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NUMERICAL MODELLING OF TUBE FIXING IN THE HEAT EXCHANGER OF A NUCLEAR POWER PLANT

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1. INTRODUCTION

The work deals with numerical solution of problems which arise in the explosive tube fixing in the heat exchanger of the nuclear power plant [1], [2], see Fig. 1.

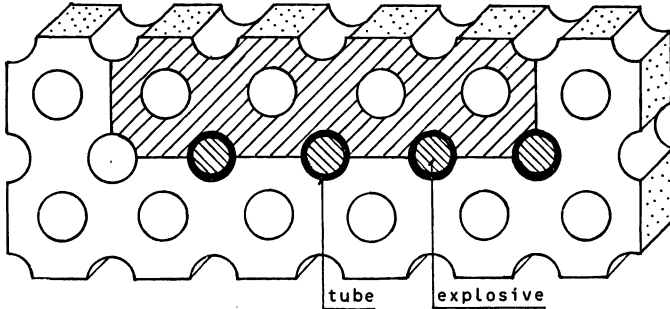


Fig. 1 Part of heat exchanger

In order to reduce the computational cost two simplifying assumptions were made: only a small section of the exchanger wall was considered; twodimensional behaviour was considered (the ratio of the size of the section to the diameter of the exchanger is small). For the computation the shaded domain Ω with the boundary Γ in Fig. 1 was taken with boundary conditions indicated in Fig. 2.

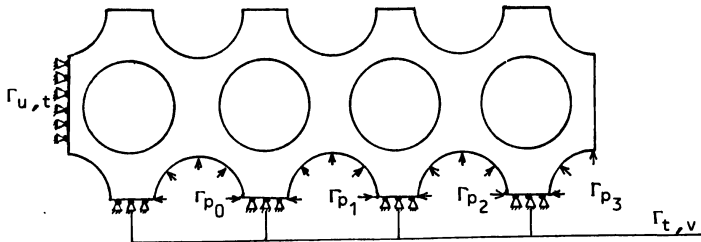


Fig. 2 Computational domain Ω

The influence of boundary conditions (depending on explosive behaviour) on the stress field was studied for simpler geometry. The results were used to assess time dependent stress field for various variants (different spacing of holes, enlarged holes) of the configuration in Fig. 1. Tension in material was evaluated to detect domains of possible damage.

2. BASIC EQUATIONS

This dynamical elastoplastic problem is described by nonlinear partial differential equations of the second order, hyperbolic type [3]

$$\sigma_{ij,j} = \rho \ddot{u}_i \text{ in } \Omega \times I, \quad i=1,2 \quad (1)$$

with boundary conditions

$$\sigma_{ij} n_j = p(t-kd) \text{ on } \Gamma_{p_k} \times I, k=0,1,2,3 \quad (1.a)$$

$$u_1 = 0, \sigma_{2j} n_j = 0 \text{ on } \Gamma_{u,t} \times I \quad (1.b)$$

$$u_2 = 0, \sigma_{1j} n_j = 0 \text{ on } \Gamma_{t,u} \times I \quad (1.c)$$

$$\sigma_{ij} n_j = 0 \text{ on } (\Gamma - \Gamma_{u,t} - \Gamma_{t,v} - \bigcup_k \Gamma_{p_k}) \times I, i=1,2 \quad (1.d)$$

($I=[0,T]$, σ_{ij} is the stress, ϵ_{ij} is the strain, u_i is the displacement, $i=1,2$; ρ is mass density and p is the time dependent normal load, d is the time delay ($3\mu\text{s}$ in our case)) and initial conditions

$$u_i(x,0) = 0, \dot{u}_i(x,0) = 0 \text{ in } \Omega, i=1,2. \quad (1.e)$$

We shall solve (1) by finite element method [3], [4], where we can use the Principle of Virtual Work and the d'Alembert's principle:

$$\int_{\Omega} \sigma_{ij}^n \delta \epsilon_{ij}^n d\Omega + \int_{\Omega} \rho \ddot{u}_i^n \delta u_i^n d\Omega - \sum_{k=0}^3 \int_{\Gamma_{p_k}} \delta u_i^n p^n n_i d\Gamma = 0. \quad (2)$$

In (2) a superscript n refers to a quantity sampled at time $t = t_n$.

