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Aplikace matematiky, Vol. 23 (1978), No. 3, 199–207

Persistent URL: `http://dml.cz/dmlcz/103745`

REMARK TO THE COMPARISON OF SOLUTION PROPERTIES OF LOVE'S EQUATION WITH THOSE OF WAVE EQUATION

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(Received November 8, 1976)

Love's partial differential equation is derived in [2], [3] by the energy method and under the assumptions that the kinetic energy per unit of length is

$$T_1 = \tfrac{1}{2}F\varrho[(u_t)^2 + \mu^2k^2(u_{tx})^2] \tag{1}$$

and the potential energy per unit of length is

$$V_1 = \tfrac{1}{2}EF(u_x)^2 , \tag{2}$$

where F is an area of cross-section, k is a cross-section radius of gyration about the central line.

Using in (2) the corrected form of tension we have

$$V_1 = \tfrac{1}{2}Fu_x(Eu_x + \varrho\mu^2k^2u_{xtt}) \tag{3}$$

and the variational equation of motion is

$$\delta\int_{t_1}^{t_2}\mathrm{d}t\int_0^L\{\tfrac{1}{2}F\varrho[(u_t)^2 + \mu^2\,k^2(u_{tx})^2] - \tfrac{1}{2}Fu_x(Eu_x + \varrho\mu^2k^2u_{xtt})\}\,\mathrm{d}x = 0\,. \tag{4}$$

If we form variations we obtain the equation of extensional vibrations of rods in the form

$$u_{tt} - \frac{E}{\varrho}u_{xx} - 2\mu^2k^2u_{xxtt} = 0\,. \tag{5}$$

This equation differs from the Love's one only by the double coefficient at the fourth derivative.

Taking in the variational equation of motion the term (1) for kinetic energy uncorrected and the term (2) for the potential energy, we obtain the classical wave equation $\varrho u_{tt} - E u_{xx} = 0$. Denoting $c^2 = E/\varrho$, $a^2 = \mu^2 k^2$, we have the equation (5) in the form

$$2a^2 u_{xxtt} + c^2 u_{xx} - u_{tt} = 0 \tag{6}$$

and Love's corrected wave equation in the form

$$a^2 u_{xxtt} + c^2 u_{xx} - u_{tt} = 0\,. \tag{7}$$

As the coefficient a^2 is very small in comparison with c^2, we can take constants a^2, $2a^2$ for a small parameter, consider the equations (6) and (7) as equations with a small parameter $\varepsilon > 0$ at the highest derivative and write these equations in the form

$$\varepsilon u_{xxtt} + c^2 u_{xx} - u_{tt} = 0\,. \tag{8}$$

In what follows let us compare some solution properties of the equation (8) with those of the classical wave equation

$$c^2 u_{xx} - u_{tt} = 0\,. \tag{9}$$

Let us consider the differential equation (8) with initial conditions

$$u(0, x) = \varphi_0(x) \quad u_t(0, x) = \varphi_1(x) \quad \text{if} \quad x \in [0, L] \tag{10}$$

and with boundary conditions

$$\begin{aligned} &\alpha_0\, u(t, 0) + \beta_0[\varepsilon\, u_{xtt}(t, 0) + c^2\, u_x(t, 0)] = \varphi_0(t) \\ &\alpha_1\, u(t, L) + \beta_1[\varepsilon\, u_{xtt}(t, L) + c^2\, u_x(t, L)] = \varphi_1(t) \quad \text{if} \quad t \in [0, T]\,, \end{aligned} \tag{11}$$

where $\varepsilon > 0$ is a small parameter and $c^2 > 0$, α_0, α_1, β_0, β_1, $T > 0$, $L > 0$ are given constants. We assume that the functions $\varphi_0(x)$, $\varphi_1(x)$ have in $[0, L]$ continuous derivatives up to the third order and piecewise continuous derivatives of the fourth order and that

$$\begin{aligned} \varphi_0(0) &= \varphi_0'(0) = \varphi_0''(0) = \varphi_0'''(0) = 0\,, \\ \varphi_0(L) &= \varphi_0'(L) = \varphi_0''(L) = \varphi_0'''(L) = 0\,, \\ \varphi_1(0) &= \varphi_1'(0) = \varphi_1''(0) = \varphi_1'''(0) = 0\,, \\ \varphi_1(L) &= \varphi_1'(L) = \varphi_1''(L) = \varphi_1'''(L) = 0\,. \end{aligned}$$

