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A SYNCHRONIZATION FOR COMPOSED CHANNELS BY MEANS OF A RANDOM CODING

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1. INTRODUCTION

The purpose of this paper is to prove that the total ergodicity of channels, required in the earlier papers [2], [3], [4], yielding a solution of the synchronization problem (cf. [5]), is not necessary to obtain a solution of this problem. In this paper a solution of the problem for a class of composed (i.e. nonergodic) channels is given.

We have chosen to follow the terminology and notation employed in [5]; it is assumed that the reader is familiar with [5].

Throughout the paper we shall assume that the alphabets A, B, C are a finite non-empty abstract sets.

Two memoryless channels (cf. Sec. 6 of [5]) ν^1, ν^2 are said to be different ($\nu^1 \neq \nu^2$) if there is $\mathfrak{a} \in \mathfrak{A}$ and $E \in \mathscr{B}$ such that $\nu^1(E \mid \mathfrak{a}) \neq \nu^2(E \mid \mathfrak{a})$.

By saying "composed channel ν" we shall understand the following two elements:

(I) A set of positive numbers $\{\beta_1, \beta_2, \ldots, \beta_m\}$, where $m \in I^+$, $m > 1$, and

$$\sum_{i=1}^{m} \beta_i = 1 . \tag{1.1}$$

(II) A set of mutually different memoryless channels $\{\nu^1, \nu^2, \ldots, \nu^m\}$ such that

$$\nu(E \mid \mathfrak{a}) = \sum_{i=1}^{m} \beta_i \nu^i(E \mid \mathfrak{a}) \quad \text{for every} \quad \mathfrak{a} \in \mathfrak{A}\,, \quad E \in \mathscr{B}\,. \tag{1.2}$$

It is easy to see that both memoryless and composed channels are stationary, i.e. satisfy the condition

$$\nu(T^j E \mid T^j \mathfrak{a}) = \nu(E \mid \mathfrak{a}) \quad \text{for every} \quad j \in I\,, \quad E \in \mathscr{B}\,, \quad \mathfrak{a} \in \mathfrak{A}\,, \tag{1.3}$$

and satisfy also the zero-past-history condition

$$\nu(\{\mathfrak{b} : (\mathfrak{b})_i^j = \boldsymbol{b}\} \mid \mathfrak{a}^1) = \nu(\{\mathfrak{b} : (\mathfrak{b})_i^j = \boldsymbol{b}\} \mid \mathfrak{a}^2)\,, \tag{1.4}$$

for every $n \in I^+$, $1 \leqq i \leqq j \leqq n$, $\boldsymbol{b} \in B^n$, and $\mathfrak{a}^1, \mathfrak{a}^2 \in \mathfrak{A}$, if the equality $(\mathfrak{a}^1)_i^j = (\mathfrak{a}^2)_i^j$ holds.

The source μ is said to be n-ergodic for $n \in I^+$, if the measure μ is ergodic in the usual sense with respect to the transformation T^n, i.e. if the following two conditions are satisfied:

(I) $\mu(T^n E) = \mu(E)$ for every $E \in \mathscr{C}$.

(II) If $E \in \mathscr{C}$, $T^n E = E$, $\mu(E) > 0$, then $\mu(E) = 1$.

Instead of "1-ergodic" we shall say simply "ergodic".

For every memoryless or composed channel ν, for every $n \in I^+$, $\boldsymbol{a} \in A^n$, and $\boldsymbol{b} \in B^n$ we define a number $\nu_n(b \mid a)$ by

$$(1.5)\quad \nu_n(\boldsymbol{b} \mid \boldsymbol{a}) = \nu(\{\mathfrak{b} : (\mathfrak{b})_1^n = \boldsymbol{b}\} \mid \mathfrak{a}), \quad \text{where} \quad \mathfrak{a} \in \mathfrak{A}, \quad (\mathfrak{a})_1^n = \boldsymbol{a} \quad (\text{cf. } (1.4)).$$

For every ergodic source μ and $n \in I^+$ we define

$$(1.6)\qquad \mu_n(\boldsymbol{c}) = \mu(\{\mathfrak{c} : (\mathfrak{c})_1^n = \boldsymbol{c}\}) \quad \text{for every} \quad \boldsymbol{c} \in C^n .$$

