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ON AXIOMATIC CHARACTERIZATION OF ENTROPY OF TYPE (α, β)*)

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1. INTRODUCTION

Sharma and Taneja [6, 7] introduced and characterized entropy of type (α, β) given by

$$(1.1)\qquad H_n(p_1, \ldots, p_n; \alpha, \beta) = (2^{1-\alpha} - 2^{1-\beta})^{-1} \sum_{i=1}^{n} (p_i^\alpha - p_i^\beta)\,, \quad \alpha \neq \beta\,, \quad \alpha, \beta > 0\,,$$

for a complete probability distribution $P = (p_1, \ldots, p_n)$, $p_i \geqq 0$, $\sum_{i=1}^{n} p_i = 1$ by generalizing a functional equation considered by Chaundy and McLeod [1].

The measure (1.1) satisfies a recursive relation as follows:

$$(1.2)\qquad H_n(p_1, \ldots, p_n; \alpha, \beta) - H_{n-1}(p_1 + p_2, p_3, \ldots, p_n; \alpha, \beta) =$$

$$= \frac{A_\alpha}{A_\alpha - A_\beta} (p_1 + p_2)^\alpha H_2\left(\frac{p_1}{p_1 + p_2}, \frac{p_2}{p_1 + p_2}; \alpha, 1\right) +$$

$$+ \frac{A_\beta}{A_\beta - A_\alpha} (p_1 + p_2)^\beta H_2\left(\frac{p_1}{p_1 + p_2}, \frac{p_2}{p_1 + p_2}; 1, \beta\right),$$

$$\alpha \neq \beta\,, \quad \alpha \neq 1\,, \quad \beta \neq 1\,, \quad \alpha, \beta > 0\,,$$

where $p_1 + p_2 > 0$, $A_\alpha = (2^{1-\alpha} - 1)$ and $A_\beta = (2^{1-\beta} - 1)$.

Measure (1.1) reduces to entropy of type β (or α) when $\alpha = 1$ (or $\beta = 1$) given by

$$(1.3)\qquad H_n(p_1, \ldots, p_n; 1, \beta) = H_n(p_1, \ldots, p_n; \beta) = (2^{1-\beta} - 1)^{-1} \left[\sum_{i=1}^{n} p_i^\beta - 1\right],$$

$$\beta \neq 1\,, \quad \beta > 0\,.$$

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When $\beta \to 1$, measure (1.3) reduces to Shannon's entropy [4], viz.

$$H_n(p_1, \ldots, p_n) = - \sum_{i=1}^{n} p_i \log_2 p_i \,. \tag{1.4}$$

The measure (1.3) was characterized by many authors by different approaches. Havrda and Charvát [3] characterized (1.3) by an axiomatic approach. Vajda [11] characterized it by mean value considerations. Daróczy [2] studied (1.3) by a functional equation. A joint characterization of the measures (1.3) and (1.4) has been done by the author in two different ways. Firstly by a generalized functional equation having four different functions (cf. [8]) and secondly by an axiomatic approach (cf. [9]). Functional measures of type β have also been obtained by Sharma and the author [5].

In this communication, we characterize the measure (1.1) by taking certain axioms parallel to those considered earlier by Havrda and Charvát [3] along with the recursive relation (1.2). Some properties of this measure are also studied.

2. SET OF AXIOMS

For characterizing a measure of information of type (α, β) associated with a probability distribution $P = (p_1, \ldots, p_n)$, $p_i \geqq 0$, $\sum_{i=1}^{n} p_i = 1$, we introduce the following axioms:

(I) $H_n(p_1, \ldots, p_n; \alpha, \beta)$ is continuous in the region $p_i \geqq 0$, $\sum_{i=1}^{n} p_i = 1$, $\alpha, \beta > 0$;

(II) $H_2(1, 0; \alpha, \beta) = 0$; $H_2(\frac{1}{2}, \frac{1}{2}; \alpha, \beta) = 1$, $\alpha, \beta > 0$;

(III) $H_n(p_1, \ldots, p_{i-1}, 0, p_{i+1}, \ldots, p_n; \alpha, \beta) = H_{n-1}(p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n; \alpha, \beta)$ for every $i = 1, 2, \ldots, n$;

