B. R. Handa; Sri Gopal Mohanty

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ON DWASS' METHOD FOR DERIVING THE DISTRIBUTION OF RANK ORDER STATISTICS

B. R. HANDA, S. G. MOHANTY*)

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1. DWASS' METHOD

The problem of finding the exact distribution of rank order statistics defined on two independent random samples obtained from the same continuous distribution has generated a lot of interest in the past few years. A brief survey of the methods used in the problem was recently given by Šidák in [12]. The basic problem here is of testing the hypothesis H_0: $F = G$ against a suitable alternative, by using some appropriate statistics which are usually functions of

$$H(u) = F_m(u) - G_k(u)\,, \quad -\infty < u < \infty \tag{1}$$

where F_m and G_k are the empirical distribution functions of two independent random samples $(X_1, \ldots, X_m)$ and $(Y_1, \ldots, Y_k)$, drawn from continuous population distribution functions F and G respectively. Without loss of generality, we can take $m = \mu_1 n$, $k = \mu_2 n$ where μ_1 and μ_2 are coprime positive integers. Let $Z = (Z_1, \ldots, Z_{(\mu_1+\mu_2)n})$ be the combined order statistics of two samples. A sequence $\xi = (\xi_1, \ldots, \xi_{(\mu_1+\mu_2)n})$ of $n\mu_1$ numbers $+\mu_2$'s and $n\mu_2$ numbers $-\mu_1$'s obtained from Z by replacing Z_i, $1 \leqq i \leqq (\mu_1 + \mu_2)\,n$ by $+\mu_2$ or $-\mu_1$ according as it is a X or a Y observation, is called a rank order sequence. A random variable which is a function of ξ is called a rank order statistic. Note that H defined in (1) is a function of ξ. The following important result is well known.

Lemma 1. *The possible* $\binom{(\mu_1 + \mu_2)\,n}{\mu_2 n}$ *rank order sequences are equally likely under* H_0.

*) This work was done when the second author visited the Indian Institute of Technology, Delhi in 1975.

The possible ξ sequences can be put into 1-1 correspondence with the set of lattice paths from $(0, 0)$ to $(\mu_1 n, \mu_2 n)$ in a plane, by taking the ith step of the path a unit horizontal or a unit vertical according as ξ_i is a $+\mu_2$ or a $-\mu_1$, $i = 1, \ldots, (\mu_1 + \mu_2) n$. Thus the probability distribution of any rank order statistic under H_0 involves the enumeration of lattice paths with appropriate constraints. Basically, we are using a restricted simple random walk with dependent steps. Here, various path counting techniques are useful and hence the approach becomes strictly combinatorial.

In contrast to this approach, recently Dwass [3] gave a new method based on a simple random walk with independent steps, by using which he derived the distributions of various rank order statistics in a unified manner, when the sample sizes are equal, i.e., $\mu_1 = \mu_2 = 1$. For the sake of completeness, we describe the basic results of this method.

Consider the unrestricted simple random walk (SRW) $\{S_i\}_{i=0}^{\alpha}$, where $S_0 = 0 = W_0$, $S_j = \sum_{i=0}^{j} W_i$ and W_i are identically and independently distributed random variables with common distribution

(2) $$P(W_i = +1) = p\,, \quad P(W_i = -1) = q\,, \quad p + q = 1\,.$$

Lemma 2. *For any $p \in (0, 1)$,*

$$P(W_1 = w_1, \ldots, W_{2n} = w_{2n} \mid \sum_{i=1}^{2n} W_i = 0) = \frac{1}{\binom{2n}{n}}\,,$$

for any sequence $(w_1, \ldots, w_{2n})$ of n numbers $+1$'s and n numbers -1's.

Combining Lemma 1 with $\mu_1 = \mu_2 = 1$ and Lemma 2, we conclude that

(3) $$P(W_1 = w_1, \ldots, W_{2n} = w_{2n} \mid \sum_{i=1}^{2n} W_i = 0) = P(\xi_1 = w_1, \ldots, \xi_{2n} = w_{2n})\,,$$

which connects the rank order statistics with the SRW. The event of returning to zero is known to be transient, when $p \neq \frac{1}{2}$. Thus if we take $p < \frac{1}{2}$, the SRW returns to zero only finitely many times and the probability of ever returning to zero is $2p$. Let T be the time for the last return to zero and V be a random variable on the SRW which is completely determined by $W_1, \ldots, W_T$, $T > 0$. Then using (3), we get the following lemma.

