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HASSE DIAGRAMS FOR PARABOLIC GEOMETRIES

LUKÁŠ KRUMP AND VLADIMÍR SOUČEK

1. Introduction

Invariant differential operators on manifolds with a given parabolic structure have been intensively studied last years. General introduction to the concept of parabolic geometries can be found in [12]. Construction of basic data of a parabolic geometry from a given geometrical data is described in [5].

The motivating example of parabolic geometry is conformal geometry. A systematic discussion of conformally invariant differential operators can be found, for example, in [13]. A general overview of basic facts on invariant operators in the case of a general parabolic geometry is available in [14].

Invariant operators can be divided into two basic classes, standard and nonstandard. Another possible division is into regular or singular cases. Standard regular operators are coming in repeated patterns called Bernstein-Gelfand-Gelfand (BGG) sequences. In flat case, these sequences are in fact complexes. These complexes were constructed first in homogeneous case in terms of the dual language of homomorphisms of (generalized) Verma modules, see [1], [2]. A geometrical construction of BGG sequences in a curved case was given in [9] and in [3]. Explicit form of many of regular standard operators is described in [4].

Every BGG sequence has a pattern characteristic for considered parabolic geometry. This pattern is given by the corresponding Hasse diagram. It is hence useful to have at hand efficient means how to compute the form of the Hasse diagram in individual cases. The main aim of the paper is to describe an alternative way how to characterize Hasse diagrams and to give its more efficient description in the case of low gradings. The most important case is the case of AHS structures ([6], [7], [8]), i.e., the $|1|$-graded case.

2. Notation

This section introduces notation used in the paper. The basic data for a parabolic geometry is a simple Lie algebra $\mathfrak{g}$ with a Cartan subalgebra $\mathfrak{h}$ and a $|k|$-grading $\mathfrak{g} = \mathfrak{g}_{-k} \oplus \ldots \oplus \mathfrak{g}_k$. There is a decomposition of the subalgebra $\mathfrak{g}_0 = \mathfrak{g}_0^s \oplus \mathfrak{g}_0'$, where $\mathfrak{g}_0^s$

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is semisimple with Cartan subalgebra $\mathfrak{h}^s$. Let $\mathfrak{p} = \mathfrak{g}_0 \oplus \ldots \oplus \mathfrak{g}_k$ denote the corresponding parabolic subalgebra of $\mathfrak{g}$ and $\mathfrak{p}_+ = \mathfrak{g}_1 \oplus \ldots \oplus \mathfrak{g}_k$ its positive part.

If $\mathfrak{g}'$ is a subalgebra of $\mathfrak{g}$, the symbol $\Delta(\mathfrak{g}')$ denotes all roots such that their root spaces belong to $\mathfrak{g}'$. The symbols Δ^+, Δ^- denote the positive and negative roots of the whole graded Lie algebra $\mathfrak{g}$.

The reflection $\sigma_\alpha(\beta)$ with respect to a root α is given by $\sigma_\alpha(\beta) = \beta - \langle \beta, \check{\alpha} \rangle \alpha$, with $\check{\alpha} = \frac{2\alpha}{|\alpha|^2}$. The Weyl group W of $\mathfrak{g}$ is generated by the reflections σ_{α_i} for α_i simple roots. As usually, we denote by δ the half sum of all positive roots of $\mathfrak{g}$.

3. Three alternative descriptions of the Hasse diagram

The main aim of this section is to reformulate the standard definition of the Hasse diagram for a chosen parabolic subalgebra in an alternative way, which is more suitable for computations. The Hasse diagram is a labeled graph (with labels in a set Δ_+ of positive roots). Throughout the section, let a semisimple Lie algebra $\mathfrak{g}$ and its Weyl group W be fixed. For $w \in W$, the symbol $|w|$ denotes the length of w, i.e. the least integer k such that $w = \sigma_{\alpha_1} \circ \cdots \circ \sigma_{\alpha_k}$, where α_i are simple roots.

Let us recall first the classical definition of the Hasse diagram for a given simple Lie algebra $\mathfrak{g}$, which can be found in [1].

3.1. Hasse diagram of the Weyl group.

Definition 3.1. The Hasse diagram $H \equiv H(\mathfrak{g})$ of the Lie algebra $\mathfrak{g}$ is a labeled oriented graph with labels in Δ^+. The set of its vertices is the Weyl group W. Let $w, w' \in W$ are two vertices of H. There is an arrow from w to w' labeled by $\alpha \in \Delta^+$, if $w' = \sigma_\alpha \circ w$ and $|w'| = |w| + 1$.

