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RECTANGULAR THIN ELASTIC PLATE WITH EDGES "REMAINING STRAIGHT" DURING THE DEFORMATION

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1. INTRODUCTION

Numerical solutions of the v. KÁRMÁN quasilinear partial differential equations of a thin elastic plate appear in the literature mostly in the cases when the plate is rectangular, simply supported, loaded by a uniform perpendicular load or a uniform membrane compression or a shear load with its edges remaining straight during the deformation. The boundary conditions assumed approach the boundary conditions of the plate situated in the system of rectangular edge stiffened plates covering the whole x, y plane with stiffeners flexible in torsion and in bending in the plane of the middle surface of the plate and stiff in the plane perpendicular to the plate. The normal stiffness of the ribs is negligible. In this paper we shall deal with the boundary value problems for differential equations of a plate subjected to the load and boundary conditions which include the above mentioned cases. It is assumed that a rectangular plate is loaded by a perpendicular load (defined in Section 3) and by membrane forces given by a biharmonic function. Each edge of the plate is simply supported or clamped and the membrane effects due to the deflection of the plate do not alter its curvature. It was shown in [4] that a biharmonic function giving bounded stresses in infinity yields only uniform compression or/and uniform shear load. Hence in presence of other membrane loads the assumed boundary conditions approach the boundary conditions of the plate which lies within the finite system of rectangular plates with edge stiffeners of the type specified above.

The boundary value problem is formulated in Section 2. It is shown that the boundary conditions can be given completely in terms of the deflection function and the stress function. When deriving this form of the boundary conditions it is assumed that the functions used are sufficiently regular. In the next section the variational solution of the problem is defined. The last section contains the treatment of two special cases, namely the buckling problem and the bending problem. A bifurcation theorem is proved in the first case and an existence theorem in the other.

As there exists a number of recent papers on the v. Kármán equations such as [1], [2], [3], [6] the author omits the details of some of the proofs preferring the references to analogous proofs in literature.

2. FORMULATION OF THE PROBLEM

The v. Kármán differential equations of a thin elastic plate are

$$\text{(2.1.a)} \qquad D\Delta\Delta w - t\left[\frac{\partial^2\Phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2\Phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2\Phi}{\partial x\,\partial y}\frac{\partial^2 w}{\partial x\,\partial y}\right] - q = 0\,,$$

$$\text{(2.1.b)} \qquad -\frac{t}{E}\,\Delta\Delta\Phi - t\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right] = 0\,, \quad x, y \in \Omega\,.$$

Here $\Delta\Delta$ denotes the biharmonic operator, D is the flexural rigidity of the plate, t – thickness, q – intensity of the perpendicular load, E – modulus of elasticity, w – deflection, Φ – the Airy stress function, $\Omega = (0, a) \times (0, b)$ is a rectangular domain. We assume that the stress function $\Phi = \Phi_1 + \lambda\Phi_0$ where Φ_0 is a biharmonic function characterizing the membrane loading of the plate and the parameter λ is the measure of the load.

The boundary conditions on w are

$$\text{(2.2)} \qquad w = 0|_\Gamma\,,$$

$$\frac{\partial w}{\partial n} = 0 \quad \text{or} \quad \frac{\partial^2 w}{\partial n^2} = 0 \quad \text{on each edge of the plate}\,.$$

n denotes the direction of the normal to the boundary Γ. The vanishing of the first derivative together with $w = 0$ on a part of the boundary means that the plate is clamped here, otherwise it is simply supported. Let us denote by u_0, v_0 the displacements of the middle surface in the x and y directions respectively due to the plane stress state of the plate given by the biharmonic function $\lambda\Phi_0$. The full displacements u, v are $u = u_0 + u_1$, $v = v_0 + v_1$. The linearity of the functional relationship of u, v on the stress function implies that u_1, v_1 depend only on w and Φ_1. In order to fix the plate in its middle surface it is assumed that at the point $x = 0$, $y = 0$

$$\text{(2.3)} \qquad u_1 = u_0 = v_1 = v_0 = \frac{\partial v_1}{\partial x} = \frac{\partial v_0}{\partial x} = 0\Big|_{\substack{x=0\\y=0}}\,.$$