(IV) $H_{n+1}(p_1, \ldots, p_{i-1}, v_{i_1}, v_{i_2}, p_{i+1}, \ldots, p_n; \alpha, \beta) -$

$$- H_n(p_1, \ldots, p_{i-1}, p_i, p_{i+1}, \ldots, p_n; \alpha, \beta) =$$

$$= \frac{A_\alpha}{A_\alpha - A_\beta} p_i^\alpha H_2(v_{i_1}/p_i, v_{i_2}/p_i; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} p_i^\beta H_2(v_{i_1}/p_i, v_{i_2}/p_i; 1, \beta)\,,$$

$\alpha \neq \beta$, $\alpha, \beta > 0$, $\alpha \neq 1$, $\beta \neq 1$,

for every $v_{i_1} + v_{i_2} = p_i > 0$, $i = 1, 2, \ldots, n$, where $A_\alpha = (2^{1-\alpha} - 1)$*) and $A_\beta = (2^{1-\beta} - 1)$ *).

When $\alpha = 1$ (or $\beta = 1$), the axiom (IV) reduces to the axiom (iv) used by Havrda and Charvát [3] for characterizing the measure (1.3).

*) Throughout this paper we shall adopt the notation A_α for $(2^{1-\alpha} - 1)$ and A_β for $(2^{1-\beta} - 1)$.

Theorem 2.1. *If $\alpha \neq \beta$, $\alpha, \beta > 0$, then the axioms* (I)–(IV) *determine a measure given by*

$$H_n(p_1, \ldots, p_n; \alpha, \beta) = (A_\alpha - A_\beta)^{-1} \sum_{i=1}^{n} (p_i^\alpha - p_i^\beta), \tag{2.1}$$

where A_α and A_β are the functions of the parameters α and β respectively as defined above.

Before proving the theorem, we prove some intermediate results based on the above axioms:

Lemma 1. *If $v_k \geqq 0$, $k = 1, 2, \ldots, m$; $\sum_{k=1}^{m} v_k = p_i > 0$, then*

$$\begin{aligned} &H_{n+m-1}(p_1, \ldots, p_{i-1}, v_1, \ldots, v_m, p_{i+1}, \ldots, p_n; \alpha, \beta) = \\ &= H_n(p_1, \ldots, p_n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} p_i^\alpha H_m(v_1/p_i, \ldots, v_m/p_i; \alpha, 1) + \\ &+ \frac{A_\beta}{A_\beta - A_\alpha} p_i^\beta H_m(v_1/p_i, \ldots, v_m/p_i; 1, \beta). \end{aligned} \tag{2.2}$$

Proof. To prove the lemma, we proceed by induction. For $m = 2$, the desired statement holds (cf. axiom (IV)). Let us suppose the result is true for numbers less than or equal to m. We shall prove it for $m + 1$. We have

$$\begin{aligned} &H_{n+m}(p_1, \ldots, p_{i-1}, v_1, \ldots, v_{m+1}, p_{i+1}, \ldots, p_n; \alpha, \beta) = \\ &= H_{n+1}(p_1, \ldots, p_{i-1}, v_1, L, p_{i+1}, \ldots, p_n; \alpha, \beta) + \\ &+ \frac{A_\alpha}{A_\alpha - A_\beta} L^\alpha H_m(v_2/L, \ldots, v_{m+1}/L; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} L^\beta H_m(v_2/L, \ldots, v_{m+1}/L; 1, \beta) \\ &\qquad \text{(where } L = v_2 + \ldots + v_{m+1}) \\ &= H_n(p_1, \ldots, p_n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} p_i^\alpha H_2(v_1/p_i, L/p_i; \alpha, 1) + \\ &+ \frac{A_\beta}{A_\beta - A_\alpha} p_i^\beta H_2(v_1/p_i, L/p_i; 1, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} L^\alpha H_m(v_2/L, \ldots, v_{m+1}/L; \alpha, 1) + \\ &+ \frac{A_\beta}{A_\beta - A_\alpha} L^\beta H_m(v_2/L, \ldots, v_{m+1}/L; 1, \beta) = \\ &= H_n(p_1, \ldots, p_n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} \{p_i^\alpha H_2(v_1/p_i, L/p_i; \alpha, 1) + \\ &+ L^\alpha H_m(v_2/L, \ldots, v_{m+1}/L; \alpha, 1)\} + \\ &+ \frac{A_\beta}{A_\beta - A_\alpha} \{p_i^\beta H_2(v_1/p_i, L/p_i; 1, \beta) + L^\beta H_m(v_2/L, \ldots, v_{m+1}/L; 1, \beta)\}, \end{aligned} \tag{2.3}$$

