(t)$ a.e.;

c) $B(t) = 0$ a.e.;

where $g(t) \geqq 0$ *for* $t > 0$ *and* $g(t) \in C((0, \infty))$. *Then* $B(t)$ *can be changed on a set of zero measure so that then* $B(t)$ *is a solution of* (1).

Proof. In the case c) the lemma is obvious.

Let us consider (1) in the form

$$B^3(t) = L_1(t)\, B(t) + L_2(t)\,, \tag{2}$$

where

$$L_1(t) = \left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha}\int_0^t B^2(\tau)\, K(t - \tau)\, \mathrm{d}\tau + P_2\lambda\frac{\beta}{\alpha} - P_1\,,$$

$$L_2(t) = P_1\left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha}\int_0^t B(\tau)\, K(t - \tau)\, \mathrm{d}\tau\,.$$

Clearly $L_1(t), L_2(t) \in C(\langle 0, \infty))$ and they are absolutely continuous on $\langle 0, \infty)$. Using the Cardan formula for (2) we have

$$\begin{aligned} B(t) = {} & \{\tfrac{1}{2} L_2(t) + [\tfrac{1}{4} L_2^2(t) - \tfrac{1}{9} L_1^3(t)]^{1/2}\}^{1/3} + \\ & + \{\tfrac{1}{2} L_2(t) - [\tfrac{1}{4} L_2^2(t) - \tfrac{1}{9} L_1^3(t)]^{1/2}\}^{1/3}\,, \end{aligned} \tag{3}$$

which implies the existence of one $B_1(t)$ (trivial) or three $B_1(t)$ (trivial), $B_2(t)$, $B_3(t)$ continuous and absolutely continuous solutions of (2) on $\langle 0, \infty)$.

In case a) $B(t)$ also fulfils (2) a.e. on $\langle 0, \infty)$, so $B(t) = B_2(t)$ ($B_2(t)$ positive) a.e. on $\langle 0, \infty)$ and we can change $B(t)$ on a set of zero measure where $B(t) \neq B_2(t)$ so that $B(t) \equiv B_2(t)$ on $\langle 0, \infty)$. Analogously we proceed in case b).

If $B(t) \not\equiv 0$, then $B(t)$ must be either positive or negative on $(0, \infty)$. As $B(t)$ is absolutely continuous on $\langle 0, \infty)$ we can differentiate (1) obtaining

$$(4) \qquad B'(t) = \frac{B(t)\left[P_2\lambda - P_1 - B^2(t)\right]}{\alpha P_1 - \beta\lambda P_2 + 3\alpha\, B^2(t) - \left(\frac{\alpha}{\beta} - 1\right)\int_0^t B^2(\tau)\, K(t - \tau)\, \mathrm{d}\tau}$$

for a.e. $t \in \langle 0, \infty)$. Since $B(t)$ fulfils (1), for $t \in (0, \infty)$ we obtain

$$(5) \qquad \alpha P_1 - \beta\lambda P_2 + \alpha\, B^2(t) - \left(\frac{\alpha}{\beta} - 1\right)\int_0^t B^2(\tau)\, K(t - \tau)\, \mathrm{d}\tau =$$

$$= P_1\left(\frac{\alpha}{\beta} - 1\right)\frac{1}{B(t)}\int_0^t B(\tau)\, K(t - \tau)\, \mathrm{d}\tau > 0\,.$$

However, the continuity of $B(t)$, (4) and (5) implies $B'(t) \in C((0, \infty))$. Finally, from (4) we obtain the rest of the proof.

