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ANALYSIS OF A TWO-UNIT STANDBY REDUNDANT SYSTEM WITH THREE STATES OF UNITS

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Many authors have been interested in various two-unit redundant systems in recent years – see e.g. [4, 7–11]. Many characteristics of the behaviour of such systems have been derived. The authors mostly suppose that the state of each unit at a given moment can be described by one of only two degrees – a unit either is able to operate or not.

In this paper we shall deal with a redundant system composed of two identical units. Each unit belongs to one of three qualitative classes (states) at every moment. Units in state I or II are able to work, units in state III cannot work. In the system three is one repair facility. A unit may operate (O), wait for its repair (W), be repaired (R) or wait for its operative exploitation – be in reserve (S). Possible changes of the function of a unit are illustrated in Fig. 1 and are carried out by a switchover.

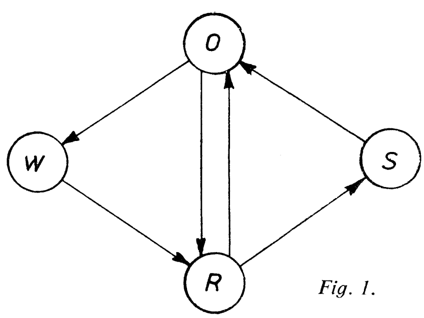

Fig. 1.

Units make their quality worse by working and improve it by being repaired. Thus at certain moments individual units are re-classified and change their states. We admit only the following state-transitions of a unit: $I \to II$, $II \to III$, $II \to I$, $III \to I$. It means that a unit in state I cannot deteriorate in such a way that it enters state III without first being in state II and that each unit is fully restored to the as-new condition (state I) upon repair.

About the organization of the system we suppose:

1) The states of units which are outside of the repair facility are monitored continuously, a unit in repair is keeping the state with which its repair started and at the moment when its repair finishes it is in state I.

2) An operating unit can stop its operation only at a moment of its change of state.

3) A repair cannot be interrupted.

4) The case of cold reserve is considered.

5) The switchover and the repair facility are perfect and instantaneous.

6) At a moment when a unit deteriorates from I to II and the other one is in state I, the former is put into repair while the latter one is switched into operation.

7) At the beginning of the operation of the system both units are in state I.

8) The system has only two states – operating and failed. The system is operating if and only if a unit is operating.

9) All random variables – time of work of a unit in state I and II and time of repair of a unit of the type $II \to I$ and $III \to I$ (denoted by $\mathscr{A}$, $\mathscr{B}$, $\mathscr{M}$ and $\mathscr{N}$, respectively) – are positive with probability 1, mutually stochastically independent and generally distributed.

The development of our system can be described as follows:

1) At the starting instant both units are in state I. We choose one of them. This one will enter state II after time $\mathscr{A}$.

2) At a moment when one unit deteriorates from I to II:

a) in the case that the other unit is in state I, the former is given into repair and the latter starts to operate;

b) in the case that the other unit is in repair (and it will stay there because of assumption 3 above), the first unit goes on operating and after time $\mathscr{B}$ it will deteriorate from II to III.

3) At a moment when one unit deteriorates from II to III:

a) in the case that the other unit is in state I, the former is given into repair and the latter starts to operate;

b) in the case that the other unit is in repair, the former starts waiting for its repair and the system interrupts its operation.

4) At a moment when a unit is waiting for its repair and a repair of the other one is finished, the former is given into repair, the latter starts to operate and the system starts its new operative period.

The aim of this paper is to find some characteristics (probabilities, distribution functions or their Laplace Stieltjes transforms, mathematical expectations) of the quality of the system described above. We consider probabilities that the first system failure occurs during a repair of a unit of the type $II \to I$ or $III \to I$, random variables time to system failure and time of a non-operating period of the system and stationary state-probabilities of the couple of units of the system.

1. NOTATION

$*$ sign of convolution,

$A(x)$ – distribution function (d.f.) of time of work of a unit in state I,

$B(x)$ – d.f. of time of work of a unit in state II,

$M(x)$ – d.f. of time of repair of a unit $II \to I$,

$N(x)$ – d.f. of time of repair of a unit $III \to I$,

$\mathscr{A}$, $\mathscr{B}$, $\mathscr{M}$, $\mathscr{N}$ – random variables with distribution functions A, B, M and N, respectively,

$\mathscr{Z}_M \;= \max\{\mathscr{M}; \mathscr{A} + \mathscr{B}\}$,

$\mathscr{Z}_N \;= \max\{\mathscr{N}; \mathscr{A} + \mathscr{B}\}$,

$$C(x) = \int_{-\infty}^{x+0} M(y)\,\mathrm{d}A(y)\,,$$

$$D(x) = \int_{-\infty}^{x+0}\left(\int_{-\infty}^{x-y+0} M(y+z)\,\mathrm{d}A(z)\right)\mathrm{d}B(y)\,,$$

$$E(x) = \int_{-\infty}^{x+0} N(y)\,\mathrm{d}A(y)\,,$$

$$F(x) = \int_{-\infty}^{x+0}\left(\int_{-\infty}^{x-y+0} N(y+z)\,\mathrm{d}A(z)\right)\mathrm{d}B(y)\,,$$

$$c = \mathsf{P}(\mathscr{A} \geqq \mathscr{M}) = \lim_{x\to\infty} C(x),$$

$$d = \mathsf{P}(\mathscr{A} + \mathscr{B} \geqq \mathscr{M}) = \lim_{x\to\infty} D(x)\,,$$

$$e = \mathsf{P}(\mathscr{A} \geqq \mathscr{N}) = \lim_{x\to\infty} E(x)\,,$$

$$f = \mathsf{P}(\mathscr{A} + \mathscr{B} \geqq \mathscr{N}) = \lim_{x\to\infty} F(x)\,,$$

α, β, γ, δ, ε, φ – Laplace Stieltjes transforms of functions A, B, C, D, E and F, respectively,

$X(t)$ – the random process describing the development of the system,

$\{\mathsf{e}_P; \mathsf{e}_S; \mathsf{e}_L; \mathsf{e}_R\}$ – the state-space of the process $X(t)$,

X_n – the chain embedded into the proces $X(t)$,

Y_n – the chain describing the phases of the development of the system,

$\mathfrak{M} = \{\mathsf{e}_S; \mathsf{e}_L\}$ – the state-space of the chain Y_n,

V – the set of all possible states of the couple of units,

$\mathscr{P}_X(i)$ for $i \in \{P; S; L; R\}$ – the condition that e_i was the initial state of the random process $X(t)$.