Using standard C^0 4-nodes isoparametric or 8-nodes serendipity finite elements [5] for spatial discretization we evaluate contributions from each element of the discretization of Ω_h of the domain Ω and then assemble them into the appropriate matrix and vectors and we obtain the equations in matrix form so that at time t_n we have

$$M \ddot{d}^n + q^n = f^n, \quad (3)$$

where M is the global mass matrix, q^n is the global vector of internal resisting nodal forces, f^n is the vector of boundary loading, \ddot{d}^n is the global vector of nodal accelerations.

For the material an elastoplastic behaviour was supposed with von Mises yield criterion and isotropic hardening and the standard

incremental method [4] with the initial stiffness method [5] was used. The mass matrix was lumped and the Gauss-Legendre rule was used for numerical integration.

The central difference approximation was adopted for temporal discretization leading to the explicit scheme:

$$d^{n+1} = M^{-1} [\Delta t^2 (-q^n + f^n) + 2Md^n - Md^{n-1}] \quad (4)$$

with the modified Courant-Fridrichs-Levy stability condition for time step length [5]:

$$\Delta t \leq \mu h_m [\rho(1+\nu)(1-2\nu) / (E(1-\nu))]^{1/2}, \quad (5)$$

where h_m is the smallest length between any two nodes in the spatial discretization, ν is Poisson's ratio, E is Young's modulus and μ is a coefficient dependent on the type of element employed.

The theory in [3], [4] and [9] implies that, for the sufficiently smooth function p and the maximum size h converging to zero in the spatial discretization, the numerical solution of (1), i.e. the nodal displacement vector d^n , converges to the solution u_i of (1).

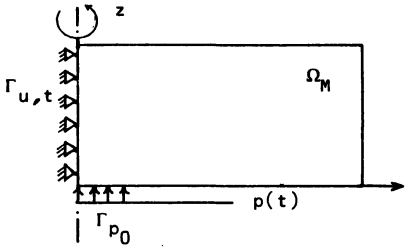
The solution of the problem (1) depends on the boundary conditions (1.a), which are usually obtained only from experiment and it is necessary to approximate them with sufficient accuracy.

3. APPROXIMATION OF BOUNDARY CONDITIONS

The boundary condition (1.a) - the function $p(t)$ defined on the interval I and equal to zero on the subinterval $[T_1, T]$ - is measured only on the subinterval $I_0 = [0, T_0]$, $T_0 < T_1$. Therefore it is necessary to extrapolate the function $p(t)$ to the subinterval $I_1 = [T_0, T_1]$ in a sufficiently accurate way to solve (1) by FEM correctly. A rough approximation of $p(t)$ in I_1 could lead to significant errors in the numerical solution of (1). This complicated problem can be solved only in cooperation with engineers and on a simpler model. Then, for the approximation of the function $p(t)$, the results of the experimental measurements described below can be used. In our case, the results were obtained in the following way. In the centre of the lower end of the standard model cylinder sample an explosive was placed and after explosion the time dependent load acting on the cylinder and the time dependent deformation in the centre of the opposite end of the sample were measured. The sample was then cut into pieces and domains of the plastic zone and damage, i.e. voids or cracks, were evaluated by means of the fractographic analysis.

The behaviour of the experimental model mentioned above is des-

cribed by the dynamic axisymmetric elastoplastic problem, similar to (1), which can be solved on a cross-section shown in Fig. 3, with the boundary condition (1.a), ($k=0$) on Γ_{p_0} , the boundary condition (1.b) p_0 on $\Gamma_{u,t}$ and the boundary condition (1.d) on $\Gamma_M - \Gamma_{u,t} - \Gamma_{p_0}$, where Γ_M is the boundary of Ω_M .



This twodimensional dynamic elastoplastic problem was solved by FEM in the same way as the problem (1). For the spatial discretization 300 8-nodes serendipity isoparametric elements with 975 nodes were used.

Fig. 3 Computational domain Ω_M

In Fig. 4 we present the first step and the last two steps of the heuristic optimization algorithm [7]. In this algorithm we minimize the differences between the experimental values described above and the values obtained from the numerical solution by means of FEM.