The boundary conditions (11) can be considered homogeneous

$$\begin{aligned} &\alpha_0\, u(t, 0) + \beta_0[\varepsilon\, u_{xtt}(t, 0) + c^2\, u_x(t, 0)] = 0\,, \\ &\alpha_1\, u(t, L) + \beta_1[\varepsilon\, u_{xtt}(t, L) + c^2\, u_x(t, L)] = 0 \quad \text{if} \quad t \in [0, T]\,, \end{aligned} \tag{12}$$

because the transformation

$$u(t, x) = Z(t, x) + \frac{1}{\alpha_0}\left(1 - 3\frac{x^2}{L^2} + 2\frac{x^3}{L^3}\right)\psi_0(t) + \frac{1}{\alpha_1}\left(3\frac{x^2}{L^2} - 2\frac{x^3}{L^3}\right)\psi_1(t)$$

if $\alpha_0 \neq 0$, $\alpha_1 \neq 0$ and the transformation

$$u(t, x) = Z(t, x) + \frac{x^2}{2L} q(t) - \left(\frac{x^2}{2L} - x\right) p(t) \quad \text{if} \quad \alpha_0 = 0\,, \quad \alpha_1 = 0\,,$$

where $p(t)$ is a solution of the differential equation $\varepsilon\, p''(t) + c^2\, p(t) = \psi_0(t)$ and $q(t)$ is a solution of the differential equation $\varepsilon\, q''(t) + c^2\, q(t) = \psi_1(t)$, transforms the inhomogeneous conditions (11) to homogeneous conditions (12).

If $\varepsilon \to 0$, then we have the initial-boundary problem for the wave equation (9):

$$u(0, x) = \varphi_0(x)\,, \quad u_t(0, x) = \varphi_1(x) \quad \text{if} \quad x \in [0, L]\,, \tag{10a}$$

$$\begin{aligned} &\alpha_0\, u(t, 0) + \beta_0 c^2\, u_x(t, 0) = 0\,, \\ &\alpha_1\, u(t, L) + \beta_1 c^2\, u_x(t, L) = 0 \qquad \text{if} \qquad t \in [0, T]\,, \end{aligned} \tag{12a}$$

with the same assumptions about the functions φ_0, φ_1 as in the case of conditions (10).

Since the initial-boundary problem (8), (10), (12) or (9), (10a), (12a) describes the extensional vibrations of rods, let us consider in what follows two variants of boundary conditions:

$$\begin{pmatrix} \alpha_0 & \alpha_1 \\ \beta_0 & \beta_1 \end{pmatrix}: \quad \mathrm{I} := \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}; \quad \mathrm{II} := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A}$$

Let $u(t, x)$ be a solution of the problem (8), (10), (12) and $U(t, x)$ that of the problem (9), (10a), (12a). Existence and uniqueness theorems for the problem (9), (10a), (12a) are very well known, for the problem (8), (10), (12) they are proved for instance in [4].

