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ON A CERTAIN NUMBERING OF THE VERTICES OF A HYPERGRAPH

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0. By a *hypergraph* we shall mean an ordered pair $\mathscr{H} = (V, \mathscr{E})$, where V is a finite nonempty set, and $\mathscr{E}$ is a set of nonempty subsets of V (note that our concept of a hypergraph is not identical with the concept of a hypergraph in the sense of [1]). The elements of V are called *vertices* of $\mathscr{H}$ and the elements of $\mathscr{E}$ are called *edges* of $\mathscr{H}$.

Let $\mathscr{H} = (V, \mathscr{E})$ be a hypergraph. Denote $n = |V|$. Consider a sequence $(v_1, \ldots, v_n)$ such that $\{v_1, \ldots, v_n\} = V$. If for each $E \in \mathscr{E}$ there exist integers i and k, $1 \leqq i \leqq k \leqq$ $\leqq n$, with the property that

$$E = \{v_j;\ i \leqq j \leqq k\}\ ,$$

then we shall say that the sequence $(v_1, \ldots, v_n)$ is a *projectoidic arrangement* of $\mathscr{H}$. Obviously, if $(v_1, \ldots, v_n)$ is a projectoidic arrangement of $\mathscr{H}$, then the sequence $(v_n, \ldots, v_1)$ is also a projectoidic one. We shall say that $\mathscr{H}$ is a *projectoid* if there exists a projectoidic arrangment of $\mathscr{H}$. This means that $\mathscr{H}$ is a projectoid if and only if its vertices can be numbered by the integers 1, …, and n in such a way that for each $E \in \mathscr{E}$, if i, j, and k are integers, $1 \leqq i \leqq j \leqq k \leqq n$, such that both i and k are the numbers assigned to some vertices of E, then j is also the number assigned to a vertex of E.

Objects equivalent to projectoids were studied by means of the matrix theory in [3] and [7], and by means of the theory of bipartite graphs in [7]. As families of sets projectoids were studied in [2] and [6] (an applications of projectoids in the area of information retrieval was shown in [2]). In [2], [3], [6], and [7] various characterizations for projectoids (or objects equivalent to them) can be found. For the full list of "subhypergraphs" (in a certain sense) which are forbidden for projectoids the reader is referred to [6]. (Note that the terms "projectoidic" or "projectoid" have not appeared in the papers mentioned above).

It is obvious that a hypergraph with at most two edges is a projectoid. In the present paper for every hypergraph $\mathscr{H}$ we shall construct a certain set of hypergraphs with exactly three edges and show that $\mathscr{H}$ is a projectoid if and only if each hypergraph in the constructed set is. The proof of this is based on the concept of a strict separating set (see below). In the last section of the papei this result will be applied to a problem concerning directed graphs.

1.1. Let $\mathscr{A}$ be a finite nonempty set of finite nonempty sets. Then we denote by $\langle \mathscr{A} \rangle$ the hypergraph $(V', \mathscr{A})$, where

$$V' = \bigcup_{A \in \mathscr{A}} A \,.$$

Let $\mathscr{H} = (V, \mathscr{E})$ be a hypergraph. If $\mathscr{A} \subseteq \mathscr{E}$, then instead of $(V, \mathscr{E} - \mathscr{A})$ we shall write $\mathscr{H} - \mathscr{A}$. If Z is a nonempty subset of V, then we denote by $\langle Z \rangle_{\mathscr{H}}$ the hypergraph $(Z, \mathscr{E}')$, where

$$\mathscr{E}' = \{E \cap Z;\ E \in \mathscr{E} \text{ and } E \cap Z \neq \emptyset\} \,.$$

We denote by $\Omega(\mathscr{H})$ the set defined as follows:

(1) if $v \in V$, then $\{v\} \in \Omega(\mathscr{H})$;
(2) if $E \in \mathscr{E}$, then $E \in \Omega(\mathscr{H})$;
(3) if $S', S'' \in \Omega(\mathscr{H})$ and $S' \cap S'' \neq \emptyset$, then $S' \cup S'' \in \Omega(\mathscr{H})$;
(4) no other element belongs to $\Omega(\mathscr{H})$.