It was verified earlier (cf. Conclusion of [3]) that memoryless channels are n-ergodic for all $n \in I^+$, i.e. that for every probability measure ϑ on $\mathscr{A}$, for every memoryless channel ν, and $n \in I^+$, the probability measure ω defined on $\mathscr{A} \otimes \mathscr{B}$ by

$$(1.7)\qquad \omega(E) = \int_{\mathfrak{A}} \nu(\{\mathfrak{b} : (\mathfrak{a}, \mathfrak{b}) \in E\} \mid \mathfrak{a})\, \mathrm{d}\vartheta(\mathfrak{a}), \quad E \in \mathscr{A} \otimes \mathscr{B},$$

is ergodic with respect to the transformation T^n of the space $\mathfrak{A} \otimes \mathfrak{B}$ into itself. (Cf. (2.3) of [5]).

If ν is a memoryless channel, then we define the capacity $\mathbf{C}^*(\nu)$ by

$$\mathbf{C}^*(\nu) = \sup \sum_{\substack{a \in A \\ b \in B}} \log \frac{\nu_1(b \mid a)}{p(a)\, q(b)} \quad (\text{cf. } (1.5)),$$

where the supremum is taken over the set of all probability measures p on the finite space A, and where

$$q(.) = \sum_{a \in A} \nu_1(. \mid a)\, p(a)$$

is a probability measure on B.

It follows from [1] that a capacity of the composed channel ν can be defined in several different ways. We define the capacity $\mathbf{C}(\nu)$ as the supremum of entropy rates of all ergodic sources μ such that, for every $\lambda > 0$, there exists $n \in I^+$ and (n, n)-encoder φ such that

$$(1.8)\qquad e(\varphi, \mu, \nu) < \lambda \quad (\text{cf. } (4.7) \text{ of } [5]).$$

Up to the end of the paper the following convention is used: If $n, p \in I^+$ and φ is an (n, p)-encoder, then φ is said to be a random (n, p)-encoder or (n, p)-encoder according as $\mathscr{Y}_* \neq \{\emptyset, Y\}$ or $\mathscr{Y} = \{\emptyset, Y\}$ (cf. [5]). The intuitive motivation of this terminology is obvious.

Remark. It is easily verified that the value of $\mathbf{C}(v)$, for any composed channel v, does not depend on whether "(n,n)-encoder" or "random (n,n)-encoder" in its definition is used.

In the literature a source μ satisfying the condition (1.8) for every $\lambda > 0$ is usually called transmissible over the channel v. It can be shown by a simple reasoning that, for every composed channel v, the set of all transmissible (over v) sources is non-empty. Hence, the definition of $\mathbf{C}(v)$ above has alwyas a logical meaning.

Lemma. *If v is a composed channel with positive capacity $\mathbf{C}(v)$, then for every $a \in A$ there are $a_i \in A$, $b_i \in B$ such that*

$$(1.9) \qquad v_1^i(b_i \mid a_i) \neq v_1^i(b_i \mid a) \quad \textit{for every} \quad i = 1, 2, \ldots, m \quad (\text{cf. } (1.2)) .$$

Proof. By Sec. 8 of [3], by Theorem 4 of [6], and by Theorem 2 of [1], the inequality $\mathbf{C}(v) \leqq \mathbf{C}^*(v^i)$, for $i = 1, 2, \ldots, m$ and for every composed channel v, can be proved. Therefore the assumption $\mathbf{C}(v) > 0$ implies that

$$(1.10) \qquad \mathbf{C}^*(v^i) > 0 \quad \text{for} \quad i = 1, 2, \ldots, m .$$

In view of Lemma in [4] and (1.10), it follows that there are $a_i \in A$, $b_i \in B$ such that

$$v_1^i(b_i \mid a_i) > v_1^i(b_i \mid a) \quad \text{for every} \quad i = 1, 2, \ldots, m ,$$

which completes the proof.