To compare the solution properties of these problems, we use the Fourier method and the method of small parameter [5], [6]. Let us assume that $u(t, x) = y(x)\, v(t)$. Then we have

$$\frac{y''}{y} = \frac{v''}{\varepsilon v'' + c^2 v} = \text{const.} \quad \text{if} \quad y(x) \neq 0\,, \quad \varepsilon\, v''(t) + c^2\, v(t) \neq 0\,. \tag{13}$$

Denoting the constant on the right hand side by $-\lambda^2$, we obtain

$$y'' + \lambda^2 y = 0\,, \tag{14}$$

$$v'' + \frac{c^2 \lambda^2}{1 + \varepsilon \lambda^2}\, v = 0\,. \tag{15}$$

For the boundary conditions we have

$$\begin{aligned} &u(t, 0) = 0 \Rightarrow y(0) = 0\,, \\ &u(t, L) = 0 \Rightarrow y(L) = 0 \quad \text{for the variant I.} \end{aligned} \tag{16}$$

$$(17) \qquad u(t, 0) = 0 \Rightarrow y(0) = 0$$

$$\varepsilon\, u_{xtt}(t, L) + c^2\, u_x(t, L) = 0 \Rightarrow y'(L) = 0 \quad \text{for the variant II.}$$

Hence we have the sequences of eigenvalues and normalized eigenfunctions in the form:

$$(18) \qquad \lambda_n = \frac{n\pi}{L}\,; \quad y_n(x) = \sqrt{\frac{2}{L}} \sin \lambda_n x \quad \text{for the variant I.}$$

$$(19) \qquad \lambda_n = \frac{(2n+1)\,\pi}{L}\,; \quad y_n(x) = \sqrt{\frac{2}{L}} \sin \lambda_n x \quad \text{for the variant II.}$$

To each λ_n we have the differential equation

$$(20) \qquad v''(t) + \frac{c^2 \lambda_n}{1 + \varepsilon \lambda_n^2}\, v(t) = 0\,,$$

and its solution

$$v_n(t) = A_n \cos \frac{c\lambda_n t}{\sqrt{(1 + \varepsilon\lambda_n^2)}} + B_n \sin \frac{c\lambda_n t}{\sqrt{(1 + \varepsilon\lambda_n^2)}}\,.$$

If we assume that the series

$$(21) \qquad \sqrt{\frac{2}{L}} \sum_{n=0}^{\infty} \sin \lambda_n x \left[A_n \cos \frac{c\lambda_n t}{\sqrt{(1 + \varepsilon\lambda_n^2)}} + B_n \sin \frac{c\lambda_n t}{\sqrt{(1 + \varepsilon\lambda_n^2)}} \right]$$

is uniformly convergent with its derivatives up to the fourth order in $[0, L] \times [0, T]$, we can take it for the solution of the problem (8), (10), (12). From the initial conditions (10) we obtain

$$\varphi_0(x) = u(0, x) = \sqrt{\frac{2}{L}} \sum_{n=0}^{\infty} \sin \lambda_n x A_n\,,$$

$$\varphi_1(x) = u_t(0, x) = \sqrt{\frac{2}{L}} \sum_{n=0}^{\infty} \sin \lambda_n x B_n \frac{c\lambda_n}{\sqrt{(1 + \varepsilon\lambda_n^2)}}\,.$$

With regard to the assumptions about the functions $\varphi_0(x)$ and $\varphi_1(x)$ we have only one development of these functions in series of normalized eigenfunctions $\sqrt{(2/L)}\,.\,\sin \lambda_n x$; these series and their derivatives up to the second order are uniformly convergent.

Hence

$$A_n = \sqrt{\frac{2}{L}} \int_0^L \sin \lambda_n \xi\, \varphi_0(\xi)\, d\xi\,,$$

$$B_n = \frac{\sqrt{(1 + \varepsilon\lambda_n^2)}}{c\lambda_n} \sqrt{\frac{2}{L}} \int_0^L \sin \lambda_n \xi\, \varphi_1(\xi)\, d\xi\,.$$