Lemma 3. *The distribution of $W_1, \ldots, W_T$ given $T = 2n$ is the same as that of $(\xi_1, \ldots, \xi_{2n})$.*

Thus, we can define a rank order statistic V^* as $V^* \equiv (V(W_1, \ldots, W_T) \mid T = 2n)$ so that the probability distribution of V^* and $(V \mid T = 2n)$ are same. Conversely, when V^* is given, we can define the corresponding V on SRW.

Finally, we state the main theorem which provides the distribution of rank order statistic V^*.

Theorem 1. *Let ϕ be a function of V. For $p < \frac{1}{2}$ we have*

$$\frac{E(\phi(V))}{1 - 2p} = \sum_{n=0}^{\infty} E(\phi(V^*)) \binom{2n}{n} (pq)^n . \tag{4}$$

Observe that ϕ could be the characteristic function of a set A and in that case $E(\phi(V)) = P(V \in A)$.

Examining these results, we note that the Dwass' method proceeds in three basic steps.

Step I. Given a rank order statistic V^*, the corresponding random variable V on the SRW is constructed.

Step II. The distribution of V is determined on the unrestricted SRW. This is comparatively simpler to derive than it is on restricted SRW.

Step III. A power series expansion of $P(V \in A)/(1 - 2p)$ in powers of pq is written down, where $P(V \in A)$ is computed in step II. The coefficient of $\binom{2n}{n}(pq)^n$ in the expansion yields the required distribution of V^*, i.e., $P(V^* \in A)$. This step results from Theorem 1 which connects the distribution on unrestricted SRW with the corresponding distribution on restricted SRW.

The following observations are in order.

1. The construction of the random walk (which is the SRW in Dwass' case) is determined by H in (1) and corresponds to the rank order sequence.
2. The crucial point of the method is step II.
3. An explicit expression for the probability of ever returning to zero (which happens to be $2p$) is needed in (4). Another important expression is the explicit expression of the probability of ever reaching some integer r, which is $(p/q)^r$ for the SRW. This expression is basic in computation of distributions of many random variables on the unrestricted SRW. Examples are: $N(r)$ – the number of indices for which the SRW crosses the height r; Q – the number of indices i for which $S_i = D^+$ where $D^+ = \max(0, S_1, S_2, \ldots)$.

As has been demonstrated by Dwass in [3], his method works satisfactorily when the sample sizes are equal because almost all results on the associated SRW are available in Feller's book [5]. It is further corroborated by Aneja and Sen [1], who using the Dwass' method in combination with the usual combinatorial argument derived several other distributions not already done by Dwass. The question arises whether the method or its variation can be used effectively in other situations. It has been partially answered by the authors [9], when the sample sizes are unequal with

$\mu_1 = \mu$ and $\mu_2 = 1$. In Section 2, we consider the general case of unequal sample sizes and point out the difficulties associated with the Dwass' method. The successful extension to the case of $\mu_1 = \mu$, $\mu_2 = 1$ leads us in a natural way to the consideration in Section 3, of the problem of deriving distributions of rank order statistics defined on $k + 1$ independent random samples of sizes $n_1, \ldots, n_k$, $\sum_{i=1}^{k} \mu_i n_i$, (μ_i's being positive integers) from the same continuous distribution. The last section contains some general remarks on the method.

2. TWO-SAMPLE CASE WITH UNEQUAL SAMPLE SIZES

Let the sample sizes, as before, be $\mu_1 n$ and $\mu_2 n$. The unrestricted random walk which is constructed on the basis of (1), is now the one such that W_i, $i \geqq 1$, takes values $-\mu_1$ and μ_2 instead of -1 and $+1$ with probability q and p respectively. This random walk is referred to as the unrestricted generalized random walk 1 (GRW 1). A lattice path graph can be associated with this random walk in a familiar way, by starting from the origin and then taking a unit horizontal step or a unit vertical step at the ith stage, according as W_i is $+\mu_2$ or $-\mu_1$.

Extending everything in the usual way, we find that the main theorem of Dwass has the following modification.