Taking into account (2.3) and the assumption that the u_1, v_1 part of u, v does not alter the curvature of the edges of the plate we write the following boundary conditions:

$$\text{(2.4)} \qquad \frac{\partial u_1}{\partial y} = C_1\Big|_{x=0}\,, \quad \frac{\partial u_1}{\partial y} = C_2\Big|_{x=a}\,, \quad \frac{\partial v_1}{\partial x} = 0\Big|_{y=0}\,, \quad \frac{\partial v_1}{\partial x} = C_3\Big|_{y=b}\,.$$

Obviously, the edges of the plate remain straight if and only if the edges of the corresponding plane stress state problem ($u = u_0$, $v = v_0$, $w = 0$) remain straight. Furthermore, we assume that the shear stress due to Φ_1 is zero on the boundary, i.e.

$$\frac{\partial^2 \Phi_1}{\partial x \, \partial y} = 0 \bigg|_{\Gamma}, \tag{2.5}$$

and that the normal stress resultants on the edges of the plate corresponding to Φ_1 satisfy

$$t \int_0^b \frac{\partial^2 \Phi_1}{\partial y^2} \, \mathrm{d}y = 0 \bigg|_{x=0,a}, \quad t \int_0^a \frac{\partial^2 \Phi_1}{\partial x^2} \, \mathrm{d}x = 0 \bigg|_{y=0,b}. \tag{2.6}$$

In general, the moment resultants on the edges resulting from Φ_1 are not zero, they arise as reactions to the restraints (2.4). None the less, their sum over all the edges is zero as follows from the definition of the stress function. Since the conditions (2.6) involve the stress function Φ_1 and since (2.5) holds, the balance of the boundary forces in the directions of x and y implies that the values of the normal stress resultants on the opposite edges are equal. Thus, at most two of the conditions (2.6) are independent.

Integrating the differential relations between u_1, v_1 and w, Φ_1 given by

$$\varepsilon_x^* = \frac{\partial u_1}{\partial x}, \quad \varepsilon_y^* = \frac{\partial v_1}{\partial y}, \quad \gamma_{xy}^* = \frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x}, \tag{2.7}$$

$$\varepsilon_x^* = \frac{1}{E}\left(\frac{\partial^2 \Phi_1}{\partial y^2} - \mu \frac{\partial^2 \Phi_1}{\partial x^2}\right) - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,$$

$$\varepsilon_y^* = \frac{1}{E}\left(\frac{\partial^2 \Phi_1}{\partial x^2} - \mu \frac{\partial^2 \Phi_1}{\partial y^2}\right) - \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \quad \gamma_{xy}^* = -\frac{2(1+\mu)}{E} \frac{\partial^2 \Phi_1}{\partial x \, \partial y} - \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}$$

(μ denotes Poisson's ratio) and taking into account (2.3), (2.4) we express u_1, v_1 in the form

$$u_1(x, y) = \int_0^x \varepsilon_x^*(t, y) \, \mathrm{d}t + \int_0^y \left[\gamma_{xy}^*(0, s) - \int_0^s \frac{\partial \varepsilon_y^*(0, t)}{\partial x} \, \mathrm{d}t\right] \mathrm{d}s, \tag{2.8.a}$$

$$v_1(x, y) = \int_0^y \varepsilon_y^*(x, t) \, \mathrm{d}t + \int_0^x \left[\gamma_{xy}^*(s, 0) - \int_0^s \frac{\partial \varepsilon_x^*(t, 0)}{\partial y} \, \mathrm{d}t\right] \mathrm{d}s - xC_1$$

and

$$u_1(x, y) = \int_a^x \varepsilon_x^*(t, y) \, \mathrm{d}t + \int_0^y \left[\gamma_{xy}^*(a, s) - \int_0^s \frac{\partial \varepsilon_y^*(a, t)}{\partial x} \, \mathrm{d}t\right] \mathrm{d}s + u_1(a, 0), \tag{2.8.b}$$

$$v_1(x, y) = \int_b^y \varepsilon_y^*(x, t) \, \mathrm{d}t + \int_0^x \left[\gamma_{xy}^*(s, b) - \int_0^s \frac{\partial \varepsilon_x^*(t, b)}{\partial y} \, \mathrm{d}t\right] \mathrm{d}s - xC_1 + v_1(0, b).$$