2. PROBABILITIES OF TYPES OF THE FIRST SYSTEM FAILURE

The behaviour of the system in question can be described by means of a random process $X(t)$ with four states (e_P, e_S, e_L, e_R), which can change its state only a moments of the following three types: 1) when a unit deteriorates from *I* to *II* and the other one is in state *I*; 2) when a unit deteriorates from *II* to *III*; 3) when a repair of a unit is finished and the other unit is in state *III* (hence it waits for its repair). Let t_0 be such a moment. We define that at t_0 the process $X(t)$ enters the state:

e_P – if at t_0 the development of the system starts and both units are in state *I*;
e_S – if t_0 is a time instant of the type 1;
e_L – if t_0 is a time instant of the type 3 or if t_0 is a time instant of the type 2 and the other unit is in state *I* at t_0;
e_R – if t_0 is a time instant of the type 2 and the other unit is not in state *I* at t_0.

In such a way the state of the process $X(t)$ has been determined with probability 1 at each moment except the moments when $X(t)$ changes its state. Let us define for the sake of completeness that the trajectories of $X(t)$ are right-continuous. Changes of states of $X(t)$ having positive probability are illustrated in Fig. 2.

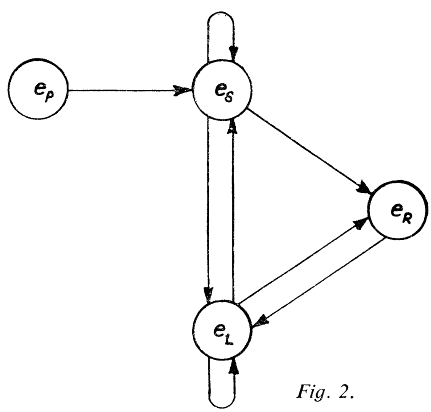

Fig. 2.

It is easy to see that the moments when the process $X(t)$ enters the state e_S or e_L have the property that the development of $X(t)$ after t_0 does not depend on the history of $X(t)$ until t_0 because at t_0, a unit starts to operate and the other one is given into repair and because of the assumption 9 about the organization of the system. On the other hand, let the process $X(t)$ enter the state e_R at t_0. Then at t_0 a unit starts to wait for its repair and a repair of the other one is in progress, i.e. it started before t_0 and will be finished after t_0. The sojourn time of $X(t)$ in the state e_R (from t_0) is hence equal to the time necessary for the completion of the repair of the second unit at t_0 and is thus dependent both on the preceding state of $X(t)$ (i.e. on the type of the repair of the second unit) and on the sojourn time of $X(t)$ in the preceding state. Altogether we obtain that the process $X(t)$ has the semi-Markov property on each time interval where it is operating.

Let X_n be the random chain embedded into the process $X(t)$, i.e. $X_n = \mathsf{e}_i$ if and only if $X(t)$ enters the state e_i, $i \in \{P; S; L; R\}$, after its n-th change of state. We know that if $X_n = \mathsf{e}_R$ then $P(X_{n+1} = \mathsf{e}_L) = 1$ irrespective of the values $X_1, \ldots, X_{n-1}$. Thus the transitions of the chain X_n from the states e_R have the Markov property. The semi-Markov property of the process $X(t)$ on each time interval where the system is operating implies the Markov property of the chain X_n with transitions from states e_P, e_S and e_L. Summarily, we obtain that the chain X_n is markovian. Its matrix of transition probabilities has the form

$$(2.1)\qquad \mathbf{X} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & \mathsf{P}(\mathscr{A} \geqq \mathscr{M}) & \mathsf{P}(\mathscr{A} < \mathscr{M} \leqq \mathscr{A} + \mathscr{B}) & \mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M}) \\ 0 & \mathsf{P}(\mathscr{A} \geqq \mathscr{N}) & \mathsf{P}(\mathscr{A} < \mathscr{N} \leqq \mathscr{A} + \mathscr{B}) & \mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N}) \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Let $x^{(2)}$, $y^{(2)}$, $z^{(2)}$ and $x^{(3)}$, $y^{(3)}$, $z^{(3)}$ be probabilities of events that the first system failure occurs during the repair of a unit of the type $II \to I$ or $III \to I$ under the conditions $\mathscr{P}_X(P)$, $\mathscr{P}_X(S)$ and $\mathscr{P}_X(L)$, respectively.

Supplementary assumption: We shall consider only the case that a failure of the system comes with probability 1 under each of the conditions $\mathscr{P}_X(P)$, $\mathscr{P}_X(S)$ and $\mathscr{P}_X(L)$, i.e. we shall suppose that the following condition is fulfilled:

$$(2.2)\qquad x^{(2)} + x^{(3)} = y^{(2)} + y^{(3)} = z^{(2)} + z^{(3)} = 1\,.$$

It can be easily seen that (2.2) is equivalent to

$$(2.3)\qquad (1 - c)\,.\,(1 - f) + e\,.\,(1 - d) \neq 0\,.$$

The restriction connected with this supplementary assumption is essential neither from the point of view of real systems, nor of the statements of this paper.