For $p < \mu_1/(\mu_1 + \mu_2)$,

$$\frac{E(\phi(V))}{1 - f} = \sum_{n=0}^{\infty} E(\phi(V^*)) \binom{(\mu_1 + \mu_2)\, n}{\mu_2 n} (p^{\mu_1} q^{\mu_2})^n \tag{5}$$

where f is the probability of ever returning to zero in the unrestricted GRW 1 and V, V^* are defined on the GRW 1 as before.

Consider the particular case of $\mu_1 = \mu$ and $\mu_2 = 1$. As can be seen from [9], the knowledge of two expressions

$$x^{\alpha} = \sum_{k=0}^{\infty} \frac{\alpha}{\alpha + k} \binom{\alpha + k}{k} \theta^k , \tag{6}$$

and

$$\frac{x^{\alpha+1}}{\beta - (\beta - 1)\, x} = \sum_{k=0}^{\infty} \binom{\alpha + k\beta}{k} \theta^k , \tag{7}$$

where

$$\theta x^{\beta} - x + 1 = 0 , \quad |\theta| < \left| \frac{(\beta - 1)^{\beta - 1}}{\beta^{\beta}} \right|$$

and α, β are any numbers [6], are crucial to the entire derivation of distributions, so much so that, only those probability distributions on the GRW 1 are dealt with which are derivable by the use of (6) and (7). It is important to note that the number of lattice paths from $(0, 0)$ to $(r + \mu n, n)$ which do not touch the line $x = \mu y + r$ except at the end, is equal to

$$\frac{r}{r + (\mu + 1) n} \binom{r + (\mu + 1) n}{n}.$$

In fact, these power series expansions yield the probability of ever returning to zero and that of ever reaching the integer r respectively as

$$f = (\mu + 1) p^\mu q y^\mu ,$$
$$(py)^r \quad \text{for} \quad r \geqq 1 , \quad \mu p^{\mu - 1} q y^\mu \quad \text{for} \quad r = -1 ,$$

when $p < \mu/(\mu + 1)$, where y is a positive root of the polynomial

(8) $$p^\mu q y^{\mu + 1} - y + 1 = 0 .$$

When $\mu = 1$, these expressions have extremely simple form which made the work on equal-sample-size case relatively easy.

The problem becomes very intricate for general μ_1 and μ_2. Let $U(s)$ and $f(s)$ denote the probability generating functions for the return time to zero and the time for the first return to zero in the unrestricted GRW 1. It is well known that $f = f(1)$ is obtainable from $f(s) = (U(s) - 1)/U(s)$, if an explicit expression in closed form for $U(s)$ is available. In our case,

(9) $$U(s) = \sum_{n=0}^{\infty} \binom{(\mu_1 + \mu_2) n}{\mu_2 n} p^{\mu_1 n} q^{\mu_2 n} s^{(\mu_1 + \mu_2) n}$$

which converges for

$$\frac{(\mu_1 + \mu_2)^{\mu_1 + \mu_2}}{\mu^{\mu_1} \mu^{\mu_2}} p^{\mu_1} q^{\mu_2} |s|^{\mu_1 + \mu_2} < 1 .$$

Unfortunately, no explicit sum of this infinite series is apparently known. Next' we may try to compute $f(s)$ directly, for which the following combinatorial result is needed.

The number a_n of lattice paths $(0, 0)$ to $(\mu_1 n, \mu_2 n)$ not passing through any of the points $(\mu_1 k, \mu_2 k)$, $k = 1, \ldots, n - 1$, is given by

(10) $$a_n = (-1)^{n-1} \det_{n \times n} (c_{ij})$$

where

$$c_{ij} = \binom{(\mu_1 + \mu_2)(j - i + 1)}{(j - i + 1) \mu_2}.$$

Note that a_n satisfies the recurrence relation

(11) $$\sum_{i=1}^{n} \binom{(\mu_1 + \mu_2)(n - i)}{(n - i) \mu_2} a_i = \binom{(\mu_1 + \mu_2) n}{\mu_2 n},$$

and therefore (10) is proved because the expression of a_n in (10) satisfies (11). Clearly

$$f(s) = \sum_{n=1}^{\infty} a_n p^{\mu_1 n} q^{\mu_2 n} s^{(\mu_1 + \mu_2)n} \tag{12}$$

for which again a closed expression does not seem to be obtainable. Thus, we are unable to proceed with the Dwass method in the present situation. However, the derivation of (10) by the use of combinatorial technique suggests that even if the Dwass method fails, the usual combinatorial argument cited in Section 1, might rescue us in obtaining the desired probability distribution. It is befitting to mention that recently Steck and Simmons in [13] have derived the joint distribution of $D^+_{m,k} = \max_i H(Z_i)$ and

$$R^+_{m,k} = \inf \{ i : H(Z_i) = D^+_{m,k} \}$$

by the combinatorial method cited at the beginning of Section 1.