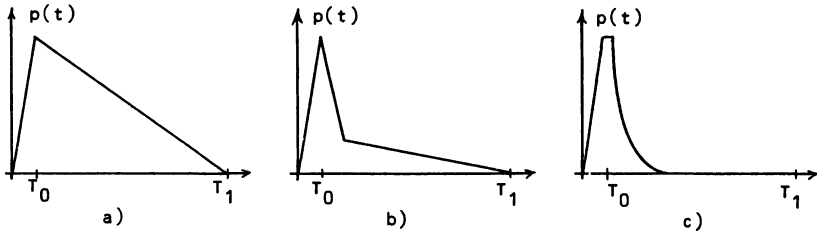


Fig. 4 Approximation of the function $p(t)$

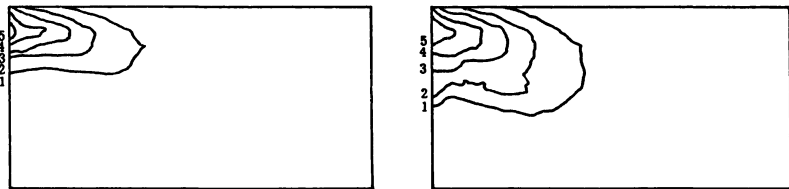


Fig. 5 Contour Lines of σ_{zz} at $7 \mu s$

Fig. 5 illustrates, for the cases b), c) of Fig. 4, the contour lines of σ_{zz} at times when it reaches its maximum. Only tensile values are taken into account since they cause the observed damage in the form of horizontal microcracks. The values refer to the multiples of $1/2\sigma_s$, where σ_s is the static yield stress.

4. DISCUSSION OF NUMERICAL RESULTS

The numerical solution of (1) with the sufficiently accurate approximation of (1.a) discussed in the previous section makes possible the correct assessment of the safety of the technology of explosive fixing of tubes in the heat exchanger of a power plant, [2]. The following Fig. 6 shows contour lines of the equivalent stress field, at time instant $4 \mu s$, for different spacing of holes (representing the geometry at the outer and inner surface of the cylindrical vessel, respectively) and for an enlarged hole (some holes must be re-drilled in the production process due to e.g. a broken drill). These results are based on computations using 2048 elements with 2317 nodes.

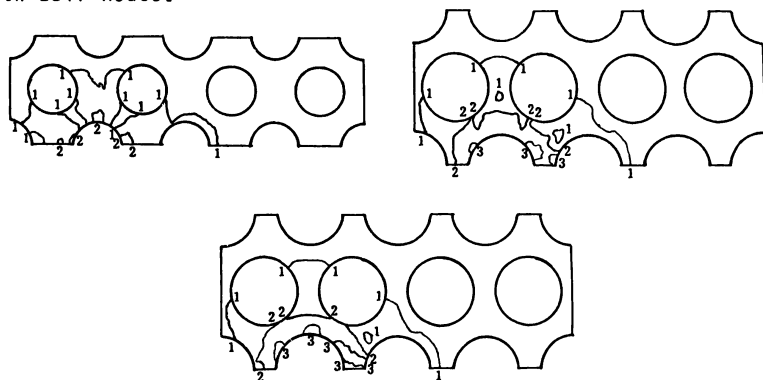


Fig. 6 Contour lines of equivalent stress (multiples of σ_s)

Fig. 6 shows the unfavourable influence of the decreasing spacing of holes on the concentration of tensile stresses near the holes' surfaces.

5. CONCLUSIONS AND REMARKS

It is shown in [7] and [8] that numerical modelling by FEM using results of physical experiments [1] can be very efficient for the solution of technological problems such as choosing the explosive parameters (modification of boundary conditions (1.a)) and suitable timing of the explosive sequence [8] in order to avoid local cracking or intolerable material damage around the tubes. The numerical experiments, however, must be based on correct verification on a simpler experimental model. Due to strong nonlinearity and complexity of the problem solved analytical solution is impossible and even the error analysis of the numerical approximation is

hardly possible due to a considerable number of simplifications and idealizations.

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