The following simple lemma will be useful below.

Lemma 3.2. *Let $w, w' \in W$, $\alpha \in \Delta^+$. Then the following conditions are equivalent:*

(1) $w' = \sigma_\alpha \circ w$,
(2) $w'(\mu) = \sigma_\alpha(w(\mu))$ *for all weights* μ *of* $\mathfrak{g}$,
(3) $w'(\delta) = \sigma_\alpha(w(\delta))$,
(4) *there exists an integer* c_α *such that* $w'(\delta) - w(\delta) = c_\alpha \alpha$; *the number* c_α *is then given by* $c_\alpha = \langle \check{\alpha}, \delta \rangle$.

Proof. (1) $\iff$ (2) $\iff$ (3) $\implies$ (4) is trivial. To prove (4) $\implies$ (3), note that $|w'(\delta)| = |w(\delta)|$. Difference between these two elements is a multiple of α, hence $w'(\delta) = \sigma_\alpha(w(\delta))$. □

3.2. The Hasse diagram of the (∗)-saturated sets.

Definition 3.3. A subset Φ of Δ^+ is called saturated, if $\alpha, \beta \in \Phi$, $\alpha + \beta \in \Delta^+$ implies $\alpha + \beta \in \Phi$. We shall say that Φ satisfies the property (∗) (or is (∗)-saturated) if both Φ and $\Phi^c = \Delta^+ - \Phi$ are saturated.

For $w \in W$, let $\Phi_w := \Delta^+ \cap w(\Delta^-)$. It is well known that a set $\Phi \subset \Delta^+$ is (∗)-saturated iff there exists $w \in W$ with $\Phi = \Phi_w$. Moreover, $|w| = \mathrm{card}(\Phi_w)$ (see [10]).

Definition 3.4. Define the Hasse diagram $G(W)$ of the $(*)$-saturated sets as follows. The set of vertices of $G(W)$ is the set of all $\Phi_w, w \in W$. There is a labeled arrow $\Phi_w \xrightarrow{\alpha} \Phi_{w'}, \alpha \in \Delta^+$, if the difference

$$\sum_{\beta \in \Phi_{w'}} \beta - \sum_{\gamma \in \Phi_w} \gamma$$

is a multiple of α and $|Ph_{w'}| > |Ph_w|$.

The following lemma can be found in [2]:

Lemma 3.5. *For every $w \in W$,*

$$w(\delta) = \delta - \sum_{\beta \in \Phi_w} \beta.$$

Theorem 3.6. *Let W be the Weyl group of $\mathfrak{g}$. Then the graphs $H(W)$ and $G(W)$ are equivalent as Δ^+-labeled graphs. More precisely, the mapping $w \mapsto \Phi_w$ is a bijection and for every $w, w' \in W$, $\alpha \in \Delta^+$, we have*

$$w \xrightarrow{\alpha} w' \iff \Phi_w \xrightarrow{\alpha} \Phi_{w'}.$$

Proof. Recall first that $\mathrm{card}(\Phi_{w'}) = \mathrm{card}(\Phi_w) + 1$ iff $|w'| = |w| + 1$.

Suppose first that there is an arrow $\Phi_w \xrightarrow{\alpha} \Phi_{w'}$, i.e. that the difference $\sum_{\beta \in \Phi_{w'}} \beta - \sum_{\gamma \in \Phi_w} \gamma$ is a multiple of α. Then by Lemma 3.5, the same is true also for $w'(\delta) - w(\delta)$ and it is sufficient to use Lemma 3.2.

On the other hand, suppose that $w' = \sigma_\alpha \circ w$ for some $w, w' \in W$. Then clearly $\sum_{\beta \in \Phi_w} \beta - \sum_{\beta \in \Phi_{w'}} \beta = w'(\delta) - w(\delta)$ is a multiple of α. □

It is useful to reformulate the condition for an arrow between two nodes in the Hasse diagram of $(*)$-saturated sets in a more suitable way. To do so, we first prove the following lemma.