We have seen just now that the Dwass method cannot in general be used in deriving the distributions of the usual rank order statistics which are functions of H defined in (1). However, we can find the distributions of a different class of statistics which are functions of

$$m\, F_m(u) - k\, G_k(u) \tag{13}$$

by modifying the method in a different direction. The motivation for considering (13) is to construct the associated random walk such that the subsequent derivations become possible unlike the previous case. The modified rank order sequence is the sequence of m numbers $+1$'s and k numbers -1's obtained from the combined order statistics of two samples. In this event, the associated unrestricted random walk is the SRW of Section 1. Furthermore, observe that the new rank order sequence introduces the change so that the restricted random walk ends at r (without loss of generality, we assume $m \geqq k$, $m - k = r$) at the $(m + k)$th step rather than at zero, „as was the case so far. Therefore, T is taken to be the time for the last return to r. In this case, Theorem 1 takes the form

$$\frac{E(\phi(V))}{1 - 2p} = \sum_{n=0}^{\infty} E(\phi(V^*)) \binom{2n + r}{n} p^{n+r} q^n \tag{14}$$

for $p < \frac{1}{2}$. Since all relevant results on the SRW are known, the rest of the derivations is straightforward. In fact using this idea, Aneja [2] and Očka [10] have derived various distributions. Unfortunately however, these rank order statistics are not very useful for testing the hypothesis $H_0 : F = G$ unless m and k are very close. Suppose m is almost equal to μk, μ being a positive integer. Then we may consider

$$H^*(u) = m\, F_m(u) - \mu k\, G_k(u) \tag{15}$$

as a variation of (1) similar to (13) and derive similar distributions by following the necessary modifications in [9]. In this way, we may however utilize the Dwass' method for every situation. The statistic (13) was suggested by Reimann and Vincze in [11].

3. A MULTI-SAMPLE CASE

It is possible to generalize the power series expansions (6) and (7) involving binomial coefficients to the case involving multinomial coefficients, either by using Lagrange inversion formula for several variables [7] or by direct analysis [8]. For simplicity of presentation, we give below the results for trinomial coefficients.

$$x^{\alpha} = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\alpha}{\alpha + \beta_1 k_1 + \beta_2 k_2} \binom{\alpha + \beta_1 k_1 + \beta_2 k_2}{k_1, k_2} \theta_1^{k_1} \theta_2^{k_2} \tag{16}$$

and

$$\frac{x^{\alpha}}{1 - \theta_1 \beta_1 x^{\beta_1 - 1} - \theta_2 \beta_2 x^{\beta_2 - 1}} = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \binom{\alpha + \beta_1 k_1 + \beta_2 k_2}{k_1, k_2} \theta_1^{k_1} \theta_2^{k_2} \tag{17}$$

where

$$\binom{a}{b, c} = \frac{a(a-1)\dots(a-b-c+1)}{b!\, c!}$$

and

$$\theta_1 x^{\beta_1} + \theta_2 x^{\beta_2} - x + 1 = 0\,. \tag{18}$$

The convergence region for these power series can be determined by using the standard results on hypergeometric series of several variables (see [4], Chapter 5). As earlier, we remark that the coefficients in the expansions have interpretation in terms of lattice paths in higher dimensions which is significant in our discussion.

Taking analogy from earlier situations in two-dimension, we are now in a position to derive closed expressions for probabilities of certain relevant events on the unrestricted random walk defined by

$$\begin{aligned} P(W_i = +1) &= p \\ P(W_i = -\mu_1) &= q_1 \\ P(W_i = -\mu_2) &= q_2 \end{aligned} \tag{19}$$

with $p + q_1 + q_2 = 1$. We refer to this random walk as GRW 2 which is naturally an extension of GRW 1. The availability of these new results encourages us to extend the Dwass' method, in order to derive the distributions of rank order statistics defined on three independent random samples, which is the content of this section. Observe that we are moving in a backward direction, i.e., given the GRW 2, we want to construct the corresponding rank order statistics. Later, we examine their utility in the context of the problem of testing hypotheses.