We denote by $S_\lambda(B)(t)$ the operator

$$S_\lambda(B)(t) = \left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha}\, B(t)\int_0^t B^2(\tau)\, K(t - \tau)\, \mathrm{d}\tau +$$

$$+ P_1\left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha}\int_0^t B(\tau)\, K(t - \tau)\, \mathrm{d}\tau + \left(P_2\lambda\frac{\beta}{\alpha} - P_1\right)B(t)\,.$$

Then the equation (1) can be written in the form

$$(6) \qquad B^3(t) = S_\lambda(B)(t)\,.$$

Let us define

$$(7) \qquad \lambda_{cr} = \frac{\alpha}{\beta}\frac{P_1}{P_2}\,.$$

Lemma 2. *The operator S_λ is monotone for $\lambda \geqq \lambda_{cr}$ on the subset of nonnegative elements in $L_\infty((0, T))$ for all $T < \infty$; i.e., if $B_1(t) \geqq B_2(t) \geqq 0$ for a.e. $t \in (0, T)$, then $S_\lambda(B_1)(t) \geqq S_\lambda(B_2)(t) \geqq 0$ for a.e. $t \in (0, T)$.*

The proof of Lemma 2 is evident.

Definition. *The function $B_1(t)$ $(B_2(t))$ from $C((0, T))$ is an upper (lower) solution of (6) if the inequality*

$$(8) \qquad S_\lambda(B_1)(t) \leqq B_1^3(t)$$

$$(9) \qquad (S_\lambda(B_2)(t) \geqq B_2^3(t))$$

takes place for all $t \in (0, T)$ and for all $T < \infty$.

Remark 1. If there exists a solution $B(t)$ of (1) with

$$\lim_{t \to \infty} B(t) = B(\infty)$$

than the Paley-Wiener theorem

$$\left(\lim_{t\to\infty}\int_0^t B(\tau) f(t-\tau)\,d\tau = B(\infty)\int_0^\infty f(\tau)\,d\tau\right) \text{ implies}$$

$$B^3(\infty) = \left(\frac{\alpha}{\beta}-1\right)\frac{\beta}{\alpha}B^3(\infty) + P_1\left(\frac{\alpha}{\beta}-1\right)\frac{\beta}{\alpha}B(\infty) + \left(P_2\frac{\beta}{\alpha}\lambda - P_1\right)B(\infty).$$

Thus $B(\infty) = 0$ or $B^2(\infty) = P_2\lambda - P_1$ holds.

3. EXISTENCE AND BIFURCATION OF A SOLUTION

Theorem 1. *If* $\lambda < \lambda_{cr}$ *(*λ_{cr} *is from* (7)*) then* $B(t) \equiv 0$ *is the unique solution of* (1).

Proof. We consider λ in the form

$$\lambda = \frac{P_1}{P_2}\frac{\alpha}{\beta}(1-\varepsilon),$$

where $\varepsilon > 0$. Let us assume that there exists a nontrivial solution $B(t)$ of (1). Then for $t_1 = \inf\{t > 0;\ B(t) \neq 0\}$ there exists $\delta' > 0$ such that $B(t) > 0$ in $(t_1, t_1 + \delta')$ (the case $B(t) < 0$ can be handled analogously). It is clear that for each $r > 0$ we can choose $0 < \delta' < r$ with the above property. Moreover, there exists δ ($\delta' \geqq \delta > 0$) such that

$$\max_{(t_1, t_1+\delta)} B(t) = B(t_1+\delta).$$

Then for $t = t_1 + \delta$ we obtain

$$B^3(t_1+\delta) - \left(\frac{\alpha}{\beta}-1\right)\frac{1}{\alpha}B(t_1+\delta)\int_{t_1}^{t_1+\delta} B^2(\tau)\exp\left[-\frac{1}{\beta}(t_1+\delta-\tau)\right]d\tau +$$

$$+ P_1\varepsilon B(t_1+\delta) - P_1\left(\frac{\alpha}{\beta}-1\right)\frac{1}{\alpha}\int_{t_1}^{t_1+\delta} B(\tau)\exp\left[-\frac{1}{\beta}(t_1+\delta-\tau)\right]d\tau = 0.$$

Thus

$$B^3(t_1+\delta) - \left(\frac{\alpha}{\beta}-1\right)\frac{\beta}{\alpha}B^3(t_1+\delta)\left[1-\exp\left(-\frac{1}{\beta}\delta\right)\right] + \tag{10}$$

$$+ P_1\varepsilon B(t_1+\delta) - P_1\left(\frac{\alpha}{\beta}-1\right)\frac{\beta}{\alpha}B(t_1+\delta)\left[1-\exp\left(-\frac{1}{\beta}\delta\right)\right] \leqq 0.$$