Theorem 1. *The probabilities* $x^{(2)}$, $y^{(2)}$, $z^{(2)}$, $x^{(3)}$, $y^{(3)}$ *and* $z^{(3)}$ *have the values*

$$(2.4)\qquad x^{(2)} = y^{(2)} = \frac{(1 - d)\,.\,(1 + e - f)}{(1 - c)\,.\,(1 - f) + e\,.\,(1 - d)}\,,$$

$$(2.5)\qquad z^{(2)} = \frac{(1 - d)\,.\,e}{(1 - c)\,.\,(1 - f) + e\,.\,(1 - d)}\,,$$

$$(2.6)\qquad x^{(3)} = y^{(3)} = \frac{(d - c)\,.\,(1 - f)}{(1 - c)\,.\,(1 - f) + e\,.\,(1 - d)}\,,$$

$$(2.7)\qquad z^{(3)} = \frac{(1 - c)\,.\,(1 - f)}{(1 - c)\,.\,(1 - f) + e\,.\,(1 - d)}\,.$$

Proof. The Markov property of the chain X_n implies following equations

(2.8) $x^{(2)} = y^{(2)}$,

(2.9) $y^{(2)} = \mathsf{P}(\mathcal{A} \geqq \mathcal{M}) \,.\, y^{(2)} + \mathsf{P}(\mathcal{A} < \mathcal{M} \leqq \mathcal{A} + \mathcal{B}) \,.\, z^{(2)} + \mathsf{P}(\mathcal{A} + \mathcal{B} < \mathcal{M})$,

(2.10) $z^{(2)} = \mathsf{P}(\mathcal{A} \geqq \mathcal{N}) \,.\, y^{(2)} + \mathsf{P}(\mathcal{A} < \mathcal{N} \leqq \mathcal{A} + \mathcal{B}) \,.\, z^{(2)}$,

and

(2.11) $x^{(3)} = y^{(3)}$,

(2.12) $y^{(3)} = \mathsf{P}(\mathcal{A} \geqq \mathcal{M}) \,.\, y^{(3)} + \mathsf{P}(\mathcal{A} < \mathcal{M} \leqq \mathcal{A} + \mathcal{B}) \,.\, z^{(3)}$,

(2.13) $z^{(3)} = \mathsf{P}(\mathcal{A} \geqq \mathcal{N}) \,.\, y^{(3)} + \mathsf{P}(\mathcal{A} < \mathcal{N} \leqq \mathcal{A} + \mathcal{B}) \,.\, z^{(3)} + \mathsf{P}(\mathcal{A} + \mathcal{B} < \mathcal{N})$.

The solution of the systems of equations (2.8) to (2.10) and (2.11) to (2.13) has the form (2.4) to (2.7).

3. TIME TO SYSTEM FAILURE

We denote the random variables "time to system failure under the conditions $\mathcal{P}_X(P)$, $\mathcal{P}_X(S)$ and $\mathcal{P}_X(L)$" by $\mathcal{P}$, $\mathcal{S}$ and $\mathcal{L}$, respectively. The semi-Markov property of the process $X(t)$ implies the relations

$$\mathcal{P} = \mathcal{A} + \mathcal{S}\,, \tag{3.1}$$

$$\mathcal{S} = \begin{cases} \mathcal{T}_{SS} + \mathcal{S}\,, & \text{if } \mathcal{A} \geqq \mathcal{M}\,, \\ \mathcal{T}_{SL} + \mathcal{L}\,, & \text{if } \mathcal{A} < \mathcal{M} \leqq \mathcal{A} + \mathcal{B}\,, \\ \mathcal{T}_{SR}\,, & \text{if } \mathcal{A} + \mathcal{B} < \mathcal{M}\,, \end{cases} \tag{3.2}$$

$$\mathcal{L} = \begin{cases} \mathcal{T}_{LS} + \mathcal{S}\,, & \text{if } \mathcal{A} \geqq \mathcal{N}\,, \\ \mathcal{T}_{LL} + \mathcal{L}\,, & \text{if } \mathcal{A} < \mathcal{N} \leqq \mathcal{A} + \mathcal{B}\,, \\ \mathcal{T}_{LR}\,, & \text{if } \mathcal{A} + \mathcal{B} < \mathcal{N}\,, \end{cases} \tag{3.3}$$

where $\mathcal{T}_{ij}$ for $i \in \{S; L\}$ and $j \in \{S; L; R\}$ is the random variable sojourn time of the process $X(t)$ in the state e_i under the condition that after this time $X(t)$ will enter the state e_j, the right hand sides are sums of independent random variables and the meaning of the symbols $\mathcal{A}$, $\mathcal{B}$, $\mathcal{M}$ and $\mathcal{N}$ is as follows: $\mathcal{M}(\mathcal{N})$ is the time of the repair which started at the moment when the system was activated in the state $\mathsf{e}_S(\mathsf{e}_L)$; $\mathcal{A}$ and $\mathcal{B}$ are the times of work in state I and II of that unit which started to operate at the moment when the system was activated. Let $P(x)$, $S(x)$ and $L(x)$ be the distribution functions of $\mathcal{P}$, $\mathcal{S}$ and $\mathcal{L}$, respectively, and let $\pi(t)$, $\sigma(t)$ and $\lambda(t)$ be their Laplace Stieltjes transforms.

Now we calculate the distribution functions of the random variables $\mathcal{T}_{ij}$:

$$\mathsf{P}(\mathcal{T}_{SS} \leqq x) = P(\mathcal{A} \leqq x / \mathcal{A} \geqq \mathcal{M}) =$$

$$= \frac{\int_{-\infty}^{x+0} \mathsf{P}(\mathscr{A} \geqq \mathscr{M} / \mathscr{A} = y)\, \mathrm{d}A(y)}{\mathsf{P}(\mathscr{A} \geqq \mathscr{M})} = \frac{C(x)}{\mathsf{P}(\mathscr{A} \geqq \mathscr{M})}$$

and similarly

$$\mathsf{P}(\mathscr{T}_{LS} \leqq x) = \frac{E(x)}{\mathsf{P}(\mathscr{A} \geqq \mathscr{N})},$$

$$\mathsf{P}(\mathscr{T}_{SL} \leqq x) = \mathsf{P}(\mathscr{A} + \mathscr{B} \leqq x / \mathscr{A} < \mathscr{M} \leqq \mathscr{A} + \mathscr{B}) =$$

