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ON SOME TYPES OF KERNELS OF A CONVERGENCE *l*-GROUP

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In the paper [9], two types of kernels in lattice ordeted groups which were defined by means of properties of sequences were investigated.

In the present paper the notion of a convergence lattice ordered group (or, shorter, a convergence l-group) is applied in the same sense as in [7]. This notion was studied also in [5], [6], [8], [10] and [11]. Particular cases of convergence l-groups were dealt with in [3] and [20].

Let G be a convergence l-group. Assume that p is a condition concerning convex l-subgroups of G. A convex l-subgroup H of G is said to be a p-kernel of G if H is the largest element of the system consisting of those convex l-subgroups of G which satisfy the condition p. If p is given, then the question arises whether the p-kernel exists.

The existence of some types of p-kernels will be investigated below. All these kernels are related to properties of sequences which were dealt with in the literature on convergence structures (for a more detailed notation, cf. below).

As an illustration, let us mention the following result. Let G be a lattice ordered group and let $c(G)$ be the system of all convex l-subgroups of G. Each element H of $c(G)$ is viewed as a convergence l-group with respect to the o-convergence. H will be said to *satisfy the condition* (M) if, whenever (x_n) is a sequence in H which o-converges to 0, then there exists a sequence (k_n) of positive integers such that $k_n \to \infty$ and $k_n x_n \to_0 0$. It will be proved below that in each lattice ordered group G the M-kernel does exist.

The condition (M) was dealt with by several authors; e.g., it was applied for defining the notion of regular vector lattices (cf. B. Z. Vulich [23], W. A. J. Luxemburg and A. C. Zaanen [15]).

1. PRELIMINARIES

The standard notions and notation for lattice ordered groups will be used (cf., e.g., [2] and [13]). The group operation in a lattice ordered group will be written additively.

Let N be the set of all positive integers. The direct product $\prod_{n\in N} G_n$, where $G_n = G$

for each $n \in N$, will be denoted by G^N. The elements of G^N are written as $(g_n)_{n \in N}$, or simply (g_n). (Instead of n, the symbols i, j of k are sometimes used.) If there exists $g \in G$ such that $g_n = g$ for each $n \in N$, then we write $(g_n) = \text{const } g$.

(g_n) is said to be a *sequence in G*. The notion of a subsequence has the usual meaning.

We recall the notion of the convergence l-group (cf. [8], Definition 1.4, Lemma 1.9 and Theorem 1.10; cf. also [7] and [10]).

A subset α of $(G^N)^+$ will be said to be *G-normal* if for each $g \in G$ the relation

$$-\text{const } g + \alpha + \text{const } g \subseteq \alpha$$

is valid.

Let α be a convex G-normal subsemigroup of the semigroup $(G^N)^+$ such that the following conditions are satisfied:

(I) If $(g_n) \in \alpha$, then each subsequence of (g_n) belongs to α.

(II) Let $(g_n) \in (G^N)^+$. If each subsequence of (g_n) has a subsequence belonging to α, then (g_n) belongs to α.

(III) Let $g \in G$. Then const g belongs to α if and only if $g = 0$.

Under these assumptions α is said to be a *convergence in G*. The pair $(G; \alpha)$ is called a *convergence l-group*. If no misunderstanding can occur, then we often write G instead of $(G; \alpha)$.

For $(g_n) \in G^N$ and $g \in G$ we put $g_n \to_\alpha g$ if and only if $(|g_n - g|) \in \alpha$. If the convergence α is fixed, then we often write $g_n \to g$ instead of $g_n \to_\alpha g$.

In view of Theorem 1.10, [8], a convergence group is a FLUSH convergence structure (for this notion cf., e.g., the monograph [19]).

Let X be a nonempty set and let $\beta \neq \emptyset$ be a subset of $X^N \times X$. The set β will be said to be a *convergence structure on X*. If $((x_n), x) \in \beta$, then we write $x_n \to x$. Hence, under the above notation, the set

$$\alpha_0 = \{((g_n), g) \colon g_n \to_\alpha g\}$$

is a convergence structure on G.

If A is a nonempty subset of G, then we always consider it to be equipped with the convergence structure $(A^N \times A) \cap \alpha_0$.

A set equipped with a convergence structure will be called a *convergence space.*

A condition p concerning convergence l-groups will be called trivial if each convergence l-group satisfies the condition p.

