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In: Michal Greguš (ed.): Equadiff 5, Proceedings of the Fifth Czechoslovak Conference on Differential Equations and Their Applications held in Bratislava, August 24-28, 1981. BSB B.G. Teubner Verlagsgesellschaft, Leipzig, 1982. Teubner-Texte zur Mathematik, Bd. 47. pp. 297--300.

Persistent URL: http://dml.cz/dmlcz/702310

ON THE METHOD OF THE SOLUTION OF ELASTICITY THEORY PROBLEM FOR THE INHOMOGENEOUS AND ANISOTROPIC BODY

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The problems of the theory of elasticity for an anisotropic and inhomogeneous body has been investigated by Sent-Venan, Foight, S.G. Lekhnitsky, S.A. Ambartsumyan, V.A. Lomakin, G.B. Coltchin and oth.

The detailed survey of the problem one can find in works [1] - [5].

In the present paper the methods of the solution of two class of problems for a common-type anisotropic and inhomogeneous body is suggested.

1. We shall consider a generalized torsion of the cylindrical prismatic homogeneous bar with rectilinear common-type anisotropy (21 or 18 elastic constants) [1]. Here is assumed, that the bar is in equillibrium on the action of a load distributed on the foreheads and torsional moment M_t. It is assumed too, that the lateral surface is free from an external force and volume forces are neglected. For the sake of simplicity the cross-section domain (Ω) assumed to be bounded and simple-connected.

Then the components of stress tensor can be founded by the formula

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y},$$
$$\tau_{xz} = \frac{\partial \Psi}{\partial y}, \quad \tau_{yz} = -\frac{\partial \Psi}{\partial x}, \tag{1.1}$$
$$\sigma_z = \frac{M_t}{a_{33}}\left(\frac{a_{34}}{I_2}x - \frac{a_{35}}{I_1}y\right) - \frac{1}{a_{33}}\left(a_{13}\sigma_x + \cdots + a_{36}\tau_{xy}\right),$$

provided functions F (x,y) and Ψ (x,y) to be known. These function satisfy the following system of equations.

$$L_4 F + L_3 \Psi = 0$$
$$L_3 F + L_2 \Psi = -2\vartheta + \frac{M_t}{2a_{33}}\left(\frac{a_{34}^2}{I_2} + \frac{a_{35}^2}{I_1}\right), \tag{1.2}$$

with the boundary conditions on Γ contour of Ω domain

$$-\quad \frac{\partial F}{\partial x}\Big|_\Gamma = 0, \quad \frac{\partial F}{\partial y}\Big|_\Gamma = 0, \quad \Psi\Big|_\Gamma = 0 . \tag{1.3}$$

Here L_4, L_3, L_2 are elliptic-type differential operators

$$L_4 = \beta_{22}\frac{\partial^4}{\partial x^4} - 2\beta_{26}\frac{\partial^4}{\partial x^3 \partial y} + (2\beta_{12}+\beta_{66})\frac{\partial^4}{\partial x^2 \partial y^2} - 2\beta_{16}\frac{\partial^4}{\partial x \partial y^3} + \beta_{11}\frac{\partial^4}{\partial y^4}$$

$$L_3 = -\beta_{24}\frac{\partial^3}{\partial x^3} + (\beta_{25}+\beta_{46})\frac{\partial^3}{\partial x^2 \partial y} - (\beta_{14}+\beta_{56})\frac{\partial^3}{\partial x \partial y^2} + \beta_{15}\frac{\partial^3}{\partial y^3}, \tag{1.4}$$

$$L_2 = \beta_{44}\frac{\partial^2}{\partial x^2} - 2\beta_{45}\frac{\partial^2}{\partial x \partial y} + \beta_{55}\frac{\partial^2}{\partial y^2},$$

where β_{ij} - elastic constants satisfy well-known unequallity [4]

$$\beta_{ii} > 0, \quad \beta_{ii}\beta_{jj} - \beta_{ij}^2 > 0, \quad (i,j = 1,2,4,5,6). \tag{1.5}$$

Further we assume that

$$\beta_{14} = \epsilon\beta_1, \quad \beta_{24} = \epsilon\beta_2, \quad \beta_{34} = \epsilon\beta_3, \quad \beta_{46} = \epsilon\beta_4, \quad \beta_{15} = \epsilon\beta_5, \tag{1.6}$$
$$\beta_{25} = \epsilon\beta_6, \quad \beta_{35} = \epsilon\beta_7, \quad \beta_{56} = \epsilon\beta_8, \quad (\epsilon < 1),$$

where ϵ is a small parameter and $\epsilon = 0$ if the plane of cross-section is a plane of elastic symmetry.