$$= \frac{\int_{-\infty}^{x+0} \left(\int_{-\infty}^{x-y+0} [M(y+z) - M(z)]\, \mathrm{d}A(z) \right) \mathrm{d}B(y)}{\mathsf{P}(\mathscr{A} < \mathscr{M} \leqq \mathscr{A} + \mathscr{B})} =$$

$$= \frac{D(x) - (B * C)(x)}{\mathsf{P}(\mathscr{A} < \mathscr{M} \leqq \mathscr{A} + \mathscr{B})}$$

and similarly

$$\mathsf{P}(\mathscr{T}_{LL} \leqq x) = \frac{F(x) - (B * E)(x)}{\mathsf{P}(\mathscr{A} < \mathscr{N} \leqq \mathscr{A} + \mathscr{B})},$$

$$\mathsf{P}(\mathscr{T}_{SR} \leqq x) = \mathsf{P}(\mathscr{A} + \mathscr{B} \leqq x / \mathscr{A} + \mathscr{B} < \mathscr{M}) =$$

$$= \frac{\int_{-\infty}^{x+0} \left(\int_{-\infty}^{x-y+0} [1 - M(y+z)]\, \mathrm{d}A(z) \right) \mathrm{d}B(y)}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})} =$$

$$= \frac{(A * B)(x) - D(x)}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})}$$

and similarly

$$\mathsf{P}(\mathscr{T}_{LR} \leqq x) = \frac{(A * B)(x) - F(x)}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N})}.$$

After passing to the Laplace Stieltjes transforms we get from the formula (3.1)

$$\pi(t) = \alpha(t) \,.\, \sigma(t) \tag{3.4}$$

and from (3.2) and (3.3)

$$\sigma(t) = \gamma(t) \,.\, \sigma(t) + [\delta(t) - \beta(t) \,.\, \gamma(t)] \,.\, \lambda(t) + \alpha(t) \,.\, \beta(t) - \delta(t)\,, \tag{3.5}$$

$$\lambda(t) = \varepsilon(t) \,.\, \sigma(t) + [\varphi(t) - \beta(t) \,.\, \varepsilon(t)] \,.\, \lambda(t) + \alpha(t) \,.\, \beta(t) - \varphi(t)\,. \tag{3.6}$$

Theorem 2. *The Laplace Stieltjes transforms of the distributions of the random variables* $\mathscr{P}$, $\mathscr{S}$ *and* $\mathscr{L}$ *have the form*

$$\pi(t) = \left[\alpha \,.\, \frac{(\alpha\beta - \delta) \,.\, (1 - \varphi + \beta\varepsilon) + (\alpha\beta - \varphi) \,.\, (\delta - \beta\gamma)}{(1 - \gamma) \,.\, (1 - \varphi + \beta\varepsilon) - \varepsilon \,.\, (\delta - \beta\gamma)} \right]_t, \tag{3.7}$$

$$(3.8)\qquad \sigma(t) = \left[\frac{(\alpha\beta - \delta) . (1 - \varphi + \beta\varepsilon) + (\alpha\beta - \varphi) . (\delta - \beta\gamma)}{(1 - \gamma) . (1 - \varphi + \beta\varepsilon) - \varepsilon . (\delta - \beta\gamma)}\right]_t ,$$

$$(3.9)\qquad \lambda(t) = \left[\frac{(1 - \gamma) . (\alpha\beta - \varphi) + \varepsilon(\alpha\beta - \delta)}{(1 - \gamma) . (1 - \varphi + \beta\varepsilon) - \varepsilon . (\delta - \beta\gamma)}\right]_t ,$$

where α, β, γ, δ, ε and φ are the Laplace Stieltjes transforms of the functions A, B, C, D, E and F determined in Section 1.

Theorem 3. *Let the random variables $\mathcal{A}$ and $\mathcal{B}$ have finite mathematical expectations. Then the mathematical expectations of the random variables $\mathcal{P}$, $\mathcal{S}$ and $\mathcal{L}$ have the form*

$$(3.10)\qquad \mathsf{E}\mathcal{P} = \mathsf{E}\mathcal{A} + \mathsf{E}\mathcal{B} + \frac{(1 - c + d + e - f) . \mathsf{E}\mathcal{A} + (d - c) . \mathsf{E}\mathcal{B}}{(1 - c) . (1 - f) + e . (1 - d)} ,$$

$$(3.11)\qquad \mathsf{E}\mathcal{S} = \mathsf{E}\mathcal{B} + \frac{(1 - c + d + e - f) . \mathsf{E}\mathcal{A} + (d - c) . \mathsf{E}\mathcal{B}}{(1 - c) . (1 - f) + e . (1 - d)} ,$$

$$(3.12)\qquad \mathsf{E}\mathcal{L} = \frac{(1 - c + e) . \mathsf{E}\mathcal{A} + (1 - c) . \mathsf{E}\mathcal{B}}{(1 - c) . (1 - f) + e . (1 - d)} .$$

4. TIME OF NON-OPERATING STATE OF THE SYSTEM

We denote the random variables "the length of time of the first non-operating period of the system under the conditions $\mathcal{P}_X(P)$, $\mathcal{P}_X(S)$ and $\mathcal{P}_X(L)$" by $\mathcal{O}_P$, $\mathcal{O}_S$ and $\mathcal{O}_L$, respectively. Let us note that from the semi-Markov property of the process $X(t)$ and from Figure 2 the following two results are obvious:

1) The random variables $\mathcal{O}_P$ and $\mathcal{O}_S$ have the same distribution.
2) The distribution of $\mathcal{O}_L$ and of the length of time of the second and all further non-operating periods of the system under an arbitrary condition about its initial state are the same.

Hence we can restrict our interest only to the variables $\mathcal{O}_P$ and $\mathcal{O}_L$.