Since $1 > 1 - \beta/\alpha > 0$ and $1 \geqq 1 - \exp(-(1/\beta)\,\delta)$ we have

$$B^3(t_1+\delta) > \left(\frac{\alpha}{\beta}-1\right)\frac{\beta}{\alpha}B^3(t_1+\delta)\left[1-\exp\left(-\frac{1}{\beta}\delta\right)\right].$$

Then we obtain from (10)

$$P_1 \varepsilon B(t_1 + \delta) - P_1 \left(\frac{\alpha}{\beta} - 1\right) \frac{\beta}{\alpha} B(t_1 + \delta) \left[1 - \exp\left(-\frac{1}{\beta}\delta\right)\right] < 0\,;$$

from this inequality we deduce

$$(11) \qquad 0 < \exp\left(-\frac{1}{\beta}\delta\right) < 1 - \frac{\alpha\varepsilon}{\alpha - \beta} = v < 1\,.$$

The last inequality takes place for

$$(12) \qquad \varepsilon < 1 - \frac{\beta}{\alpha}\,.$$

Thus for $\varepsilon \geqq 1 - \beta/\alpha$ there exists no nontrivial solution of (1). In the case (12) we obtain from (11)

$$\delta > -\beta \ln v = \gamma > 0\,,$$

which leads to a contradiction, since δ can be chosen arbitrarily small.

Consider

$$(13) \qquad S_{\lambda_{cr}}(B)(t) = B^3(t)\,; \quad t \in \langle 0, \infty)$$

where λ_{cr} is from (7).

In the next two lemmas we construct a lower and an upper solution of (13).

Lemma 3. *The function*

$$B_0(t) = \left[P_1\left(\frac{\alpha}{\beta} - 1\right)\right]^{1/2} \left[1 - \exp\left(-\frac{1}{2\alpha}t\right)\right]; \quad t \in \langle 0, \infty)$$

is a lower solution of (13).

Proof. Substituting $B_0(t)$ in (13) successively we obtain

$$S_{\lambda_{cr}}(B_0)(t) - B_0^3(t) = \left[P_1\left(\frac{\alpha}{\beta} - 1\right)\right]^{3/2} (2\alpha^2 - \beta\alpha)^{-1}\,.$$

$$.\,\beta \exp\left(-\frac{1}{2\alpha}t\right)\left\{2\alpha\left[1 - \exp\left(-\frac{1}{2\alpha}t\right)\right] - \beta\left[1 - \exp\left(-\frac{1}{\beta}t\right)\right]\right\}.$$

Differentiating

$$L(t) = 2\alpha\left[1 - \exp\left(-\frac{1}{2\alpha}t\right)\right] - \beta\left[1 - \exp\left(-\frac{1}{\beta}t\right)\right]$$

we obtain

$$L'(t) = \exp\left(-\frac{1}{2\alpha}t\right) - \exp\left(-\frac{1}{\beta}t\right).$$

Then $L'(t) > 0$ for $t \in (0, \infty)$ since $\alpha > \beta$. Thus we obtain

$$S_{\lambda_{cr}}(B_0)(t) \geqq B_0^3(t)$$

because $L'(0) = L(0) = 0$.

By elementary computation we can verify

Lemma 4. *The function*

$$B_\infty(t) = \left[P_1\left(\frac{\alpha}{\beta} - 1\right)\right]^{1/2} \; ; \quad t \in \langle 0, \infty)$$

is an upper solution of (13).

The main result is

Theorem 2. *There exist at least three different monotone solutions of* (13) *satisfying* $B(0) = 0$.