The substitution of the appropriate form of u_1, v_1 from (2.8.a) or (2.8.b) into the first, second and fourth formulas from (2.4) and the subsequent evaluation of the resulting expressions at the corners $x = 0$, $y = 0$; $x = a$, $y = 0$; $x = 0$, $y = b$ yields $C_1 = C_2 = C_3 = 0$. Then from the first and the third equality in (2.4) and (2.8.a), (2.2), (2.5) there follows that

$$\frac{\partial^3 \Phi_1}{\partial x^3} = 0 \Big|_{x=0}, \quad \frac{\partial^3 \Phi_1}{\partial y^3} = 0 \Big|_{y=0}.$$

Furthermore, using the remaining equalities in (2.4) we can obtain similarly as before conditions on the third derivatives of $\Phi_1$at the edges $x = a$ and $y = b$. Summing the results obtained we write

$$\frac{\partial^3 \Phi_1}{\partial n^3} = 0 \Big|_{\Gamma}. \tag{2.9}$$

The boundary condition (2.9) was derived by P. F. PAPKOVICH [7] under the additional assumption that the functions w and Φ are symmetric. Without using the symmetricity assumption the condition (2.9) could also be obtained from the formulas expressing $\partial^2 u/\partial y^2$, $\partial^2 v/\partial x^2$ in the form given in [4] and from (2.2), (2.5).

Let us now integrate (2.5) over each edge of the boundary. Then

$$\int_0^y \frac{\partial^2 \Phi_1}{\partial x\, \partial y}\, dy = 0 \Big|_{x=0,a}, \quad \Rightarrow \quad \frac{\partial \Phi_1}{\partial x} = C_4 \Big|_{x=0}, \quad \frac{\partial \Phi_1}{\partial x} = C_5 \Big|_{x=a},$$

and

$$\int_0^x \frac{\partial^2 \Phi_1}{\partial x\, \partial y}\, dx = 0 \Big|_{y=0,b}, \quad \Rightarrow \quad \frac{\partial \Phi_1}{\partial y} = C_6 \Big|_{y=0}, \quad \frac{\partial \Phi_1}{\partial y} = C_7 \Big|_{y=b}.$$

The integration of two independent conditions from (2.6) yields $C_4 = C_5$ and $C_6 = C_7$. As the addition of an arbitrary linear polynomial function to Φ does not affect the stress state of the plate we can take the conditions obtained for the first derivatives of Φ_1 in the form

$$\frac{\partial \Phi_1}{\partial n} = 0 \Big|_{\Gamma}. \tag{2.10}$$

It could be easily shown that (2.2), (2.9) and (2.10) imply the conditions (2.2), (2.4), (2.5), (2.6). In what follows we shall consider the equations (2.1) with the boundary conditions (2.2), (2.9), (2.10).

3. THE VARIATIONAL SOLUTION

To investigate the boundary value problem (2.1), (2.2), (2.9), (2.10) we shall make use of some subspaces of the well known Sobolev space $W_2^2(\Omega)$ (see for example [5]). Let us denote by $\dot{W}_2^2(\Omega)$ and $\tilde{W}_2^2(\Omega)$ the spaces defined as the closure in the norm of $W_2^2(\Omega)$ of the set of smooth functions defined in $\bar{\Omega}$ vanishing on the simply supported part of the boundary and in the neighborhood of the clamped part or satisfying the conditions (2.10), respectively.

We assume that q belongs to $[\dot{C}(\bar{\Omega})]'$ which is the dual space to the space $\dot{C}(\bar{\Omega})$ of continuous functions defined in $\bar{\Omega}$ and vanishing on the boundary. The value of $q \in [\dot{C}(\bar{\Omega})]'$ in $u \in \dot{C}(\bar{\Omega})$ is denoted by $\langle q, u \rangle$. Such assumption on q includes the set $L_1(\Omega)$ and the Dirac measure $\delta_{(x_0, y_0)}$ (see [6] for more details).