Let the first system failure occur at t_0, then the process $X(t)$ enters the state e_R at t_0. This transition can come either from e_S or from e_L. Let it come from e_S and let the last change of state of $X(t)$ before t_0 occur at a moment t_1. Thus at the same instant t_1 a unit began to operate in state I and a repair of the other one from state II started. Before this repair is finished such two deteriorations of the first unit occured that it changed its state from I to II and from II to III. At the moment t_0 of the second deterioration the system interrupts its operation. Hence

$$t_0 = t_1 + \mathcal{A} + \mathcal{B} .$$

On the other hand, the repair of the second unit will be finished at $t_1 + \mathcal{M}$ and the system will operate again since this moment. The non-operating period lasts from $t_1 + \mathcal{A} + \mathcal{B}$ to $t_1 + \mathcal{M}$. Thus under the condition that the first system failure occurs during a repair of a unit from state *II* we have

$$\mathcal{O}_P = \mathcal{O}_L = \mathcal{M} - \mathcal{A} - \mathcal{B}\,, \tag{4.1}$$

where the random variables $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{M}$ must fulfil the inequality $\mathcal{A} + \mathcal{B} < \mathcal{M}$. Under the condition that the first system failure occurs during a repair of a unit from state *III* we similarly have

$$\mathcal{O}_P = \mathcal{O}_L = \mathcal{N} - \mathcal{A} - \mathcal{B}\,, \tag{4.2}$$

where the random variables $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{N}$ must fulfil the inequality $\mathcal{A} + \mathcal{B} < \mathcal{N}$. We obtain

$$\begin{aligned} \mathsf{P}(\mathcal{O}_P \leqq t) = {} & x^{(2)} \,.\, \mathsf{P}(\mathcal{M} - \mathcal{A} - \mathcal{B} \leqq t / \mathcal{A} + \mathcal{B} < \mathcal{M}) + \\ & + x^{(3)} \,.\, \mathsf{P}(\mathcal{N} - \mathcal{A} - \mathcal{B} \leqq t / \mathcal{A} + \mathcal{B} < \mathcal{N})\,, \end{aligned} \tag{4.3}$$

$$\begin{aligned} \mathsf{P}(\mathcal{O}_L \leqq t) = {} & z^{(2)} \,.\, \mathsf{P}(\mathcal{M} - \mathcal{A} - \mathcal{B} \leqq t / \mathcal{A} + \mathcal{B} < \mathcal{M}) + \\ & + z^{(3)} \,.\, \mathsf{P}(\mathcal{N} - \mathcal{A} - \mathcal{B} \leqq t / \mathcal{A} + \mathcal{B} < \mathcal{N})\,. \end{aligned} \tag{4.4}$$

Theorem 4. *Let* $\mathsf{P}(\mathcal{A} + \mathcal{B} < \mathcal{M}) > 0$ *and* $\mathsf{P}(\mathcal{A} + \mathcal{B} < \mathcal{N}) > 0$. *Then for every* $t < 0$

$$\mathsf{P}(\mathcal{O}_P \leqq t) = \mathsf{P}(\mathcal{O}_L \leqq t) = 0 \tag{4.5}$$

and for every $t \geqq 0$

$$\mathsf{P}(\mathcal{O}_P \leqq t) = 1 + x^{(2)} \,.\, \frac{g(t) - 1}{\mathsf{P}(\mathcal{A} + \mathcal{B} < \mathcal{M})} + x^{(3)} \,.\, \frac{h(t) - 1}{\mathsf{P}(\mathcal{A} + \mathcal{B} < \mathcal{N})}\,, \tag{4.6}$$

and

$$\mathsf{P}(\mathcal{O}_L \leqq t) = 1 + z^{(2)} \,.\, \frac{g(t) - 1}{\mathsf{P}(\mathcal{A} + \mathcal{B} < \mathcal{M})} + z^{(3)} \,.\, \frac{h(t) - 1}{\mathsf{P}(\mathcal{A} + \mathcal{B} < \mathcal{N})}\,, \tag{4.7}$$

where the numbers $x^{(2)}$, $x^{(3)}$, $z^{(2)}$ and $z^{(3)}$ *have been determined by Theorem* 1 *and the functions* $g(t)$ *and* $h(t)$ *have for all* $t \geqq 0$ *the following expressions*

$$g(t) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} M(t + y + z)\, \mathrm{d}B(z) \right) \mathrm{d}A(y)\,, \tag{4.8}$$

$$h(t) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} N(t + y + z)\, \mathrm{d}B(z) \right) \mathrm{d}A(y)\,. \tag{4.9}$$

Proof. The random variables $\mathcal{O}_P$ and $\mathcal{O}_L$ are evidently non-negative. This fact proves (4.5) for all negative t. For all non-negative t we have

$$\mathsf{P}(\mathcal{M} - \mathcal{A} - \mathcal{B} \leqq t / \mathcal{A} + \mathcal{B} < \mathcal{M}) =$$

$$= \frac{\int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} [M(t + y + z) - M(y + z)] \, \mathrm{d}B(z) \right) \mathrm{d}A(y)}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})} =$$

$$= \frac{g(t) - \mathsf{P}(\mathscr{M} \leqq \mathscr{A} + \mathscr{B})}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})}$$

and consequently,

$$\text{(4.10)} \qquad \mathsf{P}(\mathscr{M} - \mathscr{A} - \mathscr{B} \leqq t / \mathscr{A} + \mathscr{B} < \mathscr{M}) = 1 + \frac{g(t) - 1}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})} .$$

Similarly

$$\text{(4.11)} \qquad \mathsf{P}(\mathscr{N} - \mathscr{A} - \mathscr{B} \leqq t / \mathscr{A} + \mathscr{B} < \mathscr{N}) = 1 + \frac{h(t) - 1}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N})} .$$

The relations (4.6) and (4.7) can be obtained by substituting from (4.10) and (4.11) into (4.3) and (4.4) with help of (2.2).