Proof. The operator $S_{\lambda_{cr}}$ is monotone on the set of nonnegative functions in $L_\infty((0, \infty))$ (see Lemma 2). With respect to Lemmas 3 and 4 we can construct a nondecreasing sequence $\{B_i^3(t)\}_{i=0}^{\infty}$ where $B_{i+1}^3(t) = S_{\lambda_{cr}}(B_i)(t)$ $(t \in \langle 0, \infty))$ for $i = 0, 1, \ldots$. Since $S_{\lambda_{cr}}(B_i)(t) \geqq B_i^3(t)$, we have

$$0 \leqq B_0(t) \leqq B_i(t) \leqq B_{i+1}(t) \leqq B_\infty(t) \tag{14}$$

for $t \in \langle 0, \infty)$; $i = 1, 2, \ldots$, where $B_\infty(t)$ is an upper solution. Taking the limit for $i \to \infty$ in (14) we arrive at a nonnegative function $B^*(t) \in L_\infty((0, \infty))$ satisfying (13) for all $t \in (0, \infty)$. Owing to Lemma 1, $B^*(t)$ is a solution of (1). The monotonicity of $B^*(t)$ can be proved from (4) on the basis of (14).

Since

$$S_{\lambda_{cr}}(-B)(t) = -S_{\lambda_{cr}}(B)(t)$$

we conclude that $B^{**}(t) = -B^*(t)$ is a nonpositive solution of (13). Evidently the zero function is a solution of (13) as well.

From the physical point of view it is convenient to put $B(t) = 0$ for $t < 0$ and $B(t) \,(\gtrless)\, 0$ for $t > 0$ in the case $\lambda = \lambda_{cr}$. We use a similar convention in the case $\lambda > \lambda_{cr}$ as well.

Now we investigate the existence of solutions of (1) for $\lambda > \lambda_{cr}$. In this case we can write

$$\lambda = \frac{P_1}{P_2} \frac{\alpha}{\beta}(1 + \varepsilon) \; ; \quad \varepsilon > 0 \, ,$$

and proceed analogously as in the case $\lambda = \lambda_{cr}$. In order to construct an upper solution we can apply Remark 1.

Lemma 5. *The function*

$$B_\infty(t) = \left\{P_1\left[\frac{\alpha}{\beta}(1+\varepsilon) - 1\right]\right\}^{1/2} \quad ; \quad t \in \langle 0, \infty)$$

is an upper solution of (6) *and the function*

$$B_0(t) = (P_1\varepsilon)^{1/2} \; ; \quad t \in \langle 0, \infty)$$

is a lower solution of (6).

The assertion of Lemma 5 is evident.

Analogously to Theorem 2 we can prove

Theorem 3. *There exist at least three different monotone solutions of* (1) *for* $\lambda > \lambda_{cr}$.

In the following theorem we prove the uniqueness of the corresponding branches for $\lambda > \lambda_{cr}$.

Theorem 4. *Let* $\lambda > \lambda_{cr}$. *Then there exist only three different bounded solutions of* (1).

Proof. First we prove the uniqueness of the positive solution of (1). The uniqueness of the negative solution can be proved analogously.

Let us prove the theorem by contradiction. Let $B_1(t)$, $B_2(t)$ be two different solutions. Let us denote

$$H(t) = B_1(t) - B_2(t) \,.$$

We have

$$H(t) = \left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha}\left\{B_1^2(t) + B_1(t)\,B_2(t) + B_2^2(t) - \right.$$

$$\left. - \left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha}\int_0^t B_1^2(\tau)\exp\left[-\frac{1}{\beta}(t-\tau)\right]\mathrm{d}\tau - P_1\varepsilon\right\}^{-1} .$$

$$\cdot \int_0^t H(\tau)\,\{B_2(t)\,[B_1(\tau) + B_2(\tau)] + P_1\}\exp\left[-\frac{1}{\beta}(t-\tau)\right]\mathrm{d}\tau\,,$$

which can be rewritten in the form

$$H(t) = \int_0^t K(t,\tau)\,H(\tau)\,\mathrm{d}\tau = 0\,, \tag{15}$$

where

$$K(t,\tau) = \left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha}\left\{B_1^2(t) + B_1(t)\,B_2(t) + B_2^2(t) - \right.$$

$$\left. - \left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha}\int_0^t B_1^2(\tau)\exp\left[-\frac{1}{\beta}(t-\tau)\right]\mathrm{d}\tau - P_1\varepsilon\right\}^{-1} .$$

$$\cdot \int_0^t \{B_2(t)\,[B_1(\tau) + B_2(\tau)] + P_1\} \exp\left[-\frac{1}{\beta}(t-\tau)\right] \mathrm{d}\tau \,.$$