It follows from (1) that the hypergraphs $\langle \Omega(\mathscr{H}) \rangle$ and $(V, \Omega(\mathscr{H}))$ are identical. It is obvious that there exists exactly one partition $\mathscr{P}$ of V with the properties that (a) if $U \in \mathscr{P}$, then $U \in \Omega(\mathscr{H})$; and (b) if $E \in \mathscr{E}$, then there exists $W \in \mathscr{P}$ such that $E \subseteq W$. If $V' \in \mathscr{P}$, then we shall say that $\langle V' \rangle_{\mathscr{H}}$ is a component of $\mathscr{H}$. We say that $\mathscr{H}$ is connected if it has exactly one component. Clearly, $\mathscr{H}$ is connected if and only if $V \in \Omega(\mathscr{H})$. Let $\mathscr{A} \subseteq \mathscr{E}$; we say that $\mathscr{A}$ is a separating set of $\mathscr{H}$ if $\mathscr{H} - \mathscr{A}$ is not connected. We say that a separating set $\mathscr{A}$ of $\mathscr{H}$ is strict if no proper subset of $\mathscr{A}$ is a separating set of $\mathscr{H}$.

Proofs of the following four propositions will be left to the reader:

Propostion 1. *Let $\mathscr{H} = (V, \mathscr{E})$ be a projectoid, and let $V' \subseteq V$ and $\mathscr{E}' \subseteq \mathscr{E}$, where $V' \neq \emptyset \neq \mathscr{E}'$. Then both $\langle V' \rangle_{\mathscr{H}}$ and $\langle \mathscr{E}' \rangle$ are projectoids.*

Proposition 2. *Let $\mathscr{H}$ be a hypergraph. Then every projectoidic arrangement of $\mathscr{H}$ is a projectoidic arrangement of $\langle \Omega(\mathscr{H}) \rangle$.*

Proposition 3. *A hypergraph $\mathscr{H}$ is a projectoid if and only if $\langle \Omega(\mathscr{H}) \rangle$ is.*

Proposition 4. *Let S_1, S_2, and S_3 be three finite nonempty sets. Then $\langle \{S_1, S_2, S_3\} \rangle$ is a projectoid if and only if the following conditions hold:*

(1) *if there exists a permutation p on $\{1, 2, 3\}$ such that $S_{p(1)} \cap (S_{p(2)} - S_{p(3)}) \neq \emptyset \neq S_{p(1)} \cap (S_{p(3)} - S_{p(2)})$, then $S_{p(2)} \cap S_{p(3)} \subseteq S_{p(1)}$;*
(2) *if the sets $S_1 \cap S_2$, $S_2 \cap S_3$, and $S_3 \cap S_1$ are nonempty, then there exists a permutation q on $\{1, 2, 3\}$ such that $S_{q(1)} \subseteq S_{q(2)} \cup S_{q(3)}$.*

We now state the main result of this paper:

Theorem 1. *Let $\mathscr{H}$ be a hypergraph. Then it is a projectoid if and only if for any three elements S_1, S_2, and S_3 of $\Omega(\mathscr{H})$, $\langle\{S_1, S_2, S_3\}\rangle$ is a projectoid.*

1.2. Proof of Theorem 1. Denote $\mathscr{H} = (V, \mathscr{E})$ and $|V| = n$.

(A) Assume that $\mathscr{H}$ is a projectoid. According to Proposition 3, $\langle\Omega(\mathscr{H})\rangle$ is a projectoid. It follows from Proposition 1 that for any three $S_1, S_2, S_3 \in \Omega(\mathscr{H})$, $\langle\{S_1, S_2, S_3\}\rangle$ is a projectoid.

(B) Assume that foı any three $S_1, S_2, S_3 \in \Omega(\mathscr{H})$, $\langle\{S_1, S_2, S_3\}\rangle$ is a pıojectoid. We shall prove that $\mathscr{H}$ is a projectoid.