Since the series

$$(22)\qquad u(t,x) = \frac{2}{L}\sum_{n=0}^{\infty} \sin \lambda_n x \left\{ \left[\int_0^L \sin \lambda_n \xi \, \varphi_0(\xi)\, \mathrm{d}\xi \right] \cos \frac{c\lambda_n t}{\sqrt{(1+\varepsilon\lambda_n^2)}} + \right.$$
$$\left. + \left[\frac{\sqrt{(1+\varepsilon\lambda_n^2)}}{c\lambda_n} \int_0^L \sin \lambda_n \xi \, \varphi_1(\xi)\, \mathrm{d}\xi \right] \sin \frac{c\lambda_n t}{\sqrt{(1+\varepsilon\lambda_n^2)}} \right\}$$

is majorized by the series

$$\frac{2}{L}\sum_{n=0}^{\infty} \sin \lambda_n x \int_0^L \sin \lambda_n \xi \, \varphi_0(\xi)\, \mathrm{d}\xi + \frac{2\sqrt{\varepsilon}}{Lc}\sum_{n=0}^{\infty} \sin \lambda_n x \int_0^L \sin \lambda_n \xi \, \varphi_1(\xi)\, \mathrm{d}\xi$$

which is uniformly convergent in $[0, L]$, (22) is also uniformly convergent in $[0, L] \times \times [0, T]$. The assumptions imply (see [1]) that the series

$$(23)\qquad u_{xx}(t,x) = -\frac{2}{L}\sum_{n=0}^{\infty} \lambda_n^2 \sin \lambda_n x \left\{ \left[\int_0^L \sin \lambda_n \xi \, \varphi_0(\xi)\, \mathrm{d}\xi \right] \cos \frac{c\lambda_n t}{\sqrt{(1+\varepsilon\lambda_n^2)}} + \right.$$
$$\left. + \left[\frac{\sqrt{(1+\varepsilon\lambda_n^2)}}{c\lambda_n} \int_0^L \sin \lambda_n \xi \, \varphi_1(\xi)\, \mathrm{d}\xi \right] \sin \frac{c\lambda_n t}{\sqrt{(1+\varepsilon\lambda_n^2)}} \right\}$$

is also uniformly convergent in $[0, L] \times [0, T]$. The series

$$(24)\qquad u_{tt}(t,x) = -\frac{2}{L}\sum_{n=0}^{\infty} \frac{c\lambda_n^2}{1+\varepsilon\lambda_n^2} \sin \lambda_n x \left\{ \left[\int_0^L \sin \lambda_n \xi \, \varphi_0(\xi)\, \mathrm{d}\xi \right] \cos \frac{c\lambda_n t}{\sqrt{(1+\varepsilon\lambda_n^2)}} + \right.$$
$$\left. + \left[\frac{\sqrt{(1+\varepsilon\lambda_n^2)}}{c\lambda_n} \int_0^L \sin \lambda_n \xi \, \varphi_1(\xi)\, \mathrm{d}\xi \right] \sin \frac{c\lambda_n t}{\sqrt{(1+\varepsilon\lambda_n^2)}} \right\}$$

is uniformly convergent in $[0, L] \times [0, T]$ because it is majorized by $c^2 u(t, x)/\varepsilon$. Similarly the series

$$(25)\qquad u_{xxtt} = \frac{2}{L}\sum_{n=0}^{\infty} \frac{c^2\lambda_n^2}{1+\varepsilon\lambda_n^2} \sin \lambda_n x \left\{ \left[\int_0^L \sin \lambda_n \xi \, \varphi_0(\xi)\, \mathrm{d}\xi \right] \cos \frac{c\lambda_n t}{\sqrt{(1+\varepsilon\lambda_n^2)}} + \right.$$
$$\left. + \left[\frac{\sqrt{(1+\varepsilon\lambda_n^2)}}{c\lambda_n} \int_0^L \sin \lambda_n \xi \, \varphi_1(\xi)\, \mathrm{d}\xi \right] \sin \frac{c\lambda_n t}{\sqrt{(1+\varepsilon\lambda_n^2)}} \right\}$$

is uniformly convergent in $[0, L] \times [0, T]$. Therefore (22) is a solution of the problem (8), (10), (12) for variant (I) of boundary conditions.