2. THE DIAGONAL CONDITIONS (P), (SD) AND (PSD)

Let X be a nonempty set equipped with a convergence structure. For each $i \in N$ let S_i be a sequence of elements of X; we denote $S_i = (x_{ij})$ $(j = 1, 2, \ldots)$. Let $j(1), j(2), j(3), \ldots$ be positive integers, $j(1) < j(2) < j(3) < \ldots$. Then the sequence

$$(x_{i,j(i)}) \quad (i = 1, 2, 3, \ldots) \tag{1}$$

is said to be a diagonal sequence of the system $S = \{S_i\}$ $(i \in N)$. A subsequence of the sequence (1) is called a *diagonal subsequence of the system S*.

Let $V = (v_i)$ be a sequence in X and let $v \in X$, $v_i \to v$. If for each $i \in I$ the relation $x_{ij} \to v_i$ $(j = 1, 2, 3, \ldots)$ is valid, then (S, V, v) is said to be an *s-system* (cf. [4]).

Consider the following conditions for X:

(D) For each s-system (S, V, v) there exists a diagonal sequence of S converging to v.

(SD) For each s-system (S, V, v) there exists a diagonal subsequence of S converging to v.

(PSD) For each s-system (S, V, v) with $v_i = v$ for each $i \in N$ there exists a diagonal subsequence of S converging to v.

These conditions were investigated, e.g., in [12], [14] (the condition (D)), [18] (the condition (SD)), and [17] (the condition (PSD)).

2.1. Lemma. *All the conditions (D), (SD) and (PSD) are nontrivial for abelian convergence l-groups.*

Proof. Clearly $(D) \Rightarrow (SD) \Rightarrow (PSD)$. Hence it suffices to construct an abelian convergence l-group G which does not satisfy the condition (PSD).

Let R be the additive group of all reals with the natural linear order. For all m, $fn \in N$ let $G_{mn} = R$ and let $G = \prod_{(m,n) \in N \times N} G_{mn}$. We denote by α the set of all sequences f_k in G which have the following properties:

(i) for each $(m, n) \in N \times N$, $0 \leqq f_k(m, n) \to 0$ $(k = 1, 2, 3, \ldots)$ in R (with respect to the usual topology of R);

(ii) there exist $k_0, m_0 \in N$ such that $f_k(m, n) = 0$ for each $m > m_0$, $k > k_0$ and for each $n \in N$.

It is obvious that α satisfies the conditions (I), (II) and (III) from Section 1. Hence α is a convergence on the lattice ordered group G.

For each $i, j \in N$ let $f_{ij} \in G$ be such that $f_{ij}(m, n) = 1/j$ if $i = m$, and $f_{ij}(m, n) = 0$ otherwise. Let $V =$ const 0, $v = 0$. Put

$$S = \{(f_{ij})_{j=1,2,3,\ldots}\}_{i=1,2,3,\ldots} \,.$$

Then (S, V, v) is an s-system such that $v_i = v$ for each $i \in N$. No diagonal subsequence of S satisfies (ii), hence no such subsequence converges to v. Thus G does not satisfy the condition (PSD).

Let G be a convergence l-group and let $x, y \in G$. We put

$$x \varrho_D y$$

if the interval $[x \wedge y, x \vee y]$ of G satisfies the condition (D).

2.2. Theorem. *Let G be a convergence l-group. The following conditions are equivalent:*

(i) *ϱ_D is a congruence relation of the lattice ordered group G.*

(ii) *If $a, b, c \in G$, $a \leqq b \leqq c$, $a \varrho_D b$ and $b \varrho_D c$, then $a \varrho_D c$.*

For proving 2.2 we need some lemmas. Let $x, y \in G$ be such that $x \varrho_D y$ is valid. Put $x \wedge y = q, x \vee y = r$. Hence $[q, r]$ satisfies the condition (D).

2.3. Lemma. *Let $z \in G$. Then $z \vee x \varrho_D z \vee y$ and $z \wedge x \varrho_D z \wedge y$.*

Proof. Put $q_1 = q \vee z, r_1 = r \vee z$. We have to verify that the interval $[q_1, r_1]$ satisfies the condition (D).