Note: If $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M}) = 0$ and $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N}) > 0$, then by Theorem 1

$$x^{(2)} = z^{(2)} = 0 ,$$

$$x^{(3)} = z^{(3)} = 1$$

and it is easy to find that for all $t \geqq 0$

$$\text{(4.12)} \qquad \mathsf{P}(\mathscr{O}_P \leqq t) = \mathsf{P}(\mathscr{O}_L \leqq t) = 1 + \frac{h(t) - 1}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N})} .$$

On the other hand, if $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M}) > 0$ and $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N}) = 0$ then similarly for all $t \geqq 0$

$$\text{(4.13)} \qquad \mathsf{P}(\mathscr{O}_P \leqq t) = \mathsf{P}(\mathscr{O}_L \leqq t) = 1 + \frac{g(t) - 1}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})} .$$

The case $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M}) = \mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N}) = 0$ is not possible because of Supplementary assumption (2.3).

Theorem 5. *Let* $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M}) > 0$ *and* $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N}) > 0$ *and let the random variables* $\mathscr{A}$, $\mathscr{B}$, $\mathscr{M}$ *and* $\mathscr{N}$ *have finite mathematical expectations. Then the mathematical expectations of the variables* $\mathscr{O}_P$ *and* $\mathscr{O}_L$ *have the forms*

$$\text{(4.14)} \qquad \mathsf{E}\mathscr{O}_P = \frac{x^{(2)} . (\mathsf{E}\mathscr{Z}_M - \mathsf{E}\mathscr{A} - \mathsf{E}\mathscr{B})}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})} + \frac{x^{(3)} . (\mathsf{E}\mathscr{Z}_N - \mathsf{E}\mathscr{A} - \mathsf{E}\mathscr{B})}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N})} ,$$

$$\text{(4.15)} \qquad \mathsf{E}\mathscr{O}_L = \frac{z^{(2)} . (\mathsf{E}\mathscr{Z}_M - \mathsf{E}\mathscr{A} - \mathsf{E}\mathscr{B})}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})} + \frac{z^{(3)} . (\mathsf{E}\mathscr{Z}_N - \mathsf{E}\mathscr{A} - \mathsf{E}\mathscr{B})}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N})} ,$$

where the numbers $x^{(2)}$, $x^{(3)}$, $z^{(2)}$ *and* $z^{(3)}$ *have been given by Theorem* 1 *and the random variables* $\mathscr{Z}_M$ *and* $\mathscr{Z}_N$ *have been defined in Section* 1.

Proof. The distribution functions of the random variables $\mathcal{O}_P$ and $\mathcal{O}_L$ have been given in (4.4) to (4.6). We have

$$(4.16)\quad \mathsf{E}\mathcal{O}_P = \int_0^\infty \left[x^{(2)} \cdot \frac{1 - g(t)}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})} + x^{(3)} \cdot \frac{1 - h(t)}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N})}\right] \mathrm{d}t =$$

$$= \frac{x^{(2)}}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})} \cdot \int_0^\infty [1 - g(t)]\, \mathrm{d}t + \frac{x^{(3)}}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N})} \cdot \int_0^\infty [1 - h(t)]\, \mathrm{d}t\,,$$

where

$$(4.17)\quad \int_0^\infty [1 - g(t)]\, \mathrm{d}t = \int_0^\infty \left(\int_{-\infty}^\infty [1 - M(t + y)]\, \mathrm{d}(A * B)(y)\right) \mathrm{d}t =$$

$$= \int_{-\infty}^\infty \left(\int_{y-0}^\infty [1 - M(z)]\, \mathrm{d}z\right) \mathrm{d}(A * B)(y) =$$

$$= \int_{-\infty}^\infty \left(\int_{-\infty}^{z+0} [1 - M(z)]\, \mathrm{d}(A * B)(y)\right) \mathrm{d}z =$$

$$= \int_0^\infty [1 - M(z)]\,(A * B)(z)\, \mathrm{d}z = \mathsf{E}\mathscr{Z}_M - \mathsf{E}\mathscr{A} - \mathsf{E}\mathscr{B}$$

and similarly

$$(4.18)\quad \int_0^\infty [1 - h(t)]\, \mathrm{d}t = \mathsf{E}\mathscr{Z}_N - \mathsf{E}\mathscr{A} - \mathsf{E}\mathscr{B}\,.$$

By substituting from (4.17) and (4.18) into (4.16) we get (4.14). The formula (4.15) can be proved in a similar way.

Note. If $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M}) = 0$ and $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N}) > 0$ then

$$(4.19)\quad \mathsf{E}\mathcal{O}_P = \mathsf{E}\mathcal{O}_L = \frac{\mathsf{E}\mathscr{Z}_N - \mathsf{E}\mathscr{A} - \mathsf{E}\mathscr{B}}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N})}$$

and if $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M}) > 0$ and $\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{N}) = 0$ then

$$(4.20)\quad \mathsf{E}\mathcal{O}_P = \mathsf{E}\mathcal{O}_L = \frac{\mathsf{E}\mathscr{Z}_M - \mathsf{E}\mathscr{A} - \mathsf{E}\mathscr{B}}{\mathsf{P}(\mathscr{A} + \mathscr{B} < \mathscr{M})}\,.$$

5. STATIONARY STATE-PROBABILITIES OF THE COUPLE OF UNITS

Let us observe two regenerative events — those of the random process $X(t)$ (described in Section 2) entering the states e_S and e_L. Let us denote

$$\mathfrak{M} = \{\mathrm{e}_S; \mathrm{e}_L\}\,.$$

The time interval between two successive regenerative events i and j, where $i, j \in \mathfrak{M}$, will be called the phase of the type i. The random chain Y_n which describes the type of phases is clearly markovian with the matrix of transition probabilities

$$(5.1) \qquad \begin{pmatrix} \mathsf{P}(\mathscr{A} \geqq \mathscr{M}), & \mathsf{P}(\mathscr{A} < \mathscr{M}) \\ \mathsf{P}(\mathscr{A} \geqq \mathscr{N}), & \mathsf{P}(\mathscr{A} < \mathscr{N}) \end{pmatrix} = \begin{pmatrix} c, & 1 - c \\ e, & 1 - e \end{pmatrix}.$$