Analogously we obtain the solution of the problem (9), (10a), (12a):

$$(26)\qquad U(t,x) = \frac{2}{L}\sum_{n=0}^{\infty} \sin \lambda_n x \left\{ \left[\int_0^L \sin \lambda_n \xi \, \varphi_0(\xi)\, \mathrm{d}\xi \right] \cos c\lambda_n t + \right.$$
$$\left. + \left[\frac{1}{c\lambda_n} \int_0^L \sin \lambda_n \xi \, \varphi_1(\xi)\, \mathrm{d}\xi \right] \sin c\lambda_n t \right\},$$

where λ_n are the same as in (18) and (19). For the corrected tension we have

$$(27)\quad \frac{\sigma(t, x, \varepsilon)}{\varrho} = \frac{2c^2}{L} \sum_{n=0}^{\infty} \frac{\lambda_n}{1 + \varepsilon\lambda_n^2} \cos \lambda_n x \left\{ \left[\int_0^L \sin \lambda_n \xi \, \varphi_0(\xi) \, d\xi \right] \cos \frac{c\lambda_n t}{\sqrt{(1 + \varepsilon\lambda_n^2)}} + \right.$$
$$\left. + \left[\frac{\sqrt{(1 + \varepsilon\lambda_n^2}}{c\lambda_n} \int_0^L \sin \lambda_n \xi \, \varphi_1(\xi) \, d\xi \right] \sin \frac{c\lambda_n t}{\sqrt{(1 + \varepsilon\lambda_n^2)}} \right\},$$

and in the case of classical solution we obtain

$$(28)\quad \frac{\sigma(t, x)}{\varrho} = \frac{2c^2}{L} \sum_{n=0}^{\infty} \lambda_n \cos \lambda_n x \left\{ \left[\int_0^L \sin \lambda_n \xi \, \varphi_0(\xi) \, d\xi \right] \cos c\lambda_n t + \right.$$
$$\left. + \left[\frac{1}{c\lambda_n} \int_0^L \sin \lambda_n \xi \, \varphi_1(\xi) \, d\xi \right] \sin c\lambda_n t \right\}.$$

Since the series (25) and (27) are uniformly convergent in $[0, L] \times [0, T]$ for each $\varepsilon \in [0, \varepsilon_0]$, where $\varepsilon_0 > 0$ is an arbitrary constant, and because all functions on the right hand side are continuous in this domain, we obtain

$$\lim_{\varepsilon \to 0} u(t, x) = U(t, x), \quad \lim_{\varepsilon \to 0} \sigma(t, x, \varepsilon) = \sigma(t, x),$$

so that for small $\varepsilon > 0$ the difference between the corrected solution and that of the classical wave equation is very small.

To obtain the asymptotic behaviour of the solution of Love's equation we can also use the method of small parameter.

Theorem 1. *Consider the problem*

$$(29a)\quad \varepsilon u_{xxtt} + c^2 u_{xx} - u_{tt} = 0 \quad \text{if} \quad (t, x) \in [0, L] \times [0, T],$$

$$(29b)\quad u(0, x) = \varphi_0(x), \quad u_t(0, x) = \varphi_1(x) \quad \text{if} \quad x \in [0, L],$$

$$(30)\quad \text{variant I}: u(t, 0) = \psi_0(t), \quad u(t, L) = \psi_1(t) \quad \text{if} \quad t \in [0, T],$$

or

$$(30a)\quad \text{variant II}: u(t, 0) = \psi_0(t) \quad \varepsilon\, u_{xtt}(t, L) + c^2\, u_x(t, L) = \psi_1(t) \quad \text{if} \quad t \in [0, T],$$

where $\varphi_0(x)$, $\varphi_1(x) \in C^2[0, L]$, $\psi_0(t)$, $\psi_1(t) \in C^2[0, T]$. Then in the quadrilateral $[0, L] \times [0, T]$ we can write for the solution $u(t, x)$ of Love's equation the relation