Let (x_n) be a sequence in $[q_1, r_1]$ and let $x \in [q_1, r_1]$. Denote

$$x'_n = x_n \wedge r\,, \quad x' = x \wedge r\,. \tag{2}$$

Then we have

$$x_n = x'_n \vee q_1\,, \quad x = x' \vee q_1\,. \tag{3}$$

From (2) and (3) we obtain that

$$x_n \to x \Leftrightarrow x'_n \to x' \tag{4}$$

is valid. In view of (4), the interval $[q_1, r_1]$ satisfies (D) if and only if the interval $[q_1 \wedge r, r]$ satisfies (D). Since $[q_1 \wedge r, r] \subseteq [q, r]$, the condition (D) is valid for $[q_1 \wedge r, r]$. Therefore (D) holds for $[q_1, r_1]$ as well. Hence $x \vee z \varrho_D y \vee z$. The relation $x \wedge z \varrho_D y \wedge z$ can be verified dually.

2.4. Lemma. *Let $z \in G$. Then $z + x \varrho_D z + y$ and $x + z \varrho_D y + z$.*

Proof. Put $u_1 = z + q$, $v_1 = z + r$. Because of $(z + x) \wedge (z + y) = u_1$ and $(z + x) \vee (z + y) = v_1$, we have to verify that $[u_1, v_1]$ satisfies (D). Let (x_n) be a sequence in $[u_1, v_1]$ and let $x \in [u_1, v_1]$. Denote $x'_n = -z + x_n$, $x' = -z + x$. Then (4) holds and hence the interval $[u_1, v_1]$ satisfies (D). Thus $z + x \varrho_D z + y$. Similarly we verify that $x + z \varrho_D y + z$.

2.5. Lemma. *The following conditions are equivalent:*

(i) ϱ_D is an equivalence relation on G.

(ii) If $a, b, c \in G$, $a \leqq b \leqq c$, $a \varrho_D b$ and $b \varrho_D c$, then $a \varrho_D c$.

Proof. The implication (i) $\Rightarrow$ (ii) is obvious. Assume that (ii) is valid. The relation ϱ_D is reflexive and symmetric; it remains to verify that it is transitive.

Let $x, y, z \in G$ such that $x \varrho_D y$ and $y \varrho_D z$. Denote $q_1 = x \vee y$, $q_2 = y \vee z$, $t_1 = q_1 \wedge q_2$, $t_2 = q_1 \vee q_2$. We have $t_1 \varrho_D q_2$, because $t_1, q_2 \in [y \wedge z, y \vee z]$. Hence the relations

$$t_2 = q_2 - t_1 + q_1\,, \quad q_1 = q_2 - q_2 + q_1$$

and 2.4 imply that $q_1 \varrho_D t_2$ is valid. We have also $x \varrho_D q_1$, thus in view of (ii) we get $x \varrho_D t_2$. Moreover, $x \vee z \in [x, t_2]$ and thus $x \varrho_D x \vee z$. In a dual way we can verify that $x \varrho_D x \wedge z$ holds. By applying (ii) again we infer that $x \wedge z \varrho_D x \vee z$ is valid. Therefore $x \varrho_D z$.

From 2.3, 2.4 and 2.5 we obtain that 2.2 is valid. If the relations ϱ_{SD} and ϱ_{PSD} are defined analogously to ϱ_D, then the same method can be used.

2.6. Proposition. *Let G be a convergence l-group. Let $\varrho \in \{\varrho_{SD}, \varrho_{PSD}\}$. Then he following conditions are equivalent:*

(i) *ϱ is a congruence relation of the lattice ordered group G.*

(ii) *If a, b, c are elements of G such that $a \leqq b \leqq c$ and $a \varrho b$, $b \varrho c$, then $a \varrho c$.*

A convex l-subgroup H of G will be said to *have the property* $p(D)$ if each interval of H satisfies the condition (D). The properties $p(SD)$ and $p(PSD)$ are defined analogously. For each equivalence relation ϱ on G and $x \in G$ we denote $[x] \varrho = \{t \in G: t \varrho x\}$.