We use the following notation

$$G(t) = \max_{\tau\in\langle 0,t\rangle} H(\tau)\,,$$

$$L(t) = \left[\int_0^t K^2(t,\tau)\,\mathrm{d}\tau\right]^{1/2} \quad ; \quad t \geqq 0\,.$$

Since $B_1(t)$, $B_2(t)$ are two solutions of (1), the inequality

$$B_1^2(t) - \left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha}\int_0^t B_1^2(\tau) \exp\left[-\frac{1}{\beta}(t-\tau)\right] \mathrm{d}\tau - P_1\varepsilon =$$

$$= P_1\left(\frac{\alpha}{\beta} - 1\right)\frac{1}{\alpha}\frac{1}{B_1(t)}\int_0^t B_1(\tau) \exp\left[-\frac{1}{\beta}(t-\tau)\right] \mathrm{d}\tau > 0$$

holds for $t \geqq 0$. Hence from

$$B_1(t)\,B_2(t) + B_2^2(t) > 0$$

we conclude that $K(t,\tau)$ is well defined for $t > 0$ and

$$G(t) < \infty\,;$$

$$L(t) < \infty \quad \text{for all} \quad t > 0\,.$$

Using the Cauchy-Schwarz inequality in (15) we successively get

$$H(t) \leqq \left[\int_0^t H^2(\tau)\,\mathrm{d}\tau\right]^{1/2} \left[\int_0^t K^2(t,\tau)\,\mathrm{d}\tau\right]^{1/2} \leqq GLt^{1/2}$$

and

$$H(t) \leqq GL\left[\int_0^t K^2(t,\tau)\,\mathrm{d}\tau\right]^{1/2} \left[\int_0^t \tau\,\mathrm{d}\tau\right]^{1/2} \leqq$$

$$\leqq GL^2 t\, 2^{1/2} \leqq \ldots \leqq GL^n\left(\frac{t^n}{n!}\right)^{1/2} \quad \text{for} \quad n = 0, 1, \ldots,$$

which imply $H(t) = 0$ for $t > 0$. Thus we obtain $B_1(t) = B_2(t)$.

Corollary. *Theorems* 1–4 *imply that the point* λ_{cr} *given by* (7) *is the critical point of* (1).

Despite of the fact that we consider only the first approximation of the sought solution (w, F) of the generalized von Karman equations, our results and the properties of the solutions provide a qualitative analysis of the bifurcation for visco-elastic plates.

References

[1] *J. Brilla:* Stability problems in mathematical theory of viscoelasticity, in Equadiff IV, Proceedings, Prague, August 22—26, 1977 (ed. J. Fabera). Springer, Berlin—Heidelberg—New York 1979.
[2] *N. Distéfano:* Nonlinear Processes in Engineering. Academic press, New York, London 1974.
[3] *A. N. Kolmogorov, S. V. Fomin:* Elements of the theory of functions and functional analysis (Russian). Izd. Nauka, Moskva 1976.
[4] *J. L. Lions:* Quelques méthodes de résolution des problèmes aux limites no linéaires. Dunod, Gautier-Villars, Paris 1969.
[5] *F. G. Tricomi:* Integral equations. Interscience Publishers, New York, 1957.

Súhrn

TEÓRIA BIFURKÁCIÍ ZOVŠEOBECNENÝCH KARMANOVÝCH ROVNÍC

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Práca sa zaoberá skúmaním existencie a bifurkácie riešenia nelineárnej homogénnej Volterrovej integrálnej rovnice, ktorú sme dostali ako prvú aproximáciu pri riešení zovšeobecnených Karmanových rovníc, ktoré popisujú rovnovažné stavy obdĺžnikovej tenkejväzkopružnej dosky, na dva protiľahle okraje ktorej pôsobí konštantné rovnomerné zaťaženie závislé od parametra úmernosti zaťaženia.

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