Then the system of differential equations (1.2) may be written as

$$L_4 F + \epsilon \tilde{L}_3 \Psi = 0 \tag{1.7}$$
$$\epsilon \tilde{L}_3 F + L_2 \Psi = -2\vartheta + \epsilon^2 \tilde{A},$$

where following notations are used

$$\tilde{L}_3 = -\beta_2\frac{\partial^3}{\partial x^3} + (\beta_6+\beta_4)\frac{\partial^3}{\partial x^2 \partial y} - (\beta_1+\beta_8)\frac{\partial^3}{\partial x \partial y^2} + \beta_5\frac{\partial^3}{\partial y^3}, \tag{1.8}$$

$$\tilde{A} = \frac{M_t}{2}\left(\frac{\beta_9^2}{I_2} + \frac{\beta_{10}^2}{I_1}\right), \quad a_{34} = \epsilon\beta_9\sqrt{a_{33}}, \quad a_{35} = \epsilon\beta_{10}\sqrt{a_{33}} .$$

The solution of (1.7) is expanded in power series in small physical paramater ϵ as follows

$$\Psi = \Psi_0 + \epsilon\Psi_1 + \epsilon^2\Psi_2 + \cdots \tag{1.9}$$
$$F = F_0 + \epsilon F_1 + \epsilon^2 F_2 + \cdots \tag{1.10}$$

Then for the determination of $F_k(x,y)$, $\Psi_k(x,y)$ functions from equation (1.3) and (1.7) one can obtain the follow system of recurrent boundary problems

$$L_4 F_0 = 0 , \quad (x,y) \in \Omega ,$$
$$\frac{\partial F_0}{\partial x} = \frac{\partial F_0}{\partial y} = 0 , \quad (x,y) \in \Gamma ; \tag{1.11}$$
$$L_4 F_k = f_k(x,y) , \ \left(f_k = -\tilde{L}_3 \Psi_{k-1} \right) , \ (x,y) \in \Omega , \tag{1.12}$$
$$\frac{\partial F_k}{\partial x} = \frac{\partial F_k}{\partial y} = 0 , \ (k = 1,2,3,\cdots) , \ (x,y) \in \Gamma ;$$
$$L_2 \Psi_k = g_k(x,y) , \ (x,y) \in \Omega , \tag{1.13}$$
$$\Psi_k = 0 , \quad (x,y) \in \Gamma , \qquad (k = 1,2,3 \cdots)$$

where
$$g_0 = -2\vartheta , \ g_2 = -L_3 F_1 + \tilde{A} , \ g_k(x,y) = -L_3 F_{k-1} , \ (k=1,2,3,\dots). \tag{1.14}$$

It follows from (1.11) that $F_0 = 0$. Thus, the solution of the problem of a generalized torsion of a homogeneous bar with a very common-type anisotropy reduce to a number of problems similar to problems of bars torsion (1.13) and plane problem for a body with special-type anisotropy (with one plane of symmetry at each point-13 or 12 elastic constants).

In view of Sobolev's theorem [6] and some results of works [4,7] following theorems can be proved.

Theorem 1. There exists a solution of the boundary problem (1.13) of the form (1.9). If $\xi < \xi_0$ the series (1.9) converge absolutely and uniformly in x,y on Ω domain, the first derivative of (1.9) in x,y converge in every $L_p(\Omega)$, $(p>1)$ and the second derivative - in $L_2(\Omega)$ only.

Theorem 2. If $\xi < \xi_0$ then the solution of the problem (1.12) have the form (1.10). The first and second derivatives of this solution converge uniformly in Ω domain by x,y, the third derivative - in any $L_p(\Omega)$ $(p>1)$, and the forth - in $L_2(\Omega)$ only.

Here ξ_0 is some constant depending on coefficients and constants entering in Sobolev's theorems.

2. We consider now a generalized torsion of anisotropic bar with a weak inhomogenity [4,7]
$$\beta_{ij} = \beta_{ij}^{0} \left[\alpha_{ij} + \delta f_{ij}(x,y) \right] , \tag{2.1}$$
where β_{ij}^{0} are constants, $f_{ij}(x,y)$ - continuous known functions of x,y with second order dervatives at least, δ - some small physical parameter. Then the solution of the problem we seek as a power series in δ as follows.

$$F = F_0 + \delta F_1 + \delta^2 F_2 + \cdots$$
$$\Psi = \Psi_0 + \delta\Psi_1 + \delta^2\Psi_2 + \cdots$$

In order to determinate F_k, Ψ_k functions we obtain the system of recurrent boundary problems, corresponding to homogeneous ones. These problems can be solved using the methods mentioned above.

Finally we note, there are convergence theorems for F, Ψ series on the analogy of 1 section.

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