By applying the notion of the p-kernel as defined in the introduction, we infer from 2.2 and 2.6:

2.7. Theorem. *Let G be a convergence l-group and let $\varrho \in \{\varrho_D, \varrho_{SD}, \varrho_{PSD}\}$. Then the $p(\varrho)$-kernel in G exists if and only if the condition* (ii) *from 2.6 is satisfied. If this condition holds, then $[0] \varrho$ is the $p(\varrho)$-kernel of G.*

2.8. Open questions:

(2.8.1) *Let $\varrho \in \{\varrho_D, \varrho_{SD}, \varrho_{PSD}\}$. Does the condition* (ii) *from 2.6 hold for each convergence l-group?*

(2.8.2) *Let $X \in \{D, SD, PSD\}$. Does the X-kernel exist for each convergence l-group?*

3. THE DIAGONAL CONDITIONS (Y) AND (P)

Again, let X be a nonempty set equipped with a convergence structure and let (S, V, v) be an s-system in X.

Let $f = (x_{i,j(i)})$ and $g = (x_{i,j_1(i)})$ $(i = 1, 2, 3, \ldots)$ be diagonal sequences of the system S such that for each $i \in N$ we have $j(i) \leqq j_1(i)$; then we write $f \leqq g$. If, moreover, $f \neq g$, then we put $f < g$.

We consider the following condition:

(Y) For each s-system (S, V, v) there exists a diagonal f of S such that each diagonal g with $f < g$ converges to v.

The relation between the conditions (D) and (Y) was investigated in the papers [1], [4], [14] (this investigation was inspired by a question proposed in [12]).

3.1. Lemma. *The condition (Y) is nontrivial for abelian convergence l-groups.*

Proof. This follows from 2.1 since $(Y) \Rightarrow (D)$.

Let G be a convergence l-group. For $x, y \in G$ we put $x \varrho_Y y$ if the interval $[x \wedge y, x \vee y]$ satisfies the condition (Y).

3.2. Lemma. *Let a, b, c be elements of G such that $a \leqq b \leqq c$ and $a \varrho_Y b$, $b \varrho_Y c$. Then $a \varrho_Y c$.*

Proof. We shall apply the following notation. If f is a sequence in $[a, c]$, $f = (x_n)$, then we put $\varphi_1(f) = (x_n \wedge b)$ and $\varphi_2(f) = (x_n \vee b)$. Hence $\varphi_1(f)$ is a sequence in $[a, b]$ and $\varphi_2(f)$ is a sequence in $[b, c]$. Conversely, let $g = (x'_n)$ be a sequence in $[a, b]$ and let $h = (x''_n)$ be a sequence in $[b, c]$. We put $\psi(g, h) = (y_n)$, where $y_n = x'_n - b + x''_n$; thus $\psi(g, h)$ is a sequence in $[a, c]$. If f converges to an element x in G, then $x \in [a, c]$; moreover, $\varphi_1(f)$ converges to $x \wedge b$ and $\varphi_2(f)$ converges to $x \vee b$. Next, if g converges to x' and h converges to x'', then $\psi(g, h)$ converges to $x' - b + x''$.

Let (S, V, v) be an s-system in $[a, c]$ (under the notation as above). Put

$$x'_{ij} = x_{ij} \wedge b\,, \quad v'_i = v_i \wedge b\,, \quad V' = (v'_i)\,, \quad v' = v \wedge b\,,$$

$$x''_{ij} = x_{ij} \vee b\,, \quad v''_i = v_i \vee b\,, \quad V'' = (v''_i)\,, \quad v'' = v \vee b\,.$$

The meaning of S' and S'' is analogous. Then (S', V', v') is an s-system in $[a, b]$ and S'' is an s-system in $[b, c]$. Since $[a, b]$ satisfies the condition (Y), there exists a diagonal

$$g = (x'_{ij_1(i)}) \quad (i = 1, 2, \ldots)$$

of S' such that if g' is a diagonal of S' with $g' > g$, then g' converges to v'. Similarly, there exists a diagonal

$$h = (x''_{ij_2(i)}) \quad (i = 1, 2, \ldots)$$

of S'' such that if h' is a diagonal of S'' with $h' > h$, then h' converges to v''.

Let $(j_3(i))$ be a sequence of integers such that $j_3(1) < j_3(2) < j_3(3) < \ldots, j_1(i) \leqq j_3(i)$ and $j_2(i) \leqq j_3(i)$ for each $i \in N$. Put

$$f = (x_{i,j_3(i)}) \quad (i = 1, 2, \ldots)$$

and let f' be a diagonal of S with $f' > f$. Then we have

$$x'_{i,j_3(i)} \to v'\,, \quad x''_{i,j_3(i)} \to v''\,,$$

whence

$$x_{i,j_3(i)} = x'_{i,j_3(i)} - b + x''_{i,j_3(i)} \to v' - b + v'' = v\,.$$

Therefore $[a, c]$ satisfies (Y) and hence $a\, \varrho_Y\, c$.