Let Δ denote the Laplace operator. We introduce

Definition 3.1. *The couple* $(w, \Phi_1) \in \dot{W}_2^2(\Omega) \times \tilde{W}_2^2(\Omega)$ *is called a variational solution of the boundary value problem* (2.1), (2.2), (2.9), (2.10) *if the identities**)

$$(3.1) \quad \int_\Omega \{D \, \Delta w \, \Delta \omega + t[(\Phi_{xx} w_y - \Phi_{xy} w_x) \, \omega_y + (\Phi_{yy} w_x - \Phi_{xy} w_y) \, \omega_x]\} \, dx \, dy - \\ - \langle q, \omega \rangle = 0 \, ,$$

$$(3.2) \qquad \int_\Omega \left\{ - \frac{t}{E} \Delta \Phi_1 \, \Delta \psi - t(w_{xx} w_{yy} - w_{xy}^2) \, \psi \right\} dx \, dy = 0 \, ,$$

are satisfied for all $(\omega, \psi) \in \dot{W}_2^2(\Omega) \times \tilde{W}_2^2(\Omega)$.

Remarks. The identities (3.1), (3.2) can be formally obtained by multiplying Eqs. (2.1) by test functions $\omega \in \dot{W}_2^2(\Omega)$ and $\psi \in \tilde{W}_2^2(\Omega)$ respectively, and by integrating by parts with the boundary conditions (2.2), (2.9), (2.10).

Substituting a constant for ψ in (3.2) we obtain that

$$(3.3) \qquad \int_\Omega (w_{xx} w_{yy} - w_{xy}^2) \, dx \, dy = 0 \, ,$$

is a necessary condition of solvability of Eq. (2.1.b) for a fixed w. It could be easily proved that for every $w \in \dot{W}_2^2(\Omega)$ the condition (3.3) is satisfied. We see that if there exists a solution of (2.1.b) at given w, then there exists an infinite number of solutions differing by a constant only. In order to achieve the uniqueness of the solution we choose to introduce an additional condition

$$(3.4) \qquad \int_\Omega \Phi_1 \, dx \, dy = 0$$

on Φ_1 rather than to use the factor spaces because, as we have already mentioned, the addition of a linear polynomial function to the stress function has no effect on the solution of the problem from the mechanical point of view. Our choice of condition (3.4) causes both the w part and the Φ_1 part of the equation (2.1.b) as well as the

*) In what follows we shall use the notation $u_x = \partial u / \partial x$, $u_y = \partial u / \partial y, \ldots$

function Φ_1 to be orthogonal to the space P_0 of constant functions ($P_0 \subset \tilde{W}_2^2(\Omega)$). This enables us to introduce another definition of the variational solution which will be used in the sequel.

Definition 3.2. The couple $(w, \Phi_1) \in \dot{W}_2^2(\Omega) \times V$ ($V = \tilde{W}_2^2(\Omega) \perp P_0$) is called *a variational solution of the boundary value problem* (2.1), (2.2), (2.9), (2.10) if the identities (3.1), (3.2) are satisfied for all $(\omega, \psi) \in \dot{W}_2^2(\Omega) \times V$.

4. THE BUCKLING PROBLEM AND THE BENDING PROBLEM

The forthcoming theoretical investigation is restricted to two special cases of Eqs. (2.1) – namely, $q = 0$ or $\Phi_0 = 0$. Let us first of all treat the buckling problem – – $q = 0$. Using Friedrich's inequality [5]

$$\int_\Omega u^2 \, dx \, dy \leqq A_0 \int_\Omega (u_x^2 + u_y^2) \, dx \, dy \,, \quad A_0 > 0 \,, \quad u = 0|_\Gamma \,,$$

Poincarré's inequality [5]

$$\int_\Omega u^2 \, dx \, dy \leqq A_1 \int_\Omega (u_x^2 + u_y^2) \, dx \, dy + A_2 \left(\int_\Omega u \, dx \, dy \right)^2 , \quad A_1 > 0, \; A_2 > 0 \,,$$

and taking into account (2.2). (2.9), (2.10) and (3.4) we obtain

$$(4.1) \quad \int_\Omega (\Delta u)^2 \, dx \, dy = \int_\Omega (u_{xx}^2 + 2u_{xy}^2 + u_{yy}^2) \, dx \, dy \geqq A \int_\Omega u^2 \, dx \, dy \,, \quad A > 0$$

for every function belonging to $\dot{W}_2^2(\Omega)$ or V. The inequality (4.1) enables us to use in $\dot{W}_2^2(\Omega)$ and V equivalent norms denoted by $\|.\|_W$ and $\|.\|_V$ respectively and given by formulas

$$\|\omega\|_W = \left[D \int_\Omega (\Delta\omega)^2 \, dx \, dy \right]^{1/2} ,$$

$$\|\psi\|_V = \left[\frac{t}{E} \int_\Omega (\Delta\psi)^2 \, dx \, dy \right]^{1/2} .$$

