Jagdish Saran; Kanwar Sen

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ON THE DISTRIBUTIONS OF $R^+_{mn}(j)$ AND $(D^+_{mn}, R^+_{mn}(j))$

JAGDISH SARAN, KANWAR SEN

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1. INTRODUCTION

Let $X_1 < X_2 < \ldots < X_m$ and $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics from two independent samples of i.i.d. random variables having continuous population distribution functions F and G, respectively, and suppose $F_m(x)$ and $G_n(x)$ are the corresponding empirical distribution functions. Let $Z_1 < Z_2 < \ldots < Z_{m+n}$ denote the ordered combined sample and let R_i denote the rank of X_i in the ordered combined sample. Finally, we consider $H_0 : F = G$. The Smirnov one-sided statistic is given by

$$D^+_{mn} = \sup_t \{F_m(t) - G_n(t)\} = \left(\frac{1}{mn}\right) \max_{1 \leqq k \leqq m} (k(m+n) - mR_k)\,.$$

This follows from Theorem 2.1 of Steck (1969). If $mn\ D^+_{mn} = d$, let $R^+_{mn}(j)$ be the jth value of k for which $k(m+n) - mR_k = d$; $R^+_{mn}(1) = R^+_{mn}$. The possible values of $R^+_{mn}(j)$ are the integers $j, j+1, \ldots, m+n$. Let

$$P_{mn}(r, j) \quad = P\{R^+_{mn}(j) = r\}\ ; \quad P_{mn}(r, 1) = P_{mn}(r)\,,$$

$$Q_{mn}(d, r, j) = P\{mn\ D^+_{mn} = d,\ R^+_{mn}(j) = r\}\ ; \quad Q_{mn}(d, r, l) = Q_{mn}(d, r)\,.$$

The distribution of $R^+_{mn}(j)$ has been discussed, for $j = 1$, by several authors in certain special cases. Vincze (1957) gave a formula for $Q_{nn}(d, r)$ and proved that $P_{nn}(2r - 1) = P_{nn}(2r)$, $r = 1, 2, \ldots, n$. Sarkadi (1961) proved that $P_{nn}(r) \geqq P_{nn}(r + 1)$. Steck (1969) showed that if m and n are relatively prime then $mn\ D^+_{mn} = d$ implies at most one solution to the equation $k(m + n) - mR_k = d$. Geller (1971) proved that the limiting distribution of $R^+_{mn}/(m + n)$ is uniform on $[0, 1]$ provided $\lim\limits_{m,n\to\infty} m/n$ is finite and positive. Steck-Simmons (1973) derived a formula for $Q_{mn}(d, r)$ and showed that if p is the greatest common divisor of m and n and $q = (m + n)/p$, then $P_{mn}(r) \geqq P_{mn}(r + 1)$, $r = 1, 2, \ldots, m + n - 1$, and $\{P_{mn}(r)\}$ are equal in blocks of length q. They also proved that R^+_{mn} is uniformly distributed on the integers $1, 2, \ldots, m + n$ if m and n are relatively prime. In this paper we propose to give, for finite m and n, the exact distributions of $R^+_{mn}(j)$ and $(D^+_{mn}, R^+_{mn}(j))$ and hence generalize the results of Steck-Simmons (1973).

2. PATH REPRESENTATION

Let us represent the $(m + n)$ observations of the ordered combined sample by a lattice path from $(0, 0)$ to (n, m) with the kth step being one unit up or one unit to the right according as the kth observation in the ordered combined sample is an X or a Y. We observe here that after the kth step up, the path is at the point $(R_k - k, k)$ and that $k(m + n) - mR_k$ is m times the horizontal distance from $(R_k - k, k)$ to the diagonal $y = mx/n$. Thus $mn\, D^+_{mn}$ is m times the maximum horizontal distance from the path to the diagonal $y = mx/n$. In the sequel we shall use the word "distance" to denote "horizontal distance". Distance to the diagonal will be taken positive if the point is to the left of the diagonal and negative otherwise.