Lemmas 2.3 and 2.4 remain valid if (D) is replaced by (Y). Hence in view of 2.5 and 3.2 we obtain

3.3. Theorem. *Let G be a convergence l-group. Then ϱ_Y is a convergence relation on G.*

A convex l-subgroup H of G is said to *have the property $p(Y)$* if each interval of H satisfies the condition (Y).

3.4. Corollary. *Let G be a convergence l-group, $H = [0]\, \varrho_Y$. Then H is the $p(Y)$-kernel of G.*

An l-subgroup H_1 of G is called *closed in G* if, whenever K is a subset of H_1 such that $\sup K$ exists in G, then $\sup K$ belongs to H_1.

The following example shows that the $p(Y)$-kernel of a convergence l-group G need not be closed in G.

3.5. Example. Let G be as in the proof of 2.1. Let H be the $p(Y)$-kernel of G.

Then H consists of all elements of G with finite support; hence H fails to be closed in G. Let us remark that H is, at the same time, the $p(\varrho)$-kernel of G for each $\varrho \in \in \{D, SD, PSD\}$.

The natural question arises whether the $p(Y)$-kernel of a convergence l-group G must satisfy the condition (Y). The answer is "No"; it suffices to consider the above example. (Roughly speaking, in this example the $p(Y)$-kernel H of G is "good" with respect to intervals, but it fails to be "good" as a whole.) The same is valid provided Y is replaced by D, SD or PSD.

Again, let G be a convergence l-group and let us consider the following condition for G:

(P) If $x_{ij} \in G$, $x_i \in G$ for $i, j \in N$, and

(i) for each i, $x_{ij} \to x_i$ $(j = 1, 2, \ldots)$,

(ii) for each sequence $(p(i))$ in N $(i = 1, 2, \ldots)$ we have $x_{i,p(i)} \to x$,

then $x_i \to x$.

The condition (P) was introduced in [22].

3.6. Lemma. *The condition (P) is non-trivial for abelian convergence l-groups.*

Proof. Let G be the set of all real functions defined on the set N. The operation $+$ and the partial order on G is defined componentwise. Let α be the set of all sequences (x_n) in G^+ which have the following property: for each $t \in N$ there exists $n_0 \in N$ such that for each $n > n_0$ and each $t_1 < t_0$ the relation $x_n(t_1) = 0$ is valid. Then α is a convergence on G.

Let $i, j \in N$. We put $x_i(t) = 1/i$ for each $t \in N$. Next we set $x_{ij}(t) = 1/i$ if $t = j$, and $x_{ij}(t) = 0$ otherwise. Let $x(t) = 0$ for each $t \in N$. Then the conditions (i) and (ii) from the condition (P) are satisfied, but (x_i) does not converge to x.

For $x, y \in G$ we put $x\, \varrho_P\, y$ if the interval $[x \wedge y, x \vee y]$ fulfils the condition (P).

Now 3.2 remains valid if Y is replaced by P. Similarly, 2.3 and 2.4 remain valid if D is replaced by P. Hence we obtain

3.7. Theorem. *Let G be a convergence l-group. Then ϱ_P is a congruence relation of the lattice ordered group G.*

Let $p(P)$ be defined analogously as $p(Y)$.

3.8. Corollary. *Let G be a convergence l-group, $H = [0]\, \varrho_P$. Then H is the $p(P)$-kernel of G.*

4. THE CONDITIONS (M_b) AND (M_b^*)

In this section we assume (when not otherwise stated) that G is a vector lattice (a K-lineal in the terminology of Soviet papers (cf., e.g. [23]), or a Riesz space in the terminology of [15]), and that, at the same time, G is a convergence l-group such that whenever $x_n \to x$ in G and $\lambda_n \to \lambda$ in R, then $\lambda_n x_n \to \lambda x$. Under these conditions G will be said to be a *convergence vector lattice*.