2. EXISTENCE OF SYNCHRONIZING RANDOM ENCODERS

Theorem 1. *If v is a composed channel with positive capacity $\mathbf{C}(v)$ and μ is an ergodic source with positive entropy rate, then for every $n, p \in I^+$ and for every (n, p)-encoder φ, there is a random $(n, p + 1)$-encoder Φ synchronizing with respect to μ and v and such that*

$$(2.1) \qquad e(\Phi, \mu, v) \leqq e(\varphi, \mu, v) + \lambda(n, \mu) ,$$

where

$$(2.2) \qquad \lim_{n \to \infty} \lambda(n, \mu) = 0 .$$

If μ is moreover an independent source, then

$$(2.3) \qquad \lambda(n, \mu) < (\tfrac{1}{2})^n .$$

Remark. If $E(n, \mu)$ is a minimum n-dimensional positive set relative to μ (cf. Lemma 2, [3]) and if we put $Y = \{1, 2, \ldots, m + 1\}$ then, for an apropriate choice of aprobability measure η on $\mathscr{Y}_*$, we shall prove that the random $(n, p + 1)$-encoder Φ defined by

$$(2.4) \quad \Phi(\mathbf{c}, y) = (a, \varphi(\mathbf{c})) \in A^{p+1} \quad \text{for} \quad \mathbf{c} \in C^n - E(n, \mu), \quad y \in Y,$$

$$(2.5) \quad \Phi(\mathbf{c}, y) = (a, a_y, a_y, \ldots, a_y) \in A^{p+1} \quad \text{for} \quad \mathbf{c} \in E(n, \mu), \quad y = 1, 2, \ldots, m,$$

where a_y for $y = 1, 2, \ldots, m$ is defined in Lemma,

$$(2.6) \qquad \Phi(\mathbf{c}, m + 1) = (a, a, \ldots, a) \in A^{p+1} \quad \text{for} \quad \mathbf{c} \in E(n, \mu),$$

is synchronizing with respect to μ and ν and satisfies (2.1), (2.2), (2.3).

Proof. Let Φ be defined as in Remark, let η be an arbitrary probability measure on Y, and let

$$(2.7) \qquad \vartheta = (\mu \otimes \tilde{\eta})\, \Phi^{-1}$$

be a probability measure on $\mathscr{A}$, defined by (2.4) and (2.6) of [5] for η and Φ given above. By Lemma 2 of [2], there is $s \in I^+$, probability measures μ^j on $\mathscr{C}$, $j = 1, 2, \ldots, s$ and positive numbers α_j, $j = 1, 2, \ldots, s$, such that

$$\vartheta = \sum_{j=1}^{s} \alpha_j \vartheta^j \quad \text{for} \quad \vartheta^j = (\mu^j \otimes \tilde{\eta})\, \Phi^{-1},$$

where μ^j are n-ergodic and ϑ^j are $(p + 1)$-ergodic measures (cf. (4.11) in [5]). If we define

$$(2.9) \qquad \gamma(E) = \int_{\mathfrak{A}} \nu(E \mid \mathfrak{a})\, \mathrm{d}\vartheta(\mathfrak{a}) \quad \text{for } \vartheta \text{ defined in (2.8)}, \quad E \in \mathscr{B},$$

$$(2.10) \qquad \gamma^{ij}(E) = \int_{\mathfrak{A}} \nu^i(E \mid \mathfrak{a})\, \mathrm{d}\vartheta^j(\mathfrak{a}) \quad \text{for} \quad i = 1, \ldots, m, \quad j = 1, \ldots, s, \quad E \in \mathscr{B},$$

then it is easy to see that γ and γ^{ij} are probability measures on $\mathscr{B}$ and, moreover, that

$$(2.11) \qquad \gamma T^k = \sum_{i,j} \beta_i \alpha_j \gamma^{ij} T^k \quad \text{for every} \quad k = 0, 1, \ldots, p \quad (\text{cf. (1.2)}),$$

where $\gamma^{ij} T^k$ are for every $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, s$, $k = 0, 1, \ldots, p$, $(p + 1)$-ergodic measures (cf. Sec. 8 of [3]).