$$(21)\quad u(t, x) = e^{-a\varepsilon}\, U(t, x) + \varepsilon\, z(t, x),$$

where a is an arbitrary constant, $U(t, x)$ is a solution of the wave equation which *fulfils the initial conditions* (29b), *the boundary conditions* (30) *in the variant* I, *the boundary conditions*

$$U(t, 0) = \psi_0(t)\,, \quad U_x(t, L) = \psi_1(t) \quad \text{if} \quad t \in [0, T] \tag{30b}$$

in the variant II, *and* $z(t, x)$ *is a solution of the differential equation* $z_{xxtt} = 0$ *for which* $e^{-a\varepsilon} U_{xxtt} + c^2 z_{xx} - z_{tt} = 0$ *holds. For the variant* I *we have*

$$z(t, x) = \frac{1}{\varepsilon}(1 - e^{-a\varepsilon})\left\{\varphi_0(x) - \frac{x}{L}\varphi_0(L) + t\left[\varphi_1(x) - \frac{x}{L}\varphi_1(L)\right] - \right.$$
$$\left. - \frac{1}{L}t(1 - x)\varphi_1(0) + \frac{x}{L}\psi_1(t) + \frac{1}{L}(L - x)\left[\psi_0(t) - \psi_0(0)\right]\right\}. \tag{31}$$

For the variant II *of boundary conditions we obtain*

$$z(t, x) = \frac{1}{\varepsilon}(1 - e^{-a\varepsilon})\left\{\varphi_0(x) - x\,\varphi_0(L) + \psi_0(t) - \psi_0(0) + \right.$$
$$+ t[\varphi_1(x) - \varphi_1(0) - x\,\varphi_1'(L)] + x\,\varphi_0(L)\cos\frac{c}{\sqrt{\varepsilon}}t +$$
$$+ x\frac{\sqrt{\varepsilon}}{c}\varphi_1'(L)\sin\frac{c}{\sqrt{\varepsilon}}t + x\sin\frac{c}{\sqrt{\varepsilon}}t\int_0^t \psi_1(\tau)\cos\frac{c}{\sqrt{\varepsilon}}\tau\,d\tau -$$
$$\left. - \cos\frac{c}{\sqrt{\varepsilon}}t\int_0^t \psi_1(\tau)\sin\frac{c}{\sqrt{\varepsilon}}\tau\,d\tau\right\}. \tag{32}$$

This theorem implies also that

$$\lim_{\varepsilon \to 0} u(t, x) = U(t, x)$$

Proof. If we take $u(t, x) = e^{-a(t)\varepsilon}\,U(t, x) + \varepsilon\,z(t, x)$ as a formal solution of the equation (29a) we obtain that $a(t)$ is a constant, $U(t, x)$ is a solution of the wave equation $c^2 u_{xx} - u_{tt} = 0$ and $z(t, x)$ is a solution of the partial differential equation $z_{xxtt} = 0$ for which $e^{-a\varepsilon} U_{xxtt} + c^2 z_{xx} - z_{tt} = 0$ holds. The relations (31) and (32) follow from the initial and boundary conditions.

For the solution of the differential equation (8) we can derive a similar theorem about the behaviour of zeros as in the paper [7] for the wave equation.

Theorem 2. *Let* $t_0 > 0$, $L_0 > 0$. *Let us consider the problem*

$$\text{(B)} \qquad \varepsilon u_{xxtt} + c^2 u_{xx} - u_{tt} = 0 \quad \text{if} \quad (t, x) \in [0, L] \times [0, T]\,,$$
$$u(t_0, x) = 0\,, \quad u_t(t_0, x) = \varphi(x) \quad \text{if} \quad 0 \leqq x \leqq L\,.$$

If $T > 0$ *is given then we can choose the function* $\varphi(x)$ *so that there exists a solution of the problem* (B) *which has no zeros for* $t_0 < t < T$. *Further, there exist constants*

$L_0 > 0$, $T_0 > 0$ *such that we can choose the function* $\varphi(x)$ *for which the solution of* (B) *has no zeros in the quadrilateral* $[0, L_0] \times [0, T_0[$.