Let us consider the following conditions:

(M) If (x_n) is a sequence in G such that $x_n \to 0$, then there exists a sequence (λ_n) of reals such that $\lambda_n \to \infty$ and $\lambda_n x_n \to 0$.

(M^*) If (x_n) is a sequence in G such that $x_n \to 0$, then there exists a subsequence $(x_{n(i)})$ of (x_n) and a sequence (λ_i) of reals such that $\lambda_i \to \infty$ and $\lambda_i x_{n(i)} \to 0$.

(Cf., e.g., [16], [19], [21] and [23], Chap. VI, § 4 and 5.)

Let (M_b) be the condition which we obtain from (M) if the words "(x_n) is a sequence in G" are replaced by "(x_n) is a bounded sequence in G". Let the condition (M_b^*) have an analogous meaning. Clearly $(M_b) \Rightarrow (M_b^*)$.

4.1. Lemma. *Both the conditions (M_b) and (M_b^*) are nontrivial for convergence vector lattices.*

Proof. Cf. the example from [23], p. 178 (concerning the o-convergence).

It is easy to verify that (M_b) is valid for G if and only if the assertion of the condition (M_b) holds whenever $x_n \geqq 0$ for each $n \in N$.

4.2. Theorem. *Let G be a convergence vector lattice. Let $\{G_i\}_{i \in I}$ be the system of all convex l-subgroups of G which satisfy the condition (M_b). Put $H = \bigvee_{i \in I} G_i$. Then H satisfies the condition (M_b) as well.*

Proof. Let (c_n) be a bounded sequence in H such that $c_n \geqq 0$ for each $n \in N$ and $c_n \to 0$. Hence there is $0 < c \in H$ with $c \geqq c_n$ for each $n \in N$. There exist $m \in N$, $i(1), i(2), \ldots, i(m) \in I$ and $c'_j \in G_{i(j)}$ $(j = 1, 2, \ldots, m)$, $0 < c'_j$, such that

$$c = c'_1 + c'_2 + \ldots + c'_m .$$

Let $n \in N$. In view of $0 \leqq c_n \leqq c$ there exists elements c_{nj} of G $(j = 1, 2, \ldots, m)$ such that

$$c_n = c_{n1} + c_{n2} + \ldots + c_{nm}$$

and $0 \leqq c_{nj} \leqq c'_j$ for $j = 1, 2, \ldots, m$. Hence for each $j \in \{1, 2, \ldots, m\}$ and each $n \in N$ we have $c_{nj} \in G_{i(j)}$ and $c_{nj} \to 0$ $(n = 1, 2, \ldots)$ in $G_{i(j)}$.

Let $j \in \{1, 2, \ldots, m\}$. Because $G_{i(j)}$ satisfies the condition (M_b), there exists a sequence (λ_{jn}) $(n = 1, 2, 3, \ldots)$ of reals such that $\lambda_{jn} \to \infty$ and $\lambda_{jn} c_{nj} \to 0$.

Denote $\lambda_n = \min \{\lambda_{jn}\}$ $(j = 1, 2, \ldots, m)$. Then $\lambda_n \to \infty$ and $\lambda_n c_{nj} \to 0$ for each $j \in \{1, 2, \ldots, m\}$. Thus $\lambda_n x_n \to 0$, which completes the proof.

4.3. Corollary. *Let G be a convergence vector lattice and let H be as in 4.2. Then H is the M_b-kernel of G.*

By a slight modification of the method applied in the proof of 4.2 we obtain

4.4. Theorem. *Let G be a convergence vector lattice. Let $\{G'_i\}_{i \in I}$ be the system of all convex l-subgroups of G which satisfy the condition (M_b^*). Put $H' = \bigvee_{i \in I} G'_i$. Then H' is the M_b^*-kernel of G.*

It is easy to verify that if in the condition (M) the words "a sequence (λ_n) of reals" are replaced by "a sequence (λ_n) of positive integers", then we obtain a condition which is equivalent to (M). In this modified formulation the condition can be applied to convergence l-groups as well. The same holds with respect to the condition (M^*). Theorems 4.2 and 4.3 remain valid, if G is any convergence l-group (with the same proofs). Moreover, if G is a convergence l-group with respect to the o-convergence, then (M) coincides with (M_b) and (M^*) coincides with (M_b^*).

4.5. Open question: *Let $X \in \{M, M^*\}$. Does the X-kernel exist for each convergence l-group?*

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