where $p_i = v_1 + L > 0$.

One more application of the induction premise yields

$$H_{m+1}(v_1/p_i, \ldots, v_{m+1}/p_i; \alpha, \beta) = H_2(v_1/p_i, L/p_i; \alpha, \beta) + \tag{2.4}$$

$$+ \frac{A_\alpha}{A_\alpha - A_\beta} (L/p_i)^\alpha H_m(v_2/L, \ldots, v_{m+1}/L; \alpha, 1) +$$

$$+ \frac{A_\beta}{A_\beta - A_\alpha} (L/p_i)^\beta H_m(v_2/L, \ldots, v_{m+1}/L; 1, \beta).$$

For $\beta = 1$, (2.4) reduces to

$$H_{m+1}(v_1/p_i, \ldots, v_{m+1}/p_i; \alpha, 1) = \tag{2.5}$$

$$= H_2(v_1/p_i, L/p_i; \alpha, 1) + (L/p_i)^\alpha H_m(v_2/L, \ldots, v_{m+1}/L; \alpha, 1).$$

Similarly for $\alpha = 1$, (2.4) reduces to

$$H_{m+1}(v_1/p_i, \ldots, v_{m+1}/p_i; 1, \beta) = \tag{2.6}$$

$$= H_2(v_1/p_i, L/p_i; 1, \beta) + (L/p_i)^\beta H_m(v_2/L, \ldots, v_{m+1}/L; 1, \beta).$$

Expression (2.3) together with (2.5) and (2.6) gives the desired result.

Lemma 2. *If* $v_{ij} \geqq 0$, $j = 1, 2, \ldots, m_i$, $\sum_{j=1}^{m_i} v_{ij} = p_i > 0$, $i = 1, 2, \ldots, n$, $\sum_{i=1}^{n} p_i =$ *then*

$$H_{m_1+\ldots+m_n}(v_{11}, v_{12}, \ldots, v_{1m_1} : \ldots : v_{n1}, v_{n2}, \ldots, v_{nm_n}; \alpha, \beta) = \tag{2.7}$$

$$= H_n(p_1, \ldots, p_n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} \sum_{i=1}^{n} p_i^\alpha H_{m_i}(v_{i1}/p_i, \ldots, v_{im_i}/p_i; \alpha, 1) +$$

$$+ \frac{A_\beta}{A_\beta - A_\alpha} \sum_{i=1}^{n} p_i^\beta H_{m_i}(v_{i1}/p_i, \ldots, v_{im_i}/p_i; 1, \beta).$$

Proof of this lemma directly follows from Lemma 1.

Lemma 3. *If* $F(n; \alpha, \beta) = H_n(1/n, \ldots, 1/n; \alpha, \beta)$, *then*

$$F(n; \alpha, \beta) = \frac{A_\alpha}{A_\alpha - A_\beta} F(n; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} F(n; 1, \beta), \tag{2.8}$$

where

$$F(n; \alpha, 1) = A_\alpha^{-1}(n^{1-\alpha} - 1), \quad \alpha \neq 1, \tag{2.9}$$

$$\textit{and} \quad F(n; 1, \beta) = A_\beta^{-1}(n^{1-\beta} - 1), \quad \beta \neq 1.$$