The corresponding scalar product is denoted by $(.,.)_W$ and $(.,.)_V$. By means of the Riesz representation theorem we form as in [1] equivalent abstract operator equations to the variational identities from Definition 3.2. The corresponding calculation yields

$$(4.2) \qquad w - \lambda L w - C(w, \Phi_1) = 0 \,,$$

$$(4.3) \qquad -\Phi_1 - \tfrac{1}{2} B(w, w) = 0 \,,$$

where L is a linear, selfadjoint and compact operator acting from $\dot{W}_2^2(\Omega)$ into itself, C and B are bounded bilinear operators acting from $\dot{W}_2^2(\Omega) \times V$ into $\dot{W}_2^2(\Omega)$ and from $\dot{W}_2^2(\Omega) \times \dot{W}_2^2(\Omega)$ into V, respectively. From the form of Eq. (4.3) it follows immediately that (3.3) is a sufficient condition of solvability of (2.1.b).

In order to apply the general bifurcation theorem from [8] we substitute for Φ_1 into (4.2) and investigate the resulting equation

(4.4) $$w + \tfrac{1}{2}C(w, B(w, w)) - \lambda Lw = 0 .$$

By means of the equalities

$$B(w, \omega) = B(\omega, w) .$$

$$(C(w, B(w, w)), \omega)_W = (B(w, \omega), B(w, w))_V ,$$

it can be easily proved that

(4.5) $$F(w) - \lambda\, G(w) = \tfrac{1}{2}\|w\|_W^2 + \tfrac{1}{8}\|B(w, w)\|_V^2 - \frac{\lambda}{2}(Lw, w)_W .$$

is a potential corresponding to Eq. (4.4).

Remarks. The functional $F(w) - \lambda\, G(w)$ given by (4.5) is equal to the potential energy of the plate considered except for an additive constant.

Definition 4.1. The number λ_0 is called a *bifurcation point of the equation*

$$F'(w) - \lambda\, G'(w) = 0 ,$$

if for an arbitrary $\varepsilon > 0$ there exist such w and λ that $F'(w) = \lambda\, G'(w)$ and $|\lambda - \lambda_0| < \varepsilon$, $0 < \|w\| < \varepsilon$.

The properties of B, C and L imply that $F(w)$, $G(w)$ and likewise $F'(w)$, $G'(w)$ are uniformly Fréchet differentiable in every ball $\|w\|_W \leqq R$ where $R > 0$, G is a weakly continuous functional, $G'(0) = 0$, $G''(0) = L$ and $F'(0) = 0$, $F''(0) = I$. Further, it holds

$$\|B(w_1, w_1) - B(w_2, w_2)\|_V = \|B(w_1, w_1 - w_2) + B(w_1 - w_2, w_2)\|_V \leqq$$
$$\leqq \text{const}\,(\|w_1\|_W + \|w_2\|_W)\,\|w_1 - w_2\|_W ,$$

which yields easily the inequality

(4.6) $$\|C(w_1, B(w_1, w_1)) - C(w_2, B(w_2, w_2))\|_W \leqq KR^2\|w_1 - w_2\|_W ,$$

where $\|w_1\|_W$, $\|w_2\|_W \leqq R$ and K is a constant independent of w_1 and w_2. Having established (4.6) we can find such numbers $\nu_1, \nu_2 > 0$ that

$$(F'(w_1) - F'(w_2), w_1 - w_2)_W \geqq \nu_1\|w_1 - w_2\|_W^2$$

and

$$(F'(w), w)_W \leqq \nu_2\|w\|_W^2$$

for every w_1, w_2, w from a ball small enough (with its center at the origin). Finally, using estimates of the type (cf. [1])

$$\left|\int_\Omega \Phi_{1,xx} w_y \omega_y \, dx \, dy\right| \leqq \text{const} \|\Phi_1\|_V \|w\|_{W_4^1(\Omega)} \|\omega\|_{W_4^1(\Omega)}$$

and the compactness of the embedding of $W_2^2(\Omega)$ into $W_4^1(\Omega)$ we conclude for a weakly convergent sequence $\{w_n\}$, $w_n \rightharpoonup w$ that

$$B(w_n, w_n) \to B(w, w),$$
$$C(w_n, B(w_n, \omega)) \to C(w, B(w, \omega)), \quad \forall \omega \in \dot{W}_2^2(\Omega).$$

Hence for $w_n \rightharpoonup 0$ we have

$$F''(w_n)\,\omega \to F''(0)\,\omega, \quad \forall \omega \in \dot{W}_2^2(\Omega).$$

All the assumptions of Theorem 1. [8] (see also [9]) being fulfilled, we can now state.