Proof. I. Let λ^2 be the constant on the right hand side of (13). From (13) we obtain

$$v'' - \frac{c^2\lambda^2}{1 - \lambda^2\varepsilon} v = 0 \quad \text{if} \quad 1 - \lambda^2\varepsilon \neq 0\,, \tag{33}$$

$$y'' - \lambda^2 y = 0\,, \tag{34}$$

and from the initial conditions (10) we have

$$v(t)\, y(x) = 0 \Rightarrow v(t_0) = 0$$

and

$$v'(t_0)\, y(x) = \varphi(x)$$

which holds, for instance, in the case $v'(t_0) = 1$, $y(x) = \varphi(x)$.

a)
Assume that $1 - \lambda^2\varepsilon > 0$. Then we have the solution of (33)

$$v_0(t) = \frac{\sqrt{(1 - \varepsilon\lambda^2)}}{c\lambda} \sin\text{h} \frac{c\lambda(t - t_0)}{\sqrt{(1 - \varepsilon\lambda^2)}}\,,$$

which has no zero if $t > t_0$.

If we choose for the function $\varphi(x)$ the positive solution $y(x)$ of the equation (34), then $u(t, x) = y(x)\, v_0(t)$ is a solution of (B) which has no zero in $(t_0.\ T) \times (0, L)$.

b)
Assume that $1 - \lambda^2\varepsilon < 0$; then we have the solution of (33) in the form

$$v_1(t) = \frac{\sqrt{(\lambda^2\varepsilon - 1)}}{c\lambda} \sin \frac{c\lambda(t - t_0)}{\sqrt{(\lambda^2\varepsilon - 1)}}\,.$$

If $T_0 = t_0 + \pi \sqrt{(\lambda^2\varepsilon - 1)}/c\lambda$, then the function $v_1(t)$ has no zero in (t_0, T_0). If we take for the function $\varphi(x)$ the positive solution of (34), then the solution $u\,(t, x) = y(x)\, v_1(t)$ has no zero in $(t_0, T_0) \times (0, L)$.

II. Let $-\lambda^2$ be the constant on the right hand side of (13). Then $1 + \lambda^2\varepsilon > 0$ and the solution of

$$v'' + \frac{c^2\lambda^2}{1 + \varepsilon\lambda^2} v = 0$$

is

$$v_2(t) = \frac{\sqrt{(1 + \varepsilon\lambda^2)}}{c\lambda} \sin \frac{c\,\lambda(t - t_0)}{\sqrt{(1 + \varepsilon\lambda^2)}}\,.$$

Denoting $T_1 = t_0 + \pi \sqrt{(1 + \lambda^2\varepsilon)}/c\lambda$, then the function $v_2(t)$ has no zero in (t_0, T_1). The function $y(x) = k \sin \lambda(x - x_0)$, being a solution of the differential equation $y'' + \lambda^2 y = 0$, is positive in $(0, L)$ if $L_0 \leqq \pi/\lambda$. If we choose this solution for the function $\varphi(x)$, then the solution $u(t, x) = y(x)\, v_2(t)$ has no zero in $(t_0, T_0) \times (0, L_0)$.

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Souhrn

POZNÁMKA K POROVNÁNÍ VLASTNOSTÍ ŘEŠENÍ ROVNICE LOVEOVY S KLASICKOU VLNOVOU ROVNICÍ

VĚRA RADOCHOVÁ

V práci je porovnáno řešení Loveovy korigované rovnice s řešením klasické vlnové rovnice a odvozeny některé vztahy, vyjadřující jejich vzájemnou souvislost, pro jistou třídu okrajových podmínek, při čemž se vychází z toho, že Loveovu rovnici lze považovat za rovnici s malým parametrem u nejvyšší derivace.

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