Proof. Replacing in Lemma 2 m_i by m and putting $v_{ij} = 1/mn$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$, where m and n are positive integers, we have

$$(2.10)\qquad F(mn; \alpha, \beta) = F(m; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta}(1/m)^{\alpha-1} F(n; \alpha, 1) + \\ + \frac{A_\beta}{A_\beta - A_\alpha}(1/m)^{\beta-1} F(n; 1, \beta),$$

$$(2.11)\qquad F(mn; \alpha, \beta) = F(n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta}(1/n)^{\alpha-1} F(m; \alpha, 1) + \\ + \frac{A_\beta}{A_\beta - A_\alpha}(1/n)^{\beta-1} F(m; 1, \beta).$$

Putting $m = 1$ in (2.10) and using $F(1; \alpha, \beta) = 0$ (by axiom (II)), we get

$$F(n; \alpha, \beta) = \frac{A_\alpha}{A_\alpha - A_\beta} F(n; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} F(n; 1, \beta),$$

which is (2.8).

Comparing the right hand sides of (2.10) and (2.11), we get

$$(2.12)\quad F(m; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta}(1/m)^{\alpha-1} F(n; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha}(1/m)^{\beta-1} F(n; 1, \beta) = \\ = F(n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta}(1/n)^{\alpha-1} F(m; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha}(1/n)^{\beta-1} F(m; 1, \beta).$$

Equation (2.12) together with (2.8) gives

$$(2.13)\qquad A_\alpha\{[1 - (1/n)^{\alpha-1}] F(m; \alpha, 1) + [(1/m)^{\alpha-1} - 1] F(n; \alpha, 1)\} = \\ = A_\beta\{[1 - (1/n)^{\beta-1}] F(m; 1, \beta) + [(1/m)^{\beta-1} - 1] F(n; 1, \beta)\}.$$

Put $n = 2$ in (2.13) and use $F(2; \alpha, \beta) = H_2(\frac{1}{2}, \frac{1}{2}; \alpha, \beta) = 1$ for all $\alpha, \beta > 0$. Then

$$A_\alpha\{(1 - 2^{1-\alpha}) F(m; \alpha, 1) - (1 - (1/m)^{\alpha-1})\} = \\ = A_\beta\{(1 - 2^{1-\beta}) F(m; 1, \beta) - (1 - (1/m)^{\beta-1})\} = C \quad \text{(say)},$$

i.e.,

$$A_\alpha\{(1 - 2^{1-\alpha}) F(m; \alpha, 1) - (1 - (1/m)^{\alpha-1})\} = C,$$

where C is an arbitrary constant.

For $m = 1$, we get $C = 0$.

Thus, we have

$$F(m; \alpha, 1) = \frac{1 - m^{1-\alpha}}{1 - 2^{1-\alpha}} = A_\alpha^{-1}(m^{1-\alpha} - 1), \quad \alpha \neq 1 .$$

Similarly,

$$F(m; 1, \beta) = \frac{1 - m^{1-\beta}}{1 - 2^{1-\beta}} = A_\beta^{-1}(m^{1-\beta} - 1), \quad \beta \neq 1 ,$$

which is (2.9).

Now (2.8) together with (2.9) gives

$$(2.14) \qquad F(n; \alpha, \beta) = \frac{A_\alpha}{A_\alpha - A_\beta} F(n; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} F(n; 1, \beta) = $$
$$= (A_\alpha - A_\beta)^{-1} (n^{1-\alpha} - n^{1-\beta}) .$$

Proof of the theorem. We prove the theorem for rationals and then the continuity axiom (I) extends the result for reals. For this, let m and r_i's be positive integers such that $\sum_{i=1}^{n} r_i = m$ and if we put $p_i = r_i/m$, $i = 1, 2, \ldots, n$ then an application of Lemma 2 gives

$$(2.15) \qquad H_m(\underbrace{1/m, \ldots, 1/m}_{r_1}, \ldots, \underbrace{1/m, \ldots, 1/m}_{r_n}; \alpha, \beta) =$$
$$= H_n(p_1, \ldots, p_n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} \sum_{i=1}^{n} p_i^\alpha H_{r_i}(1/r_i, \ldots, 1/r_i; \alpha, 1) +$$
$$+ \frac{A_\beta}{A_\beta - A_\alpha} \sum_{i=1}^{n} p_i^\beta H_{r_i}(1/r_i, \ldots, 1/r_i; 1, \beta) ,$$

i.e.,

$$H_n(p_1, \ldots, p_n; \alpha, \beta) = F(m; \alpha, \beta) - \frac{A_\alpha}{A_\alpha - A_\beta} \sum_{i=1}^{n} p_i^\alpha F(r_i; \alpha, 1) -$$
$$- \frac{A_\beta}{A_\beta - A_\alpha} \sum_{i=1}^{n} p_i^\beta F(r_i; 1, \beta) .$$