A result due to Steck (1969) needed in the sequel is quoted below:

Lemma 1. *Let $b_1 \leqq b_2 \leqq \ldots \leqq b_m$ and $c_1 \leqq c_2 \leqq \ldots \leqq c_m$ be sequences of integers such that $i \leqq b_i \leqq c_i \leqq n + i$, $i = 1, 2, \ldots, m$. Then*

$$\binom{m+n}{n} P(b_i \leqq R_i \leqq c_i, \text{ all } i) = \det \left\{ \binom{c_i - b_j + j - i + 1}{j - i + 1}_+ \right\}_{m \times m},$$

where $\displaystyle\binom{x}{r}_+ = \binom{\max(x, 0)}{r}$.

3. THE DISTRIBUTION OF $R^+_{mn}(j)$

Theorem 1. *Let $p = \gcd(m, n)$, i.e., $m = ap$, $n = bp$ with $\gcd(a, b) = 1$. Then $\displaystyle\binom{m+n}{n} P_{mn}(r, j) = M_{mn}(r, j)$ given by (1).*

The theorem can be proved by considering the following lemmas.

Lemma 2. *The number of paths from $(0, 0)$ to (n, m) through the points (x_1, y_1) and (x_2, y_2), $x_1 \leqq ny_1/m$, $x_2 \leqq ny_2/m$, $x_2 \geqq x_1$, $y_2 \geqq y_1$, that attain their maximum distance from the diagonal $y = mx/n$ for the first and the j th time at (x_1, y_1) and (x_2, y_2), respectively, is the same as the number of paths from $(0, 0)$ to (n, m) through the points $(x_2 - x_1, y_2 - y_1)$ and $(n - x_1, m - y_1)$ that are never above the diagonal before $(n - x_1, m - y_1)$ and never touch the diagonal afterwards and, moreover, have exactly $(j - 1)$ contacts with the diagonal up to the point $(x_2 - x_1, y_2 - y_1)$, the $(j - 1)$ st contact occurring at $(x_2 - x_1, y_2 - y_1)$.*

Proof. Let P_1 be a path from $(0, 0)$ to (x_1, y_1), P_2 a path from (x_1, y_1) to (x_2, y_2) and P_3 a path from (x_2, y_2) to (n, m) such that the combined path $P_1P_2P_3$ attains its maximum distance from the diagonal for the first and the jth time at (x_1, y_1) and (x_2, y_2), respectively. Let P_1' be P_1 shifted up $(m - y_1)$ units and shifted right $(n - x_1)$ units. Then P_1' is a path from $(n - x_1,\ m - y_1)$ to (n, m). Similarly, let $P_2'P_3'$ be P_2P_3 shifted down y_1 units and shifted left x_1 units. Then $P_2'P_3'$ is a path from $(0, 0)$ to $(n - x_1,\ m - y_1)$ passing through the point $(x_2 - x_1,\ y_2 - y_1)$ lying on the diagonal $y = mx/n$. The paths $P_1P_2P_3$ and $P_2'P_3'P_1'$ are in one-to-one correspondence. Finally, $P_2'P_3'P_1'$ is a path from $(0, 0)$ to (n, m) through the points $(x_2 - x_1,\ y_2 - y_1)$ and $(n - x_1,\ m - y_1)$, which is never above the diagonal before $(n - x_1,\ m - y_1)$ and never touches the diagonal afterwards and, in addition, has exactly $(j - 1)$ contacts with the diagonal up to $(x_2 - x_1, y_2 - y_1)$, the $(j - 1)$st contact taking place at $(x_2 - x_1,\ y_2 - y_1)$.

Lemma 3. *The number of paths from $(0, 0)$ to (n, m) which attain their maximum distance from the diagonal $y = mx/n$ for the first and the j th time on the s th and the r th steps, respectively, is the same as the number of paths from $(0, 0)$ to (n, m) that are never above the diagonal before the $(m + n - s)$th step and never touch it afterwards and, moreover, have exactly $(j - 1)$ touches with the diagonal up to the $(r - s)$th step, the $(j - 1)$st touch occurring on the $(r - s)$th step.*