Let there be three samples of sizes n_1, n_2 and $\mu_1 n_1 + \mu_2 n_2$ obtained from the same continuous distribution. Let their empirical distribution functions be F_{n_1}, G_{n_2} and $K_{\mu_1 n_1 + \mu_2 n_2}$ respectively. In this case $H(u)$ and the rank order sequence change to

$$H^{**}(u) = K_{\mu_1 n_1 + \mu_2 n_2}(u) - \frac{\mu_1 n_1}{\mu_1 n_1 + \mu_2 n_2} F_{n_1}(u) - \frac{\mu_2 n_2}{\mu_1 n_1 + \mu_2 n_2} G_{n_2}(u) \tag{20}$$

and a sequence of $\mu_1 n_1 + \mu_2 n_2$ numbers $+1$'s, n_1 numbers $-\mu_1$'s and n_2 numbers $-\mu_2$'s. In order to write the extension of the main theorem of Dwass, we should compute the probability of ever returning to zero in the GRW 2. Letting $U(s)$ and $f(s)$ retain the same meaning as in Section 2, we can find

$$U(s) = 1/(1 - (\mu_1 + 1)\, p^{\mu_1} q_1 x^{\mu_1} s^{\mu_1 + 1} - (\mu_2 + 1)\, p^{n_2} q_2 x^{\mu_2} s^{\mu_2 + 1}) \tag{21}$$

where

$$p^{\mu_1} q_1 x^{\mu_1 + 1} s^{\mu_1 + 1} + p^{\mu_2} q_2 x^{\mu_2 + 1} s^{\mu_2 + 1} - x + 1 = 0 \tag{22}$$

in an appropriate convergence region which exists. This is done with the help of (17) and (18). Therefore, for p, q_1, q_2 determined by the convergence region,

$$f = f(1) = (\mu_1 + 1)\, p^{\mu_1} q_1 y^{\mu_1} + (\mu_2 + 1)\, p^{\mu_2} q_2 y^{\mu_2}\,, \tag{23}$$

y being a positive root of (22) with $s = 1$. Similarly by using (16) and (18), the probability of ever reaching the positive integer r is computed to be $(py)^r$. The extended version of the main theorem is

$$\frac{E(\phi(V))}{1 - f} = \sum_{n_1 = 0}^{\infty} \sum_{n_2 = 0}^{\infty} E(\phi(V^*)) \binom{t}{n_1,\, n_2} (p^{\mu_1} q_1)^{n_1} (p^{\mu_2} q_2)^{n_2} \tag{24}$$

where f is given by (23) and $t = (\mu_1 + 1)\, n_1 + (\mu_2 + 1)\, n_2$.

Now we can derive the distribution of rank order statistics of which a few illustrations are given below.

(i) $N(\mu_1, \mu_2)$ = number of indices i for which

$$H^{**}(Z_i) = 0\,, \quad i = 0, 1, \ldots, t\,, \quad Z_0 = -\infty\,;$$

$$P(N(\mu_1, \mu_2) > K) = \sum_j \frac{\binom{t - k}{n_1 - j \;\; n_2 - k + j}}{\binom{t}{n_1,\, n_2}}\,, \tag{26}$$

$$0 < k \leqq n_1 + n_2\,; \quad n_1, n_2 \geqq 0\,,$$

$= 0$ otherwise,

where the summation is from $\max(0, k - n_2)$ to $\min(k, n_1)$.