Define on $\mathfrak{B}$ a set of $\mathscr{B}$-measurable functions

$$f_r(\mathfrak{b}) = \chi_{E_r}(\mathfrak{b}); \quad E_r = \{\mathfrak{b} : (\mathfrak{b})_1 = b_r\}; \quad r = 1, 2, \ldots, m,$$

where χ is a characteristic function and b_r are defined in Lemma. It is easily verified that

$$\alpha_{ij}^{rk} = \int_{\mathfrak{B}} f_r \, \mathrm{d}\gamma^{ij} T^k(\mathfrak{b}) = \int_Y \int_{\mathfrak{C}} v_1^i(b_r \mid (\tilde{\Phi}(\mathfrak{c}, y))_k) \, \mathrm{d}\mu^j(\mathfrak{c}) \, \mathrm{d}\eta(y) =$$

$$= \sum_{y \in Y} \int_{\mathfrak{C}} v_1^i(b_r \mid (\tilde{\Phi}(\mathfrak{c}, y))_k) \, \eta(y) \, \mathrm{d}\mu^j(\mathfrak{c}) = \sum_{y \in Y} \sum_{\mathbf{c} \in C^n} v_1^i(b_r \mid (\Phi(\mathbf{c}, y))_k) \, \eta(y) \, \mu_n^j(\mathbf{c}) =$$

$$= \sum_{C^n - E(n,\mu)} v_1^i(b_r \mid (\Phi(\mathbf{c}, .))_k) \, \mu_n^j(\mathbf{c}) +$$

$$+ \mu_n^j(E(n, \mu)) \left[v_1^i(b_r \mid a) + \sum_{y=1}^{m} \eta(y) \left(v_1^i(b_r \mid a_y) - v_1^i(b_r \mid a) \right) \right],$$

where the last equality holds for every $k = 1, 2, \ldots, p$. Using (2.4) and (2.5) we obtain that $\alpha_{ij}^{r0} = v_1^i(b_r \mid a)$ for all $j = 1, 2, \ldots, s$; hence we may write α_i^r instead of α_{ij}^{r0}. Let us denote for $i, l = 1, 2, \ldots, m$; $j = 1, 2, \ldots, s$; $k = 1, 2, \ldots, p$,

$$\beta_{ij}^{kl} = \frac{1}{\mu_n^j(E(n, \mu))} \left[\alpha_i^l - \sum_{C^n - E(n,\mu)} v_1^i(b_i \mid (\Phi(\mathbf{c}, .))_k) \, \mu_n^j(\mathbf{c}) \right] - \gamma_1^i(b_i \mid a)$$

$(\mu_n^j(E(n, \mu)) > 0$ for all $j = 1, 2, \ldots, s)$. If there are i, j, k, l such that $\beta_{ij}^{kl} \neq 0$, then define a number δ by the condition:

$$0 < \delta < \min_{i,j,k,l} \left| \beta_{ij}^{kl} \neq 0 \right|$$

If $\beta_{ij}^{kl} = 0$ for all i, j, k, l, then put $\delta = 1$. In view of (1.10), there exist numbers $\eta(y)$, $y = 1, 2, \ldots, m$, such that

$$0 < \eta(y) < \frac{1}{m}$$

$$0 < \left| \sum_{y=1}^{m} \eta(y) \left(v_1^i(b_i \mid a_y) - v_1^i(b_i \mid a) \right) \right| < \delta, \quad i = 1, 2, \ldots, m$$

and, consequently, such that

$$\eta(m + 1) = 1 - \sum_{y=1}^{m} \eta(y) > 0 .$$

If the distribution η on Y satisfies this conditions it is easily verified that

$$\text{(2.12)} \qquad \alpha_{ij}^{ik} \neq \alpha_l^l \quad \text{for all} \quad i, l = 1, \ldots, m \; ; \quad k = 1, \ldots, p \; ; \quad j = 1, \ldots, s \, .$$