Proof. Consider all points (x_1, y_1) and (x_2, y_2) such that $x_1 + y_1 = s$, $x_2 + y_2 = r$, $x_1 \leqq ny_1/m$, $x_2 \leqq ny_2/m$, $1 \leqq x_1 \leqq x_2 \leqq n$, $1 \leqq y_1 \leqq y_2 \leqq m$. The set of required paths is the union of the disjoint subsets of paths through each of the possible pairs of points $\{(x_1, y_1), (x_2, y_2)\}$. By Lemma 2, the paths in each of these subsets are in one-to-one correspondence with those in the disjoint sets of paths from $(0, 0)$ to (n, m) through $(x_2 - x_1, y_2 - y_1)$ and $(n - x_1, m - y_1)$ that are never above the diagonal and, moreover, never touch the diagonal after $(n - x_1,\ m - y_1)$ and also have $(j - 1)$ touchings of the diagonal up to the point $(x_2 - x_1, y_2 - y_1)$. Hence the elements in the set of required paths are in one-to-one correspondence with the elements in the set of paths that are never above the diagonal and, moreover, never touch the diagonal after the $(m + n - s)$th step and, in addition, have exactly $(j - 1)$ contacts with the diagonal up to the $(r - s)$th step, the $(r - s)$th step forming the $(j - 1)$st contact.

Lemma 4. *The number of paths from $(0, 0)$ to (n, m) that attain their maximum distance from the diagonal for the j th time $(j \geqq 1)$ in the r th step is*

(1) $$M_{mn}(r, j) = \sum_s \varphi_{i,j-1} A_2$$

where the summation extends over all possible integral values of s for which $i = (r - s)/(a + b)$ is an integer. $\varphi_{i,j-1}$ is the coefficient of x^i in

$(1 - e^{-F_1x - F_2x^2 - \dots})^{j-1}$ *where* i *is an integer defined above and*

$$F_j = \frac{1}{j(a+b)} \binom{ja + jb}{ja},$$

A_2 *is given by* (3).

Proof. To prove this let us assume that the path attains its maximum distance from the diagonal for the first time in the sth step. Then the required number of paths can be obtained by summing the result of Lemma 3 over all possible values of s. According to Lemma 3, the fact that the $(j - 1)$st touch with the diagonal $y = mx/n$ occurs in the $(r - s)$th step implies that the point of the path, attained just after the $(r - s)$th step is taken, will lie on the diagonal $y = mx/n$. Therefore for given r, s must be so chosen that $(r - s)$ is an integer multiple of $(a + b)$ where $a = m/p$, $b = n/p$ and $p = \gcd(m, n)$. Let us assume that $(r - s) = i(a + b)$, where i is an integer $\geqq j - 1$. Thus after the $(r - s)$th step, the path will reach the point (ib, ia) lying on the diagonal.

Now the number of transformed paths in Lemma 3 is the same as the number of paths from $(0, 0)$ to (ib, ia), where $i(a + b) = r - s$, that are never above the line $y = mx/n$ and have exactly $(j - 1)$ contacts with $y = mx/n$, the $(j - 1)$st contact occurring at (ib, ia), times the number of paths from $(0, 0)$ to $(n - ib, m - ia) \equiv \equiv ((p - i)\, b, (p - i)\, a)$ that are never above the diagonal $y = mx/n$ and, moreover, never touch it after the $(m + n - r)$th step (for the second part we have taken (ib, ia) as a new origin). Call these numbers A_1 and A_2, respectively. From Section II of Bizley (1954), A_1 is given by $\varphi_{i,j-1}$, i.e.,

$$A_1 = \varphi_{i,j-1} = \text{coeff. of } x^i \text{ in } (1 - e^{-F_1x - F_2x^2 - \dots})^{j-1}, \tag{2}$$

where

$$F_j = \frac{1}{j(a+b)} \binom{ja + jb}{ja}.$$

In what follows we shall use $[x]$, $\langle x \rangle$ and $\{x\}$ to denote, respectively, the greatest integer contained in x, the smallest integer $\geqq x$ and the smallest integer $> x$.