(ii)
$$D^+(\mu_1, \mu_2) = \max_{0 \leqq i \leqq t} H^{**}(Z_i)\,;$$

(27)
$$P(D^+(\mu_1, \mu_2) > k) =$$
$$= \sum \binom{t-k}{j_1, j_2} \left[\frac{k + (\mu_1 + 1)(n_1 - j) + (\mu_2 + 1)(n_2 - k)}{k + 2(\mu_1 + 1)(n_1 - j_1) + 2(\mu_2 + 1)(n_2 - k_2)} \right] \times$$
$$\times \binom{k + 2(\mu_1 + 1)(n_1 - j_1) + 2(\mu_2 + 1)(n_2 - j_2)}{n_1 - j_1,\ n_2 - j_2}$$
$$\text{for} \quad 0 \leqq k \leqq \mu_1 n_1 + \mu_2 n_2\,; \quad n_1, n_2 \geqq 0\,,$$
$$= 0 \quad \text{otherwise,}$$

the summation being over

$$R = \{(j_1, j_2) : \mu_1 j_1 + \mu_2 j_2 \geqq k\,,\quad 0 \leqq j_1 \leqq n_1\,,\quad 0 \leqq j_2 \leqq n_2\}\,.$$

The proofs follow the same pattern as in [9]. For (27), the following power series expansion is useful:

(28)
$$\frac{p^k}{1 - f} = \sum_R \binom{t-k}{j_1, j_2} (p^{\mu_1} q_1)^{j_1} (p q_2)^{j_2}\,.$$

Distributions of other statistics are also obtainable.

The relevant testing problem corresponding to the statistics which are functions of $H^{**}(u)$ is one when one tests the hypothesis $H_0 : F = G = K$ against either the two-sided alternative $H_1 : K \neq \alpha F + (1 - \alpha) G$ or a one-sided alternative $H_1 : K > \alpha F + (1 - \alpha) G$, $\alpha = \mu_1 n_1/(\mu_1 n_1 + \mu_2 n_2)$. In practice, given any α, we can select μ_1, μ_2, n_1 and n_2 such that the ratio $\mu_1 n_1/(\mu_1 n_1 + \mu_2 n_2)$ is close to α. H_0 is suggested by the assumption that every rank order sequence should be equally likely under H_0. To end the discussion, it may be remarked, that the above testing problem, though does not appear to have much practical importance, arises as a consequence of our interest in the Dwass' method.

The extension to the case of more than three samples is obvious and therefore is omitted.

4. CONCLUDING REMARKS

Dwass in his paper [3] has expressed the belief that, for the case of equal sizes, his method was a unified one and could derive the distributions of rank order statistics in a simple manner. His belief is perhaps shared by Šidák [12]. This is indeed true, in the two-sample case with equal sizes. The method works well enough when the sample sizes are μn and n but it is unsuitable in other situations. To avoid the difficulties, one may modify the statistics or the testing problem. Never-the-less, the modifications may not appear to give rise to useful statistics or interesting testing problems (end of Section 2 and Section 3).

The main reason for the inapplicability of the Dwass method is that the associated unrestricted random walk is not always easy to study. Though this method would be hard to use, the combinatorial method cited in Section 1 may still succeed in providing the desired distributions (see Section 2).

The completion of Step II (see Section 1), which is vital in the method, sometimes is helped by combinatorial arguments (e.g., in obtaining the joint distributions of (L, N, N^+, N^*) and of (L, R, R^+, N^*) in [1]).

As observed by Dwass, his main theorem can be regarded as the generating function of probability distribution of the rank order statistic in question, by considering the power series of the variable (pq), $p < \frac{1}{2}$. This is not unfamiliar in combinatorics. In (6) and (7), powers of $\theta = (x - 1)/x^\beta$ are found to be convenient and in many situations, the exponential generating function $\sum a_n s^n/n!$ is preferred to the ordinary generating function $\sum a_n s^n$. However, there is a striking difference between the two methods. In the usual method of generating function, one most often utilizes a recurrence relation of the sequence of numbers, whereas the main feature of the Dwass' method is that it insists on the use of probabilistic arguments by exploiting the properties of the unrestricted random walk and as such the variable (pq) is not chosen in advance, but crops in as a consequence of the novel treatment.

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Souhrn

O DWASSOVĚ METODĚ PRO ODVOZENÍ DISTRIBUCÍ POŘADOVÝCH STATISTIK

B. R. HANDA, S. G. MOHANTY

V této poznámce se předkládá kritické posouzení Dwassovy metody pro odvození distribucí pořadových statistik definovaných na náhodných výběrech ze stejné spojité populace. Jsou diskutovány nové situace pro užitečnost této metody.

Authors' addresses: Prof. *B. R. Handa,* Indian Institute of Technology, Delhi, India; Prof. *S. G. Mohanty,* McMaster University, 1280 Main Street West, Hamilton, Ontario, L8S4K1.