It follows from assumption (B) that

(*) for any nonempty proper subset V' of V and for any three $S'_1, S'_2, S'_3 \in \Omega(\langle V'\rangle_{\mathscr{H}})$, $\langle\{S'_1, S'_2, S'_3\}\rangle$ is a projectoid.

If $n \leqq 2$, then $\mathscr{H}$ is a projectoid. Let $n \geqq 3$. Assume that for every hypergraph $\mathscr{H}' = (V', \mathscr{E}')$ with $|V'| < n$ and with the property that

for every three $S'_1, S'_2, S'_3 \in \Omega(\mathscr{H}')$, $\langle\{S'_1, S'_2, S'_3\}\rangle$ is a projectoid,

it has been proved that $\mathscr{H}'$ is a projectoid. It follows from (*) and from the induction assumption that

for every nonempty proper subset V' of V, $\langle V'\rangle_{\mathscr{H}}$ is a pıojectoid.

If $V \in \mathscr{E}$, then $\mathscr{H}$ is a projectoid if and only if $(V, \mathscr{E} - \{V\})$ is a projectoid. Therefore, without loss of generality we shall assume that $V \notin \mathscr{E}$. We distinguish the following cases:

(1) Assume that $\mathscr{H}$ is not connected. Then every component of $\mathscr{H}$ is a projectoid. Hence, $\mathscr{H}$ is also a projectoid.

(2) Assume that $\mathscr{H}$ is connected.

(2.1) Assume that for every strict separating set $\mathscr{A}$ of $\mathscr{H}$, there exists a vertex of $\mathscr{H}$, say a vertex $r(\mathscr{A})$, such that $\langle V - \{r(\mathscr{A})\}\rangle_{\mathscr{H}}$ is a component of $\mathscr{H} - \mathscr{A}$. Since $n \geqq 3$, we have that $r(\mathscr{A})$ is determined uniquely.

Let $\mathscr{B}$ be an arbitrary strict separating set of $\mathscr{H}$. If $B_1, B_2 \in \mathscr{B}$, then from the fact that $\langle\{B_1, B_2, V - \{r(\mathscr{B})\}\}\rangle$ is a projectoid if follows according to Proposition 4 that either $B_1 \subseteq B_2$ or $B_2 \subseteq B_1$. Hence, $\mathscr{B}$ is linearly ordered by the inclusion. We denote by $\mathscr{B}^*$ the minimum edge of $\mathscr{B}$. We have that $\mathscr{B}$ is the set of edges $E \in \mathscr{E}$ with the properties that $r(\mathscr{B}) \in E$ and $|E| \geqq 2$. This implies that if $\mathscr{B}'$ is a strict separating set of $\mathscr{H}$, then $\mathscr{B} = \mathscr{B}'$ if and only if $r(\mathscr{B}) = r(\mathscr{B}')$.

Consider a strict separating set $\mathscr{U}$ of $\mathscr{H}$. Since $V \notin \mathscr{E}$, there exists a strict separating set $\mathscr{W}$ of $\mathscr{H}$ such that $\mathscr{U}^* \notin \mathscr{W}$. For every strict separating set $\mathscr{A}$ of $\mathscr{H}$, either $\mathscr{A} = \mathscr{U}$ or $\mathscr{A} = \mathscr{W}$ (otherwise, $\langle\{V - \{r(\mathscr{A})\}, V - \{r(\mathscr{U})\}, V - \{r(\mathscr{W})\}\}\rangle$ is not a projectoid, which is a contradiction). This implies that $\mathscr{U}^* \cup \mathscr{W}^* = V$ and $\mathscr{U}^* \cap \mathscr{W}^* \neq \emptyset$. Assume that there exists $X \in \mathscr{U} \cap \mathscr{W}$. Then $\mathscr{U}^* \subseteq X$ and $\mathscr{W}^* \subseteq X$. Hence, $X = V$. Thus $V \in \mathscr{E}$, which is a contradiction. This means that $\mathscr{U} \cap \mathscr{W} = \emptyset$.

