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CORRECTION ON "THE PROPERTIES OF THE AUMANN INTEGRAL WITH APPLICATIONS TO DIFFERENTIAL INCLUSIONS AND CONTROL SYSTEMS"

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As it was pointed to us by Dr. J. Davy, theorem 4.1 of our paper [3] is incorrect as it stands. Stronger hypotheses on the orientor field $F(t, x)$ are needed. The counterexample in p. 397 of [1] shows that the continuity hypothesis on $F(t, \cdot)$ cannot be relaxed if we want to have peripheral attainability.

The next result presents a correct infinite dimensional version of theorem 4.1 of [3], partially extends theorem 7.3 of Davy and also gives us more information about the extremal solutions.

Let $T = [0, b]$, X a separable Banach space and let $S(x_0)$ be the solution set of $\dot{x}(t) \in F(t, x(t))$ a.e., $x(0) = x_0$, while $S_e(x_0)$ denotes the solution set of $\dot{x}(t) \in \in \mathrm{bd}F(t, x(t))$ a.e., $x(0) = x_0$, with $\mathrm{bd}F(t, x(t))$ denoting the boundary of $F(t, x)$).

Theorem. *If $F: T \times X \to P_{kc}(X)$ is a multifunction s.t.*
(1) *$t \to F(t, x)$ is measurable,*
(2) *$h(F(t, x'), F(t, x)) \leqq k(t) \|x' - x\|$ a.e. with $k(\cdot) \in L^1_+$,*
(3) *$|F(t, x)| \leqq a(t) + b(t) \|x\|$ a.e. with $a(\cdot), b(\cdot) \in L^1_+$,*
(4) *$\gamma(F(t, B)) \leqq r(t) \gamma(B)$ a.e. for all $B \subseteq X$ nonempty, bounded with $r(\cdot) \in L^1_+$ and with $\gamma(\cdot)$ being the ball measure of noncompactness,*

then $S(x_0) = \overline{S_e(x_0)}$ the closure taken in $C(T, X)$ and if $z \in R(t) = S(x_0)(t)$, then there exist "bang-bang" solutions $y_n(\cdot)$ s.t. $y_n(t) \to z$.

Proof. From theorems 3.1 and 4.1 of [4] we know that $S(x_0) \in P_k(C(T, X))$. Let $x(\cdot) \in S(x_0)$. Then by definition $x(t) = x_0 + \int_0^t f(s)\,ds$, $t \in T$, $f \in S^1_{F(\cdot, x(\cdot))}$. Given $\varepsilon > 0$ and using theorem 2.1 of Chuong [2] we can find $f_1 \in S^1_{\mathrm{bd}F(\cdot, x(\cdot))}$ s.t. $\|x - z_1\|_\infty < \varepsilon$, where $z_1(t) = x_0 + \int_0^t f_1(s)\,ds$, $t \in T$. Through Aumann's selection theorem, we can find $f_2: T \to X$ measurable s.t. $d(f_1(t)_1\, \mathrm{bd}F(t, z_1(t))) = \|f_1(t) - f_2(t)\|$ $t \in T$. Let $z_2(t) = x_0 + \int_0^t f_2(s)\,ds$. Then $\|z_2(t) - x(t)\| \leqq \leqq \|z_2(t) - z_1(t)\| + \|z_1(t) - x(t)\| \leqq \int_0^t \|f_2(s) - f_1(s)\|\,ds + \varepsilon \leqq \varepsilon(\int_0^t k(s)\,ds + 1)$. Suppose we have obtained $f_1 \ldots f_n \in L^1(X)$ s.t.

$$\|f_{m+1}(t) - f_m(t)\| \leqq \varepsilon\, k(t) \frac{1}{(m-1)!} (\textstyle\int_0^t k(s)\,ds)^{m-1}$$

and

$$f_{m+1}(t) \in \mathrm{bd}F(t, z_m(t)) \text{ a.e.}, \quad z_m(t) = x_0 + \int_0^t f_m(s)\,\mathrm{d}s,$$
$$m = 1, 2, \ldots, n-1.$$

Then we can write

$$\|z_{m+1}(t) - z_m(t)\| \leqq \int_0^t \|f_{m+1}(s) - f_m(s)\|\,\mathrm{d}s \leqq \varepsilon \int_0^t \frac{k(s)}{(m-1)!} \left(\int_0^s k(r)\,\mathrm{d}r\right)^{m-1} \mathrm{d}s =$$

$$= \frac{\varepsilon}{m!} \left(\int_0^t k(s)\,\mathrm{d}s\right)^m \Rightarrow \|z_{m+1}(t) - x(t)\| \leqq \varepsilon \sum_{q=1}^{m+1} \frac{1}{q!} \left(\int_0^t k(s)\,\mathrm{d}s\right)^q \leqq \varepsilon \exp \|k\|_1 .$$

Again Aumann's theorem gives us $f_{n+1} \in S^1_{\mathrm{bd}F(\cdot, z_n(\cdot))}$.

$$\|f_{n+1}(t) - f_n(t)\| \leqq h(\mathrm{bd}F(t, z_n(t)), \mathrm{bd}F(t, z_{n-1}(t))) \leqq$$
$$\leqq k(t)\, \|z_n(t) - z_{n-1}(t)\| \leqq \frac{\varepsilon}{(n-1)!} k(t) \left(\int_0^t k(s)\,\mathrm{d}s\right)^{n-1}$$

and this completes the induction.

Clearly $f_n(t) \to^s \hat{f}(t)$ in X and $\hat{f} \in L^1(X)$. Also $z_n(t) \to^s z(t) = x_0 + \int_0^t \hat{f}(s)\,\mathrm{d}s$ and $\hat{f}(t) \in \lim \mathrm{bd}F(t, z_n(t)) = \mathrm{bd}F(t, z(t))$ a.e. (hypothesis $H(F)$ (5)) $\Rightarrow z(\cdot) \in S(x_0)$. Thus in the limit we have $\|z - x\|_\infty \leqq \varepsilon \exp \|k\|_1$. Since $\varepsilon > 0$ was arbitrary we conclude that $S(x_0) = \overline{S_e(x_0)}$ the closure in $C(T, X)$. So if $z \in R(t)$, then $z = y(t) \in S(x_0)$. So we can find "bang-bang" solutions $y_n(\cdot) \in S_e(x_0)$ s.t. $y_n(t) \to z$.

Q.E.D.

References

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