Supplementary assumption (2.3) implies that $c \neq 1$. Indeed, from the positivity of the random variable $\mathscr{B}$ we obtain

$$c = \mathsf{P}(\mathscr{A} \geqq \mathscr{M}) \leqq \mathsf{P}(\mathscr{A} + \mathscr{B} \geqq \mathscr{M}) = d$$

and if $c = 1$, then $d = 1$ and

$$(1 - c) . (1 - f) + e . (1 - d) = 0 ,$$

so that the assumption (2.3) would not be fulfilled. Thus the chain Y_n has exactly one class of recurrent states. It is periodical only in the case that

$$(5.2) \qquad c = 0 \quad \text{and} \quad e = 1 .$$

But what is the meaning of (5.2)? We shall see that under the condition (5.2) the times $\mathscr{M}$ and $\mathscr{N}$ of repairs of the type $II \to I$ and of the type $III \to I$ are in the unrealistic relation

$$(5.3) \qquad \mathscr{M} > \mathscr{N} \quad \text{with probability } 1 .$$

Indeed,

$$(5.4) \qquad \mathsf{P}(\mathscr{M} > \mathscr{N}) = \mathsf{P}(\mathscr{M} > \mathscr{N}, \mathscr{A} < \mathscr{M}, \mathscr{A} \geqq \mathscr{N}) = \mathsf{P}(\mathscr{A} < \mathscr{M}, \mathscr{A} \geqq \mathscr{N}) = 1.$$

In this section we shall suppose that (5.2) does not hold, i.e., we shall assume that

$$(5.5) \qquad 1 - c + e \neq 0 .$$

Thus the Markov chain Y_n is ergodic and has a uniquely determined stationary distribution $(\pi_{e_S}, \pi_{e_L})'$, where

$$(5.6) \qquad \pi_{e_S} = \frac{e}{1 - c + e},$$

$$(5.7) \qquad \pi_{e_L} = \frac{1 - c}{1 - c + e}.$$

The random variables $\mathscr{K}_S$ and $\mathscr{K}_L$ – the lengths of phases of the types e_S and e_L, respectively – fulfil the relations

$$(5.8) \qquad \mathscr{K}_S = \begin{cases} \mathscr{A} & \text{if } \mathscr{A} \geqq \mathscr{M} , \\ \mathscr{A} + \mathscr{B} & \text{if } \mathscr{A} < \mathscr{M} \leqq \mathscr{A} + \mathscr{B} , \\ \mathscr{M} & \text{if } \mathscr{A} + \mathscr{B} < \mathscr{M} , \end{cases}$$

Table 1

The type of repair of a unit / The state of the operating unit	No unit is repaired
I	$\frac{\pi_{eS}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{M} \leqq t, \mathscr{A} > t)\,\mathrm{d}t + \frac{\pi_{eL}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{N} \leqq t, \mathscr{A} > t)\,\mathrm{d}t$
II	$\frac{\pi_{eS}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{A} \leqq t, \mathscr{A} < \mathscr{M}, \mathscr{A} + \mathscr{B} > t, \mathscr{M} \leqq t)\,\mathrm{d}t + \frac{\pi_{eL}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{A} \leqq t, \mathscr{A} < \mathscr{N}, \mathscr{A} + \mathscr{B} > t, \mathscr{N} \leqq t)\,\mathrm{d}t$
No unit is operating	0
Column sums	$\frac{\pi_{eS}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{M} \leqq t, \mathscr{A} + \mathscr{B} > t)\,\mathrm{d}t + \frac{\pi_{eL}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{N} \leqq t, \mathscr{A} + \mathscr{B} > t)\,\mathrm{d}t$

Table 1 (Continued)

$II \to I$	$III \to I$	Row sums
$\frac{\pi_{es}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{A} > t, \mathscr{M} > t)\, \mathrm{d}t$	$\frac{\pi_{eL}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{A} > t, \mathscr{N} > t)\, \mathrm{d}t$	$\frac{1}{\Delta}\, \mathsf{E}\mathscr{A}$
$\frac{\pi_{es}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{A} \leqq t, \mathscr{A} + \mathscr{B} > t, \mathscr{M} > t)\, \mathrm{d}t$	$\frac{\pi_{eL}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{A} \leqq t, \mathscr{A} + \mathscr{B} > t, \mathscr{N} > t)\, \mathrm{d}t$	$\frac{1}{\Delta}\, \mathsf{E}\mathscr{B} \cdot (1 - \pi_{es} \cdot c - \pi_{eL} \cdot e)$
$\frac{\pi_{es}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{A} + \mathscr{B} \leqq t, \mathscr{M} > t)\, \mathrm{d}t$	$\frac{\pi_{eL}}{\Delta} \cdot \int_0^\infty \mathsf{P}(\mathscr{A} + \mathscr{B} \leqq t, \mathscr{N} > t)\, \mathrm{d}t$	$\frac{\pi_{es}}{\Delta} \cdot \mathsf{E}\mathscr{Z}_M + \frac{\pi_{eL}}{\Delta} \cdot \mathsf{E}\mathscr{Z}_N - \frac{1}{\Delta}(\mathsf{E}\mathscr{A} + \mathsf{E}\mathscr{B})$
$\frac{\pi_{es}}{\Delta} \cdot \mathsf{E}\mathscr{M}$	$\frac{\pi_{eL}}{\Delta} \cdot \mathsf{E}\mathscr{N}$	1

(5.9)
$$\mathscr{K}_L = \begin{cases} \mathscr{A} & \text{if } \mathscr{A} \geqq \mathscr{N}, \\ \mathscr{A} + \mathscr{B} & \text{if } \mathscr{A} < \mathscr{N} \leqq \mathscr{A} + \mathscr{B}, \\ \mathscr{N} & \text{if } \mathscr{A} + \mathscr{B} < \mathscr{N}. \end{cases}$$

Let us calculate the distribution of $\mathscr{K}_S$:

(5.10)
$$\mathsf{P}(\mathscr{K}_S \leqq x) = \mathsf{P}(\mathscr{A} \leqq x, \mathscr{A} \geqq \mathscr{M}) +$$
$$+ \mathsf{P}(\mathscr{A} + \mathscr{B} \leqq x, \mathscr{A} < \mathscr{M}, \mathscr{M} \leqq \mathscr{A} + \mathscr{B}) + \mathsf{P}(\mathscr{M} \leqq x, \mathscr{A} + \mathscr{B} < \mathscr{M}) =$$
$$= \mathsf{P}(\mathscr{A} \leqq x, \mathscr{A} \geqq \mathscr{M}) + \mathsf{P}(\mathscr{A} + \mathscr{B} \leqq x, \mathscr{M} \leqq \mathscr{A} + \mathscr{B}) +$$
$$+ \mathsf{P}(\mathscr{M} \leqq x, \mathscr{A} + \mathscr{B} < \mathscr{M}) - \mathsf{P}(\mathscr{A} + \mathscr{B} \leqq x, \mathscr{M} \leqq \mathscr{A} + \mathscr{B}, \mathscr{A} \geqq \mathscr{M}) =$$
$$= \int_{-\infty}^{x+0} \mathsf{P}(\mathscr{A} \geqq \mathscr{M} | \mathscr{A} = y) \, \mathrm{d}A(y) + \mathsf{P}(\mathscr{Z}_M \leqq x) -$$
$$- \int_{-\infty}^{x+0} \left(\int_{-\infty}^{x-y+0} \mathsf{P}(\mathscr{A} \geqq \mathscr{M} | \mathscr{A} = z, \mathscr{B} = y) \, \mathrm{d}A(z) \right) \mathrm{d}B(y) =$$
$$= C(x) + \mathsf{P}(\mathscr{Z}_M \leqq x) - (B * C)(x).$$

The mathematical expectation of $\mathscr{K}_S$ has the form

(5.11)
$$\mathsf{E}\mathscr{K}_S = \mathsf{E}\mathscr{Z}_M - c \,.\, \mathsf{E}\mathscr{B}$$

and similarly

(5.12)
$$\mathsf{E}\mathscr{K}_L = \mathsf{E}\mathscr{Z}_N - e \,.\, \mathsf{E}\mathscr{B}\,.$$

Thus the mean length of a phase is

(5.13)
$$\Delta = \pi_{e_S} \,.\, \mathsf{E}\mathscr{K}_S + \pi_{e_L} \,.\, \mathsf{E}\mathscr{K}_L =$$
$$= \frac{1}{1 - c + e} \,.\, \left[e \,.\, \mathsf{E}\mathscr{Z}_M + (1 - c) \,.\, \mathsf{E}\mathscr{Z}_N - e \,.\, \mathsf{E}\mathscr{B} \right].$$

Let us now be interested in the possible states of the couple of units of our system. They form the set

(5.14)
$$V = \{(k;\, l);\, k, l \in \{I; II; III\}\} \setminus \{(III; I)\}\,,$$

where the first component expresses the state of the operating unit (for $k = I, II$) or the fact that no unit is operating (for $k = III$) and the second component expresses the type of the repair which is being carried out (for $l = II, III$) or the fact that no unit is being repaired (for $l = I$). The couple $(III; I)$ cannot be an element of the set V because of the assumption 5 about the organization of the system.

By the paper [1] we know that if the mathematical expectations of all the variables $\mathscr{A}$, $\mathscr{B}$, $\mathscr{M}$ and $\mathscr{N}$ are finite and if the distribution functions of the distance between two successive i-events (for both $i \in \mathfrak{M}$) are non-lattice, then the stationary probability p_j of the state j, $j \in V$, of the couple of units has the form

$$(5.15) \qquad p_j = \frac{1}{\Delta} \sum_{i \in \mathfrak{M}} \pi_i \, . \int_0^\infty Q_i(u, j) \, \mathrm{d}u \, ,$$

where $Q_i(u, j)$ is the probability that a phase is longer than u and after time u from the beginning of this phase the couple of units is in the state j under the condition that the period in question is of the type i.

The full list of formulas for computing the stationary probabilities p_j for $j \in V$ is given in Table 1. The row and column sums are very essential characteristics of availability of the system and of the level of use of the repair facility. So the stationary availability of our system has the form

$$\frac{1}{(1 - c + e) \, . \, \Delta} \left[\mathsf{E}\mathscr{A} \, . \, (1 - c + e) + \mathsf{E}\mathscr{B} \, . \, (1 - c) \right] ,$$

while the stationary probability that the repair facility is operating is

$$\frac{1}{(1 - c + e) \, . \, \Delta} \left[e \, . \, \mathsf{E}\mathscr{M} + (1 - c) \, . \, \mathsf{E}\mathscr{N} \right] ,$$

where Δ is determined by (5.13).

Another paper, which is expected to appear in this journal presently, will deal with stochastic characteristics of the behaviour of the system considered in this paper in the course of its first operating period. It will be devoted to the following random variables: the whole time of repairs of units of the type $II \to I$ (or $III \to I$), the whole time of operation of units in state I (or II) and the number of finished repairs of units of the type $II \to I$ (or $III \to I$).

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Souhrn

ANALÝZA SYSTÉMU S NEZATÍŽENOU ZÁLOHOU SLOŽENÉHO ZE DVOU PRVKŮ, KTERÉ MOHOU BÝT VE TŘECH STAVECH

Antonín Lešanovský

V článku je uvažován jistý systém s nezatíženou zálohou složený ze dvou prvků a jednoho zařízení pro jejich opravy. Prvky mohou být ve třech stavech: bezvadném (I), zhoršeném (II) a poruchovém (III). Předpokládáme, že možné jsou pouze následující změny stavu prvků: $I \to II$, $II \to III, II \to I$, $III \to I$. Oprava prvku typu $II \to I$ může být interpretována jako jeho preventivní údržba, jejíž realizace závisí na stavech obou prvků. V článku je odvozena řada charakteristik chování systému, např. rozložení a střední hodnoty doby do první poruchy systému a doby poruchového prostoje systému, stacionární pravděpodobnosti možných dvojic stavů prvků.

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