Without loss of generality we shall assume that $|\mathscr{U}^*| \geqq |\mathscr{W}^*|$. It is obvious that $\langle V - \{r(\mathscr{U})\}\rangle_{\mathscr{H}}$ is a projectoid. We denote by $(v_1, \ldots, v_{n-1})$ a projectoidic arrangement of $\langle V - \{r(\mathscr{U})\}\rangle_{\mathscr{H}}$. Since $|\mathscr{U}^* - \{r(\mathscr{U})\}| \leqq n - 2$ and $\mathscr{W}^* \in \mathscr{E}$, we have that either $v_1 \notin \mathscr{U}^*$ or $v_{n-1} \notin \mathscr{U}^*$. Without loss of generality we assume that $v_{n-1} \notin \mathscr{U}^*$. Hence, $v_{n-1} \in \mathscr{W}^*$. If $|\mathscr{W}^*| = n - 1$, then $|\mathscr{U}^*| = n - 1$, and therefore, $v_1 \in \mathscr{U}^*$. Let $|\mathscr{W}^*| \leqq n - 2$; since $v_{n-1} \in \mathscr{W}^*$ and $\mathscr{W}^* \in \mathscr{E}$, we have that $v_1 \notin \mathscr{W}^*$; hence, $v_1 \in \mathscr{U}^*$. This means that $(u, v_1, \ldots, v_{n-1})$ is a projectoidic arrangement of $\mathscr{H}$. Therefore, $\mathscr{H}$ is a projectoid.

(2.2) Assume that there exists a strict separating set $\mathscr{A}$ of $\mathscr{H}$ such that the hypergraph $\mathscr{H} - \mathscr{A}$ contains no component with $n - 1$ vertices.

(2.2.1) Assume that $\mathscr{H} - \mathscr{A}$ has at least three components. Let $\mathscr{H}_1 = (V_1, \mathscr{E}_1)$, $\mathscr{H}_2 = (V_2, \mathscr{E}_2), \ldots, \mathscr{H}_k = (V_k, \mathscr{E}_k)$ be the components of $\mathscr{H} - \mathscr{A}$. Hence, $k \geqq 3$. Since $\mathscr{A}$ is a strict separating set of $\mathscr{H}$, we have that for every i, $1 \leqq i \leqq k$, and every $A \in \mathscr{A}$, the inequality $A \cap V_i \neq \emptyset$ holds.

Assume that for every j, $1 \leqq j \leqq k$, there exists $A_j \in \mathscr{A}$ such that $V_j - A_j \neq \emptyset$. Denote

$$B_1 = A_1 \cup V_2 \cup V_3\,, \quad B_2 = A_2 \cup V_3 \cup V_1\,, \quad B_3 = A_3 \cup V_1 \cup V_2\,.$$

Clearly, $B_1, B_2, B_3 \in \Omega(\mathscr{H})$. We can see that

$$V_3 - A_3 \subseteq B_1 \cap (B_2 - B_3)\,, \quad V_2 - A_2 \subseteq B_1 \cap (B_3 - B_2)\,,$$

and

$$V_1 - A_1 \subseteq (B_2 \cap B_3) - B_1\,.$$

Since $V_j - A_j \neq \emptyset$, for $1 \leqq j \leqq 3$, it follows from Proposition 4 that $\langle\{B_1, B_2, B_3\}\rangle$ is not a projectoid, which is a contradiction. This means that there exists f, $1 \leqq f \leqq k$, such that for every $A \in \mathscr{A}$, $V_f \subseteq A$.

Let $(u_1, \ldots, u_{n-|V_f|})$ be a projectoidic arrangement of $\langle V - V_f\rangle_{\mathscr{H}}$. There exists g, $1 \leqq g \leqq k$ and $g \neq f$, such that $u_1 \in V_g$. Clearly, $u_1, \ldots, u_{|V_g|} \in V_g$. Let $(w_1, \ldots, w_{|V_f|})$ be a projectoidic arrangement of $\langle V_f\rangle_{\mathscr{H}}$. Then

$$(u_1, \ldots, u_{|V_g|}, w_1, \ldots, w_{|V_f|}, u_{|V_g|+1}, \ldots, u_{n-|V_f|})$$

is a projectoidic arrangement of $\mathscr{H}$. Hence, $\mathscr{H}$ is a projectoid.