To find A_2, we observe that the line $x + y = m + n - r$ intersects the diagonal $y = mx/n$ at $y_0 = m - mr/(m + n)$. Therefore, the required paths in A_2 are those for which $R_k - k \geqq nk/m$, $1 \leqq k \leqq [y_0]$; $R_k - k > nk/m$, $[y_0] < k < m - ia$; $R_{m-ia} = m + n - i(a + b) = m + n - r + s$. But $[y_0] = m - \langle mr/(m + n) \rangle$, thus A_2 is given by Lemma 1 with sample sizes $m' = m - ia$, $n' = n - ib$; $c_k - k = n - ib$, $k = 1, 2, \dots, m - ia$; $b_k - k = \langle nk/m \rangle$, $k \leqq [y_0]$; $b_k - k = \{nk/m\}$, $[y_0] < k < (m - ia)$; and $b_{m-ia} = m + n - r + s$. Hence

(3)

$$A_2 = \begin{array}{cccc}
\multicolumn{2}{c}{\underbrace{m - \left\langle \frac{mr}{m+n} \right\rangle \text{ columns with } \langle\cdot\rangle}} & & \underbrace{\left\langle \frac{mr}{m+n} \right\rangle - ia - 1 \text{ columns with } \{\cdot\}} \\
\multicolumn{4}{c}{\begin{vmatrix}
\binom{n - ib - \left\langle \frac{n}{m} \right\rangle + 1}{1} & \binom{n - ib - \left\langle \frac{2n}{m} \right\rangle + 1}{2} & \dots & \binom{n - ib - \left\{ \frac{(m - ia - 1)\, n}{m} \right\} + 1}{m - ia - 1} \\
1 & \binom{n - ib - \left\langle \frac{2n}{m} \right\rangle + 1}{1} & \dots & \binom{n - ib - \left\{ \frac{(m - ia - 1)\, n}{m} \right\} + 1}{m - ia - 2} \\
\vdots & \vdots & & \vdots \\
0 & 0 & \dots & \binom{n - ib - \left\{ \frac{(m - ia - 1)\, n}{m} \right\} + 1}{1}
\end{vmatrix}}
\end{array}$$

order: $(m - ia - 1) \times (m - ia - 1)$.

This proves Lemma 4 which in turn gives the exact null distribution of $R^+_{mn}(j)$.

Corollary. $j = 1 \Rightarrow R^+_{mn}(1) = R^+_{mn}$, $s = r$. *Therefore*, $i = 0$ *and hence* $M_{mn}(r, 1) = M_{mn}(r)$ *as given in* (3.2) *of Steck-Simmons* (1973).

This verifies, for $j = 1$, the distribution of R^+_{mn} derived by Steck-Simmons (1973). Thus, in this way, we can say that our result for the exact distribution of $R^+_{mn}(j)$, $j \geqq 1$, is a generalization of Steck-Simmons' result (1973).

4. JOINT DISTRIBUTION OF D^+_{mn} AND $R^+_{mn}(j)$

To derive the joint probability distribution of D^+_{mn} and $R^+_{mn}(j)$, let us first compute the probability $P(mn\, D^+_{mn} = d,\ R^+_{mn}(1) = s,\ R^+_{mn}(j) = r)$ where $r \geqq s$. For this let us consider a path from $(0, 0)$ to (n, m) through the points (x_1, y_1) and (x_2, y_2), $x_i \leqq ny_i/m$, $i = 1, 2$, $x_1 \leqq x_2$, $y_1 \leqq y_2$, that attains its maximum distance from the diagonal for the first and the jth time at (x_1, y_1) and (x_2, y_2), respectively. It corresponds to a path for which $mn\, D^+_{mn} = ny_1 - mx_1 = ny_2 - mx_2$, $R^+_{mn}(1) = x_1 + y_1$, $R^+_{mn}(j) = x_2 + y_2$. By Lemma 2 the number of such paths is the same as the number of paths from $(0, 0)$ to $(x_2 - x_1, y_2 - y_1)$ that are never above the diagonal and have $(j - 1)$ contacts with the diagonal, the $(j - 1)$st contact occurring at $(x_2 - x_1, y_2 - y_1)$, times the number of paths from $(x_2 - x_1, y_2 - y_1)$ to $(n - x_1, m - y_1)$ that are never above the diagonal, times the number of paths from $(n - x_1, m - y_1)$ to (n, m) that never touch the diagonal $y = mx/n$. Let us call these numbers B_1, B_2 and B_3, respectively.