We shall prove that the random encoder Φ is synchronizing with respect to μ and v provided that (2.12) holds. Define $E_{ij}^{rk}, E_i^r \in \mathscr{B}$ by

$$E_{ij}^{rk} = \left\{ \mathfrak{b} : \lim_{N \to \infty} \frac{1}{N} \sum_{q=0}^{N-1} f_r(T^{(j+1)q} \, \mathfrak{b}) = \alpha_{ij}^{rk} \right\},$$

for every i, j, k, r under consideration,

$$E_i^r = \left\{\mathfrak{b} : \lim_{N \to \infty} \frac{1}{N} \sum_{q=0}^{N-1} f_r(T^{(p+1)q}\mathfrak{b}) = \alpha_i^r\right\}, \quad i, r = 1, \ldots, m,$$

and put

$$E = \bigcup_{i=1}^{m} \bigcap_{r=1}^{m} E_i^r .$$

In view of Theorem 1 of [5], to prove that Φ is synchronizing with respect to μ and ν, it suffices to prove that

$$\gamma(E) = 1, \tag{2.13}$$

$$\gamma(T^k E) = 0 \quad \text{for} \quad k = 1, 2, \ldots, p \quad (\text{cf. } (2.7), (2.9)) \tag{2.14}$$

or, in view of (2.11) that

$$\gamma(E) = 1,$$

$$\gamma^{ij}(T^k E) = 0 \quad \text{for all} \quad i = 1, \ldots, m; \quad j = 1, \ldots, s; \quad k = 1, \ldots, p.$$

By the definition of E_{kr}^{ij}, E_i^r and by the ergodicity of the measures $\gamma^{ij}T^k$ proved above, we can write

$$\gamma^{ij}(E_i^r) = 1,$$

$$\gamma^{ij}(T^k E_{ij}^{rk}) = 1 \quad \text{for all} \quad r = 1, 2, \ldots, m$$

and, consequently, $\gamma(E) = 1$ as well as

$$\gamma^{ij}(T^k \bigcap_{r=1}^{m} E_{ij}^{rk}) = 1$$

for all i, j, k under consideration. To finish the proof it suffices to show that

$$E \cap (\bigcap_{r=1}^{m} E_{ij}^{rk}) = \emptyset$$

for all i, j, k under consideration. To prove the latter equality one can use (2.12) to obtain

$$E_l^l \cap E_{ij}^{ik} = \emptyset \quad \text{for} \quad l = 1, 2, \ldots, m$$

or, consequently,

$$\bigcup_{l=1}^{m} E_l^l \cap E_{ij}^{ik} = \emptyset \quad \text{for all} \quad i = 1, \ldots, m; \quad j = 1, \ldots, s; \quad k = 1, \ldots, p$$

and then to use the following relations:

$$E \subset \bigcup_{l=1}^{m} E_l^l, \quad \bigcap_{r=1}^{m} E_{ij}^{rk} \subset E_{ij}^{ik}$$

that evidently holds for all i, j, k under consideration.

Next we prove that (2.1) holds for $\lambda(n, \mu) = \mu_n(E(n, \mu))$. Let ψ be an arbitrary (p, n)-decoder (i.e. according to [5], let a measure space $(Z, \mathscr{Z}_*, \zeta)$ andtrans a formation $\psi(\mathbf{b}, z)$ of $B^p \otimes Z$ into C^n be given). Define a $(p + 1, n)$-decoder Ψ by

$$\Psi(\mathbf{b}, z) = \psi((\mathbf{b})_2^{p+1}, z) \quad \text{for all} \quad \mathbf{b} \in B^{p+1}, \quad z \in Z. \tag{2.14}$$

In view of the definition of $e(\Phi, \mu, \nu)$ in [5] and in view of (1.4), it follows that

$$e(\Phi, \mu, \nu) \leqq \sum_{C^n} G(\mathbf{c})\, \mu_n(\mathbf{c}) = \sum_{C^n - E(n,\mu)} G(\mathbf{c})\, \mu_n(\mathbf{c}) + \sum_{E(n,\mu)} G(\mathbf{c})\, \mu_n(\mathbf{c}), \tag{2.15}$$

where

$$G(\mathbf{c}) = 1 - \int_Z \int_Y \nu_{p+1}(\Psi^{-1}(\mathbf{c}, z) \mid \Phi(\mathbf{c}, y))\, \mathrm{d}\eta(y)\, \mathrm{d}\zeta(z)$$