Theorem 2.1. *The number* λ_0 *is a bifurcation point of Eqs.* (4.2), (4.3) *if and only if it is an eigenvalue of the equation*

$$\omega - \lambda L\omega = 0.$$

In the case of the bending problem ($\Phi_0 = 0$) we prove the following theorem.

Theorem 2.2. *The boundary value problem* (2.1), (2.2), (2.9), (2.10) *has at least one variational solution which coincides with the absolute minimum of the potential energy functional of the plate.*

Proof. Let us investigate the minimization problem of the potential energy functional of the plate given in the form

$$\Pi(w) = \tfrac{1}{2}\|w\|_W^2 + \tfrac{1}{8}\|B(w, w)\|_V^2 - \langle q, w\rangle. \tag{4.7}$$

The variational equation of the critical points of (4.7) together with Eq. (4.3) are evidently equivalent to the variational identities (3.1), (3.2) with $\Phi_1, \psi \in V$.

From what has been shown above the weak lower semicontinuity of $\Pi(w)$ can be proved. By means of the inequality

$$|\langle q, w\rangle| \leqq \text{const} \|w\|_W$$

we easily find a constant C such that

$$\Pi(w) + C \geqq \tfrac{1}{4}\|w\|_W^2.$$

Thus, there exists $\tilde{w} \in \dot{W}_2^2(\Omega)$ for which $\Pi(w)$ attains its absolute minimum on $\dot{W}_2^2(\Omega)$ and the couple $(\tilde{w}, \tilde{\Phi})$ where $\tilde{\Phi} = \frac{1}{2}B(\tilde{w}, \tilde{w})$ is a variational solution of the bending problem considered.

References

[1] *Berger, M. S.:* On von Kármán's equations and the buckling of a thin elastic plate, I. The clamped plate. — Comm. Pure Appl. Math. *20* (1967), 687—719.

[2] *Berger M. S.* and *Fife, P. C.:* Von Kármán's equations and the buckling of a thin elastic plate, II. Plate with general edge conditions. — Comm. Pure Appl. Math., *21* (1968), 227—241.

[3] *Knightly, G. H.:* An existence theorem for the von Kármán equations. — Arch. Rat. Mech. Anal., *27* (1967), 233—242.

[4] *Муштари, X. M. - Галимов, K. З.:* Нелинейная теория упругих оболочек. — Таткнигоиздат, Казань 1957.

[5] *Nečas, J.:* Les méthodes directes en théorie des equations elliptiques. — Academia, Prague 1967.

[6] *Nečas, J.* and *Naumann, J.:* On a boundary value problem in nonlinear theory of thin elastic plates. — Aplikace Matematiky., *19* (1974), 7—16.

[7] *Папкович, П. Ф.:* Строительная механика корабля, II. — Оборонгиз, Ленинград 1941.

[8] *Скрыпник, И. В.:* Точки бифуркации вариационных задач. — Математическая физика, В. *9*., Наукова Думка, Киев 1971, 117—123.

[9] *Скрыпник, И. В.:* О бифуркации равновесия гибких пластин. — Математическая физика, В. *13*., Наукова думка, Киев 1973, 159—161.

Súhrn

PRAVOUHLÁ TENKÁ PRUŽNÁ DOSKA S OKRAJMI „ZACHOVÁVAJÚCIMI PRIAMOČIAROSŤ" POČAS DEFORMÁCIE

ZOLTÁN SADOVSKÝ

Autor sa v článku zaoberá riešením v. Kármánových diferenciálnych rovníc tenkej pružnej pravouhlej dosky, ktorej okraje sú kĺbovo uložené alebo votknuté a od účinkov membránových napätí vyvolaných priehybom dosky nemenia svoju krivosť. Definuje sa variačné riešenie formulovanej úlohy. Pre prípad stabilitného problému je dokázaná bifurkačná veta a v prípade ohybového problému existenčná veta.

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