(2.2.2) Assume that $\mathscr{H} - \mathscr{A}$ has exactly two components, say the components $\mathscr{H}_1 = (V_1, \mathscr{E}_1)$ and $\mathscr{H}_2 = (V_2, \mathscr{E}_2)$. Obviously, $\min(|V_1|, |V_2|) \geqq 2$. Since $\mathscr{A}$ is a strict separating set of $\mathscr{H}$, we have for every $A \in \mathscr{A}$ the inequalities $A \cap V_1 \neq \emptyset \neq A \cap V_2$. Consider arbitrary $A', A'' \in \mathscr{A}$. Since both $\langle\{V_1, A' \cup V_2, A'' \cup V_2\}\rangle$ and $\langle\{V_2, A' \cup V_1, A'' \cup V_1\}\rangle$ are projectoids, we have that (a) either $A' \cap V_1 \subseteq A'' \cap V_1$ or $A'' \cap V_1 \subseteq A' \cap V_1$, and (b) $A' \cap V_2 \subseteq A'' \cap V_2$ or $A'' \cap V_2 \subseteq A' \cap V_2$. This implies that there exists $v_1 \in V_1$ and $v_2 \in V_2$ such that for every $A \in \mathscr{A}$, we have $v_1, v_2 \in A$.

Consider a projectoidic arrangement $(u_0, \ldots, u_{|V_1|})$ of $\langle V_1 \cup \{v_2\}\rangle_{\mathscr{H}}$, and a projectoidic arrangement $(w_0, \ldots, w_{|V_2|})$ of $\langle V_2 \cup \{v_1\}\rangle_{\mathscr{H}}$. It is clear that without loss of generality we may assume that $u_{|V_1|} = v_2$ and $w_0 = v_1$. It is not difficult to see that

$(u_0, \ldots, u_{|V_1|-1}, w_1, \ldots, w_{|V_2|})$ is a projectoidic arrangement of $\mathscr{H}$. Hence, $\mathscr{H}$ is a projectoid, which completes the proof of Theorem 1.

2. Let $D = (V, A)$ be a digraph in the sense of [4]. For every $v \in V$, we denote by $R(v, D)$ the set of vertices which are reachable form v (in D). Obviously, $w \in R(w, D)$, for each $w \in V$. Denote

$$\mathscr{R}(D) = \{R(v, D);\ v \in V\} .$$

We denote by $[D]$ the graph obtained from D in such a way that each arc (u, v) is replaced by the edge $\{u, v\}$. If $u, v, w \in V$, then we shall say that v is (u, w)-reachable (in D) if for every path P (in the sense of [3]) which connects u with w in [D], there exists a vertex t_P belonging to P and such that $v \in R(t_P, D)$.

Let $D = (V, A)$ be a digraph. Denote $|V| = n$. Consider a sequence $(v_1, \ldots, v_n)$ such that $\{v_1, \ldots, v_n\} = V$. We shall say that the sequence $(v_1, \ldots, v_n)$ is a *projective arrangement* of D if it is a projectoidic arrangement of the hypergraph $(V, \mathscr{R}(D))$. The term "projective" in the sense of the present paper has its origin in mathematical linguistics, namely in studying sentence structures. For some furher details the reader is referred to [5].

We shall say that a digraph D is a *project* if there exists a projective arrangement of D. It is obvious that a digraph $D = (V, A)$ is a project if and only if $(V, \mathscr{R}(D))$ is a projectoid. For example, every out-tree is a project. There exists exactly one digraph with less than five vertices which is not a project; it is the in-tree T with the property that $[T]$ is the star $K_{1,3}$.

The proof of the following proposition is easy (cf. the proof of Theorem 3.2 in [5]).