(cf. (2.9), (2.10), (2.11), (4.7) in [5]). Since by (2.4), for every $\mathbf{c} \in C^n - E(n, \mu)$, $\Phi(\mathbf{c}, y) = (a, \varphi(\mathbf{c}))$ for every $y \in Y$, we obtain using (2.14) that $G(\mathbf{c}) = G_\psi(\mathbf{c})$ for all $\mathbf{c} \in C^n - E(n, \mu)$, where

$$G_\psi(\mathbf{c}) = 1 - \int_Z \int_Y \nu_p(\psi^{-1}(\mathbf{c}, z) \mid \varphi(\mathbf{c}, y))\, \mathrm{d}\eta(y)\, \mathrm{d}\zeta(z)$$

Hence, by an evident inequality $0 \leqq G(c) \leqq 1$ and by (2.15), we can write

$$e(\Phi, \mu, \nu) \leqq \sum_{C^n - E(n,\mu)} G_\psi(\mathbf{c})\, \mu_n(\mathbf{c}) + \mu_n(E(n, \mu)) \tag{2.16}$$

for every (p, n)-decoder ψ. By the definition of $e(\varphi, \mu, \nu)$, for every $\varepsilon > 0$ there is a (p, n)-decoder ψ such that

$$\sum_{C^n} G_\psi(\mathbf{c})\, \mu_n(\mathbf{c}) \leqq e(\varphi, \mu, \nu) + \varepsilon$$

and hence, such that

$$\sum_{C^n - E(n,\mu)} G_\psi(\mathbf{c})\, \mu_n(\mathbf{c}) \leqq e(\varphi, \mu, \nu) + \varepsilon.$$

Because of that ε may be arbitrary and in view of (2.16), it follows the desired result (2.1).

The statements (2.2) and (2.3) were proved in Lemma 2 of [3].

3. CAPACITY OF UNSYNCHRONIZED COMPOSED CHANNEL

Denote by $\mathscr{M}$ the class of all ergodic sources μ for which, for every $\lambda > 0$, there is a random (n, n)-encoder φ, synchronizing with respect to μ and to a composed channel ν, such that $e(\varphi, \mu, \nu) < \lambda$. If $\mathscr{M} = \emptyset$, then we define the capacity $\mathrm{C}^\circ(\nu)$ of the unsynchronized channel ν equal to zero and, if $\mathscr{M} \neq \emptyset$, then we define

$$\mathrm{C}^\circ(\nu) = \sup_{\mu \in \mathscr{M}} H(\mu)$$

where $H(\mu)$ is entropy rate of the source μ. The following inequality follows immediately from the definition:

(3.1) $$C^{\circ}(\nu) \leqq C(\nu)$$

The aim of this section is to prove that

(3.2) $$C^{\circ}(\nu) = C(\nu)$$

holds, for every composed channel ν.

Theorem 2. *If μ is an ergodic source with positive entropy rate $H(\mu)$ and if ν is a composed channel with $H(\mu) < C(\nu)$, then for every $\lambda > 0$ there is a positive integer n_0 such that, for every $n > n_0$, there exists a random (n, n)-encoder Φ synchronizing with respect to μ and ν and such that $e(\Phi, \mu, \nu) < \lambda$.*

Proof. In view of Theorems 3.2 and 3.4 of [7] and according to the McMillan's asymptotic equipartition property, for every $\varepsilon > 0$ there is an integer $n_1 = n_1(\varepsilon) \in I^+$ such that, for every $n > n_1$, there are subsets $L_n \subset C^n$, $S_{n-1} \subset A^{n-1}$, such that $\mu_n(L_n) > 1 - \varepsilon$, $\nu_{n-1}(B_i \mid a^i) > 1 - \varepsilon$, $a^i \in S_{n-1}$, $i = 1, 2, \ldots, r$, for at least one disjoint decomposition

$$B^{n-1} = \bigcup_{i=1}^{r} B_i ,$$

where $r = \text{card}\,(S_{n-1}) > \text{card}\,(L_n)$, card denotes the cardinal number.

Let $\lambda > 0$ be an arbitrary fixed number. If we denote by $n_2 = n_2(\lambda)$ the least element of I^+ such that, for every $n > n_2$, the inequality $\lambda(n, \mu) < \lambda$ holds (cf. Theorem 1), and if we put $n_0 = \max\{n_1(\lambda/4), n_2(\lambda/2)\}$, then it is obvious that for every $n > n_0$ there exists an $(n, n-1)$-encoder φ such that $e(\varphi, \mu, \nu) < \lambda/2$. To prove Theorem 2 it remains to apply Theorem 1.