The fact that the $(j - 1)$st contact with the diagonal $y = mx/n$ occurs at the point $(x_2 - x_1, y_2 - y_1)$ implies that the point $(x_2 - x_1, y_2 - y_1)$ lies on the diagonal $y = mx/n$. Therefore, $(x_2 - x_1)$ and $(y_2 - y_1)$ must be integer multiples of b and a, respectively, where $b = n/p$, $a = m/p$ and $p = \gcd(m, n)$. Let $(x_2 - x_1, y_2 - y_1) \equiv (ib, ia)$ where $i = ((x_2 + y_2) - (x_1 + y_1))/(a + b)$ is an integer $\geqq j - 1$. Then again from Section II of Bizley (1954),

$$B_1 = \varphi_{i,j-1} = \text{coeff. of } x^i \text{ in } \left(1 - e^{-F_1 x - F_2 x^2 - \dots}\right)^{j-1} \tag{4}$$

where

$$F_j = \frac{1}{j(a + b)} \binom{ja + jb}{ja}.$$

Taking $(x_2 - x_1, y_2 - y_1)$ as a new origin, B_2 is given by Lemma 1 with sample sizes $m' = m - y_2$, $n' = n - x_2$; $c_k - k = n - x_2$ and $b_k - k = \langle nk/m \rangle$, $k = 1, 2, \dots, m - y_2$. Hence

(5)

$$B_2 = \begin{vmatrix} \dbinom{n - x_2 - \left\langle \frac{n}{m} \right\rangle + 1}{1} & \dbinom{n - x_2 - \left\langle \frac{2n}{m} \right\rangle + 1}{2} & \cdots & \dbinom{n - x_2 - \left\langle \frac{(m - y_2)\, n}{m} \right\rangle + 1}{m - y_2} \\ 1 & \dbinom{n - x_2 - \left\langle \frac{2n}{m} \right\rangle + 1}{1} & \cdots & \dbinom{n - x_2 - \left\langle \frac{(m - y_2)\, n}{m} \right\rangle + 1}{m - y_2 - 1} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \dbinom{n - x_2 - \left\langle \frac{(m - y_2)\, n}{m} \right\rangle + 1}{1} \end{vmatrix}$$

$$\text{order: } (m - y_2) \times (m - y_2)\,.$$

Since the point $(n - x_1, m - y_1)$ is y_1 units below and x_1 units to the left of (n, m), we can take (n, m) as a new origin and consider B_3 as the number of paths from $(0, 0)$ to (x_1, y_1) that are above the line $y = mx/n$. This number is given by Lemma 1 with sample sizes $m' = y_1$, $n' = x_1$; $b_k - k = 0$, $k = 1, 2, \ldots, y_1$; $c_1 = 1$ and $c_k - k + 1 \equiv Z_k = \min\,(x_1 + 1, \langle (n(k - 1))/m \rangle)$, $k = 2, 3, \ldots, y_1$. Hence

(6)

$$B_3 = \begin{vmatrix} \dbinom{\left\langle \frac{n}{m} \right\rangle}{1} & \dbinom{\left\langle \frac{n}{m} \right\rangle}{2} & \dbinom{\left\langle \frac{n}{m} \right\rangle}{3} & \cdots & \dbinom{\left\langle \frac{n}{m} \right\rangle}{y_1 - 1} \\ 1 & \dbinom{\left\langle \frac{2n}{m} \right\rangle}{1} & \dbinom{\left\langle \frac{2n}{m} \right\rangle}{2} & \cdots & \dbinom{\left\langle \frac{2n}{m} \right\rangle}{y_1 - 2} \\ 0 & 1 & \dbinom{\left\langle \frac{3n}{m} \right\rangle}{1} & \cdots & \dbinom{\left\langle \frac{3n}{m} \right\rangle}{y_1 - 3} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \dbinom{z_{y_1}}{1} \end{vmatrix},$$

$$\text{order: } (y_1 - 1) \times (y_1 - 1)\,,$$

since the first rov of the lemma determinant is $(1, 0, 0, \ldots, 0)$. Thus we have the following result:

Lemma 5. *Let $p = \gcd(m, n)$, i.e., $m = ap$, $n = bp$ with $\gcd(a, b) = 1$, $r \geqq s$ and r and s are so connected that $i = (r - s)/(a + b)$ is an integer $\geqq j - 1$. Then*

$$\binom{m+n}{n} P(mn\, D_{mn}^{+} = d,\ R_{mn}^{+}(1) = s,\ R_{mn}^{+}(j) = r) = \begin{cases} B_1 B_2 B_3 \\ 0 \end{cases} \tag{7}$$

according as there exists an integer solution to the equations $ny_1 - mx_1 = ny_2 - mx_2 = d$, $x_1 + y_1 = s$ and $x_2 + y_2 = r$ such that $0 \leqq x_1 \leqq x_2 \leqq n$, $0 \leqq y_1 \leqq y_2 \leqq m$ or not.

Corollary. *For $j = 1$, $R_{mn}^{+}(1) = R_{mn}^{+}$, $s = r$, $x_1 = x_2 = x$, say, $y_1 = y_2 = y$, say, $i = 0$. Thus in this case $B_1 = 1$ and $B_2 = N_1(x, y)$, $B_3 = N_2(x, y)$ as given in (4.1) and (4.2) of Steck-Simmons (1973), respectively. Hence (7) reduces to*

$$\binom{m+n}{n} P(mn\, D_{mn}^{+} = d,\ R_{mn}^{+} = r) = \begin{cases} N_1(x, y) \,.\, N_2(x, y) \\ 0 \end{cases} \tag{8}$$

according as there exists an integer solution to the equations $ny - mx = d$, $x + y = r$ such that $0 \leqq x \leqq n$, $0 \leqq y \leqq m$ or not.

It verifies Lemma 6 of Steck-Simmons (1973). Finally, we have

Theorem 2. *If $p = \gcd(m, n)$, i.e., $m = ap$, $n = bp$ with $\gcd(a, b) = 1$, then*

$$\binom{m+n}{n} Q_{mn}(d, r, j) = \begin{cases} \sum_{x_1} \sum_{y_1} B_1 B_2 B_3 \\ 0 \end{cases} \tag{9}$$

according as there exists an integer solution to the equations $ny_2 - mx_2 = d$, $x_2 + y_2 = r$ such that $0 \leqq x_2 \leqq n$, $0 \leqq y_2 \leqq m$ or not; the summation extends over all pairs of values (x_1, y_1) for which the following conditions hold:

(i) *x_1 and y_1 are integers with $0 \leqq x_1 \leqq x_2$, $0 \leqq y_1 \leqq y_2$,*

(ii) *$ny_1 - mx_1 = d$,*

(iii) *$r - (x_1 + y_1)$ is an integer multiple of $(a + b)$, i.e., $i = (r - (x_1 + y_1))/(a + b)$ is an integer $\geqq j - 1$.*

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Souhrn

O ROZLOŽENÍ $R_{mn}^+(j)$ A $(D_{mn}^+, R_{mn}^+(j))$

JAGDISH SARAN, KANWAR SEN

Nechť $F_m(x)$, $G_n(x)$ jsou dvě empirické funkce rozložení ve dvouvýběrovém problému. Rozdíl $F_m(x) - G_n(x)$ se mění pouze v bodech x_i, $i = 1, \ldots, m + n$, které odpovídají jednotlivým pozorováním. Nechť $R_{mn}^+(j)$ označuje index i, pro který x_i je j-tý bod, v němž $F_m(x) - G_n(x)$ dosahuje maximální hodnoty D_{mn}^+. V článku se odvozují pravděpodobnosti pro $R_{mn}^+(j)$ a pro vektor $(D_{mn}^+, R_{mn}^+(j))$ při hypotéze $H_0 : F = G$; tím se zobecňují výsledky Stecka-Simmonse (1973). Výsledky jsou odvozeny pomocí náhodných procházek.

Authors' address: Prof. *Jagdish Saran,* Prof. *Kanwar Sen,* Department of Mathematical Statistics, Faculty of Mathematics, University of Delhi, Delhi-110007, India.