Proposition 5. *Let $(v_1, \ldots, v_n)$ be a projective arrangement of a project D. Then for any three integer i, j, and k, $1 \leqq i \leqq j \leqq k \leqq n$, v_j is (v_i, v_k)-reachable.*

The following theorem is a solution of the problem which was stated by the present author at Czechoslovak Graph Theory Conference held in Brno, May 1975:

Theorem 2. *Let $D = (V, A)$ be a digraph. Then it is a project if and only if for any $v_1, v_2, v_3 \in V$, there exists a permutation p on $\{1, 2, 3\}$ such that $v_{p(2)}$ is $(v_{p(1)}, v_{p(3)})$-reachable.*

Proof. One of the implications in the statement of Theorem 2 follows immediately from Proposition 5. We shall prove the other one.

Let D not be a project. Then $(V, \mathscr{R}(D))$ is not a projectoid. According to Theorem 1, there exist distinct $S_1, S_2, S_3 \in \Omega((V, \mathscr{R}(D))$ such that $\langle\{S_1, S_2, S_3\}\rangle$ is not a projectoid. We distinguish two cases:

(1) Assume that the set $S_1 - (S_2 \cup S_3)$, $S_2 - (S_3 \cup S_1)$, and $S_3 - (S_1 \cup S_2)$ are nonempty. Consider $v_1 \in S_1 - (S_2 \cup S_3)$, $v_2 \in S_2 - (S_3 \cup S_1)$, and $v_3 \in S_3 - (S_1 \cup S_2)$. Since $S_1, S_2, S_3 \in \Omega((V, \mathscr{R}(D))$, we have that v_i, where $i = 1, 2, 3$, is not reachable from any vertex in S_j, where $j = 1, 2, 3$ and $j \neq i$. Since $\langle\{S_1, S_2, S_3\}\rangle$ is not a projectoid, it follows from Proposition 4 that $S_1 \cap S_2$, $S_2 \cap S_3$, and $S_3 \cap S_1$

are nonempty. This means that in $[D]$ there exist paths P_{12}, P_{23}, and P_{31} which connect v_1 with v_2, v_2 with v_3, and v_3 with v_1, respectively, such that each vertex of P_{12}, P_{23}, and P_{31}, belongs to $S_1 \cup S_2$, $S_2 \cup S_3$, and $S_3 \cup S_1$, respectively. Hence, for any permutation p on $\{1, 2, 3\}$, $v_{p(2)}$ is not $(v_{p(1)}, v_{p(3)})$-reachable.

(2) Assume that at least one of the sets $S_1 - (S_2 \cup S_3)$, $S_2 - (S_3 \cup S_1)$, and $S_3 - (S_1 \cup S_2)$ is empty. Without loss of generality we assume that $S_1 \subseteq S_2 \cup S_3$. If $S_1 \cap (S_2 - S_3) = \emptyset$ or $S_1 \cap (S_3 - S_2) = \emptyset$, then $S_1 \subseteq S_3$ or $S_1 \subseteq S_2$, respectively, and therefore, $\langle\{S_1, S_2, S_3\}\rangle$ is a projectoid, which is a contradiction. This means that $S_1 \cap (S_2 - S_3) \neq \emptyset \neq S_1 \cap (S_3 - S_2)$. It follows from Proposition 4 that $(S_2 \cap S_3) - S_1 \neq \emptyset$. Consider $v_{12} \in S_1 \cap (S_2 - S_3)$, $v_{13} \in S_1 \cap (S_3 - S_2)$, and $v_{23} \in (S_2 \cap S_3) - S_1$. It is clear that v_{12}, v_{13} and v_{23} are reachable from no vertex in S_3, S_2 and S_1, respectively. There exist paths P_1, P_2, and P_3 which connect v_{12} with v_{13}, v_{12} with v_{23}, and v_{13} with v_{23}, respectively, such that each vertex of P_1, P_2, and P_3, belongs to S_1, S_2, and S_3, respectively. Hence, for any permutation p on $\{1, 2, 3\}$, $v_{p(2)}$ is not $(v_{p(1)}, v_{p(3)})$-reachable.

Thus the proof of Theorem 2 is complete.

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