Corollary. For every composed channel the equality (3.2) holds.

Proof. If $C(\nu) = 0$, then (3.2) follows immediately from (3.1). If $C(\nu) > 0$, then it is sufficient to use Theorem 2 together with the well-known fact that for every non-negative number α there is an ergodic source μ with the entropy rate $H(\mu) = \alpha$.

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Souhrn

SYNCHRONIZACE SLOŽENÝCH KANÁLŮ POMOCÍ NÁHODNÉHO KÓDOVÁNÍ

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Složený sdělovací kanál je definován jako konečný soubor diskrétních stacionárních kanálů bez paměti s zadanými pravděpodobnostmi připojení jednotlivých kanálů na zdroj informace [1]. V práci se studují možnosti sdělování informace složeným kanálem pomocí blokových kódů za předpokladu, že výstup kanálu je synchronizován se vstupem a posteriori na základě přijaté zprávy. V práci je ukázána univerzální metoda, umožňující libovolný blokový (n, p)-kód, tj. libovolné zobrazení úseků délky n zprávy ze stacionárního ergodického zdroje v úseky délky p vstupní zprávy kanálu modifikovat v synchronizační náhodný $(n, p + 1)$-kód tj. v náhodné zobrazení úseků délky n původní zprávy v úseky délky $p + 1$ vstupní zprávy kanálu, které umožňuje dostatečně dlouhou přijatou zprávu rozdělit v bloky délky $p + 1$, které by „časově" odpovídaly vstupním blokům s libovolně malou pravděpodobností chyby. Nepatrné zvýšení pravděpodobnosti nesprávného dekódování uvažovaných úseků délky n původní zprávy přitom konverguje k nule, jestliže $n \to \infty$. Na základě této metody se v práci dále dokazuje, že supremum rychlostí entropie všech ergodických zdrojů, které jsou přenesitelné složeným kanálem s libovolně malou pravděpodobností chyby pomocí blokových (náhodných i deterministických) (n, n)-kódů se rovná supremu rychlostí entropie všech ergodických zdrojů, které jsou ve stejném smyslu přenesitelné pomocí synchronizačních blokových (n, n)-kódů. Kapacita složeného kanálu se tedy zachová, jestliže výstup kanálu není a priori synchronizován se vstupem. Jestliže uvážíme, že složený kanál není ergodický, pak z tohoto výsledku plyne, že ergodicita kanálů, předpokládaná ve všech dřívějších pracích zabývajících se otázkami synchronizace, není nutnou podmínkou pro existenci synchronizačních kódů ani pro zachování kapacity.

Резюме

СИНХРОНИЗАЦИЯ СОСТАВНЫХ КАНАЛОВ СВЯЗИ ПРИ ПОМОЩИ СЛУЧАЙНОГО КОДИРОВАНИЯ

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Составный канал связи задается конечным набором дискретных каналов без памяти и набором соответствующих вероятностей включения каналов в систему передачи сообщений [1]. В работе изучаются возможности передачи сообщений по составным каналам при помощи блочныхк одов в случае, когда на выходе канала неизвестен момент начала передачи, т.е. когда вход и выход синхронизируются апостериори на основе принятого сообщения. Предлагается универсальный метод, позволяющий любой блочный (n, p)-код, т.е. любое отображение блоков длинны n сообщения из стационарного и эргодического источника в блоки длинны p входного сообщения канала трансформировать в случайный синхронизирующий $(n, p + 1)$-код, т.е. в случайное отображение соответствующих блоков длинны n в блоки длинны $p + 1$, которое позволяет достаточно длинную выходную последовательность разбить на блоки длинны $p + 1$, которые „временно" соответствуют входным блокам с произвольно малой вероятностью ошибки. Некоторое увеличение вероятности ошибочного декодирования соответствующих блоков длинны n при этом стремится к нулю, если $n \to \infty$. С помощью этого метода в статье доказывается, что пропускная способность составного канала сохраняется, если выход канала не является априори синхронизированным с входом.

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