Equation (2.15) together with (2.9) and (2.14) gives

$$H_n(p_1, \ldots, p_n; \alpha, \beta) = (A_\alpha - A_\beta)^{-1} \sum_{i=1}^{n} (p_i^\alpha - p_i^\beta) , \quad \alpha \neq \beta , \quad \alpha, \beta > 0 ,$$

which is (2.1).

This completes the proof of the theorem.

3. PROPERTIES OF ENTROPY OF TYPE (α, β)

The measure $H_n(P; \alpha, \beta)$, where $P = (p_1, \dots, p_n)$, $p_i \geqq 0$, $\sum_{i=1}^{n} p_i = 1$ is a probability distribution, as characterized in the preceding section, satisfies certain properties, which are given in the following theorems:

Theorem 3.1. *The measure $H_n(P; \alpha, \beta)$ is non-negative for $\alpha, \beta > 0$.*

Definition. *We shall use the following definition of a convex function.*

A function $f(\cdot)$ over the points in a convex set R is convex $\cap$ if for all $r_1, r_2 \in R$ and $\mu \in (0, 1)$

$$\mu f(r_1) + (1 - \mu) f(r_2) \leqq f(\mu r_1 + (1 - \mu) r_2) . \tag{3.1}$$

The function f is convex $\cup$ if (3.1) holds with $\geqq$ in place of $\leqq$.

Theorem 3.2. *The measure $H_n(P; \alpha, \beta)$ is convex $\cap$ function of the probability distribution $P = (p_1, \dots, p_n)$, $p_i \geqq 0$, $\sum_{i=1}^{n} p_i = 1$, when one of the parameters α and β (>0) is greater than unity and the other is less than or equal to unity, i.e., either $\alpha > 1$, $0 < \beta \leqq 1$ or $\beta > 1$, $0 < \alpha \leqq 1$.*

Proof. Let there be r distributions

$$P_k(X) = \{p_k(x_1), \dots, p_k(x_n)\} , \quad \sum_{i=1}^{n} p_k(x_i) = 1 , \quad k = 1, 2, \dots, r , \tag{3.2}$$

associated with the random variable $X = (x_1, \dots, x_n)$.

Consider r numbers $(a_1, \dots, a_r)$ such that $a_k \geqq 0$ and $\sum_{k=1}^{r} a_k = 1$ and define

$$P_0(X) = \{p_0(x_1), \dots, p_0(x_n)\} ,$$

where

$$p_0(x_i) = \sum_{k=1}^{r} a_k \, p_k(x_i) , \quad i = 1, 2, \dots, n . \tag{3.3}$$

Obviously $\sum_{i=1}^{n} p_0(x_i) = 1$ and thus $P_0(X)$ is a bonafide distribution of X.

Let $\alpha > 1$, $0 < \beta \leqq 1$, then we have

$$\sum_{k=1}^{r} a_k \, H_n(P_k; \alpha, \beta) - H_n(P_0; \alpha, \beta) = \tag{3.4}$$

$$= \sum_{k=1}^{r} a_k \, H_n(P_k; \alpha, \beta) - (A_\alpha - A_\beta)^{-1} \{[\sum_{j=1}^{r} a_j p_j]^\alpha - [\sum_{j=1}^{r} a_j p_j]^\beta\} \leqq$$

$$\leqq \sum_{k=1}^{r} a_k \, H_n(P_k; \alpha, \beta) - (A_\alpha - A_\beta)^{-1} \{\sum_{j=1}^{r} a_j p_j^\alpha - \sum_{j=1}^{r} a_j p_j^\beta\} = 0$$

$$\text{for} \quad \alpha > 1 , \quad 0 < \beta \leqq 1 ,$$

i.e.,

$$\sum_{k=1}^{r} a_k H_n(P_k; \alpha, \beta) \leqq H_n(P_0; \alpha, \beta) \quad \text{for} \quad \alpha > 1, \quad 0 < \beta \leqq 1 .$$

By symmetry in α and β, the above result is also true for $\beta > 1, 0 < \alpha \leqq 1$.

Theorem 3.3. *The measure $H_n(P; \alpha, \beta)$ satisfies the following relations:*

(i) (*Generalized-Additive*):

$$H_{nm}(P*Q; \alpha, \beta) = G_n(P; \alpha, \beta) H_m(Q; \alpha, \beta) + G_m(Q; \alpha, \beta) H_n(P; \alpha, \beta), \quad \alpha, \beta > 0, \tag{3.5}$$

where

$$G_n(P; \alpha, \beta) = \tfrac{1}{2} \sum_{i=1}^{n} (p_i^\alpha + p_i^\beta), \quad \alpha, \beta > 0 . \tag{3.6}$$

(ii) (*Sub-Additive*): *For $\alpha, \beta > 1$, the measure $H_n(P; \alpha, \beta)$ is sub-additive, i.e.,*

$$H_{nm}(P * Q; \alpha, \beta) \leqq H_n(P; \alpha, \beta) + H_m(Q; \alpha, \beta), \tag{3.7}$$

where $P = (p_1, \ldots, p_n)$, $Q = (q_1, \ldots, q_m)$ and $P * Q = (p_1 q_1, \ldots, p_1 q_m; \ldots; p_n q_1, \ldots \ldots, p_n q_m)$, are complete probability distributions.

Proof. (i) We have

$$\begin{aligned} H_{nm}(P * Q; \alpha, \beta) &= (A_\alpha - A_\beta)^{-1} \sum_{i=1}^{n} \sum_{j=1}^{m} [(p_i q_j)^\alpha - (p_i q_j)^\beta] = \\ &= (A_\alpha - A_\beta)^{-1} \sum_{i=1}^{n} \sum_{j=1}^{m} [(p_i q_j)^\alpha - (p_i q_j)^\beta + p_i^\beta q_j^\alpha - p_i^\alpha q_j^\beta] = \\ &= (A_\alpha - A_\beta)^{-1} \left[\sum_{i=1}^{n} p_i^\alpha \sum_{j=1}^{m} (q_j^\alpha + q_j^\beta) - \sum_{j=1}^{n} q_j^\beta \sum_{i=1}^{n} (p_i^\alpha + p_i^\beta) \right] . \end{aligned} \tag{3.8}$$

. . . .

Similarly, we can write

$$H_{nm}(P * Q; \alpha, \beta) = (A_\alpha - A_\beta)^{-1} \left[\sum_{j=1}^{m} \alpha_j^\alpha \sum_{i=1}^{n} (p_i^\alpha + p_i^\beta) - \sum_{i=1}^{n} p_i^\beta \sum_{j=1}^{m} (q_j^\alpha + q_j^\beta) \right] \tag{3.9}$$

. . . .

(ii) As $G_n(P; \alpha, \beta) = \frac{1}{2} \sum_{i=1}^{n} (p_i^\alpha + p_i^\beta) \leqq 1$ for $\alpha, \beta \geqq 1$, the relation (3.5) gives the sub-additivity (3.7).

The results of this section show that the measure is suitable for applications, meeting at least partially the demand of the information theory for sub-additive measures.

Some other properties of this measure which make it a good measure of information have been mentioned in [7, 10].

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Souhrn

AXIOMATICKÁ CHARAKTERIZACE ENTROPIE TYPU (α, β)

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V článku je charakterizována entropie typu (α, β) s použitím axiomatického přístupu. Jako speciální případ je zahrnuta míra typu β, kterou již dříve studovali mnozí autoři. Sharma a Taneja ji vyšetřovali pomocí zobecnění jisté funkcionální rovnice, kterou se předtím zabývali Chaundy a McLeod. V článku se vyšetřují některé vlastnosti této míry.

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