Lemma 3.7. *Suppose that $\alpha, \beta \in \Delta^+$ and $\omega' = \sigma_\alpha \circ w$. Then:*

1. *if $\beta \in \Phi_w$, $\alpha \notin \Phi_w$ and $\sigma_\alpha(\beta) \in \Delta^-$, then $\beta \in \Phi_{w'}$;*
2. *$\alpha \in \Phi_w \cup \Phi_{w'}$ and $\alpha \notin \Phi_w \cap \Phi_{w'}$;*
3. *if $\alpha \in \Phi_{w'}$, then $\sigma_\alpha(\Phi_w - \Phi_{w'}) \subset (\Phi_{w'} - \{\alpha\}) - \Phi_w$;*
4. *$\alpha \in \Phi_w$ if and only if $|w'| > |w|$.*

Proof. (1) We know that $(w)^{-1}(\beta) \in \Delta_-$; $(w)^{-1}(\alpha) \in \Delta_+$ and we suppose that $\sigma_\alpha(\beta) \in \Delta^-$. Hence $\sigma_\alpha(\beta) = \beta - k\alpha$ with $k > 0$. Then

$$(w')^{-1}(\beta) = w^{-1}(\beta) - k\, w^{-1}(\alpha) \in \Delta^-.$$

(2) It follows immediately from

$$(\omega')^{-1}(\alpha) = w^{-1}(\sigma_\alpha(\alpha)) = -w^{-1}(\alpha).$$

(3) Consider $\beta \in (\Phi_w - \Phi_{w'})$. Hence $w^{-1}(\beta) \in \Delta_-$ and $(w')^{-1}(\beta) \in \Delta_+$. Consequently,

$$w^{-1}(\sigma_\alpha(\beta)) = (w')^{-1}(\beta) \in \Delta_+;\ (w')^{-1}(\sigma_\alpha(\beta)) = w^{-1}(\beta) \in \Delta_-.$$

Note now that if $\sigma_\alpha(\beta) \in \Delta_-$, then $\beta \in \Phi_{w'}$ by (1), which is a contradiction. Hence both β and $\sigma_\alpha(\beta)$ belong to Δ_+. But then clearly $\sigma_\alpha(\beta) \notin \Phi_w$ and $\sigma_\alpha(\beta) \in \Phi_{w'}$.
(4) From (3), it follows that if $\alpha \in \Phi_{w'}$, then $|\Phi_w - \Phi_{w'}| \leq |(\Phi_{w'} - \{\alpha\}) - \Phi_w|$. Hence $|\Phi_w| < |\Phi_{w'}|$. Using the same for w and w' interchanged, we get the claim that $|w'| > |w|$ iff $\alpha \in \Phi_{w'}$. □

Now we are able to prove the following theorem.

Theorem 3.8. *There is a labeled arrow $w \xrightarrow{\alpha} w'$, $\alpha \in \Delta^+$ iff $\alpha \in \Phi_{w'}$ and*

$$(\Phi_{w'} - \{\alpha\}) - \Phi_w = \sigma_\alpha(\Phi_w - \Phi_{w'}).$$

Proof. If $w \xrightarrow{\alpha} w'$, then $w' = \sigma_\alpha \circ w$, $\alpha \in \Phi_{w'}$ and $\sigma_\alpha(\Phi_w - \Phi_{w'}) \subset (\Phi_{w'} - \{\alpha\}) - \Phi_w$. Moreover we know that $|\Phi_{w'}| = |\Phi_w| + 1$. Hence

$$(\Phi_{w'} - \{\alpha\}) - \Phi_w = \sigma_\alpha(\Phi_w - \Phi_{w'}).$$

On the other hand, if the above two sets coincide and $\alpha \in \Phi_{w'}$, then clearly the difference $\sum_{\beta\in\Phi_{w'}} \beta - \sum_{\gamma\in\Phi_w} \gamma$ is a multiple of α, and we can use the previous theorem to get that $w \xrightarrow{\alpha} w'$. □

The condition in the theorem can be also reformulated in the following way. There is a labeled arrow $\Phi_w \xrightarrow{\alpha} \Phi_{w'}$ if and only if $\alpha \in \Phi_{w'}$ and there exists a number $k \geq 0$ and positive roots γ_i, δ_i for $1 \leq i \leq k$ such that

$$\Phi_{w'} - \Phi_w = \{\alpha, \delta_1, \dots \delta_k\},$$
$$\Phi_w - \Phi_{w'} = \{\gamma_1, \dots \gamma_k\},$$

and

$$\sigma_\alpha(\gamma_i) = \delta_i$$

for every $1 \leq i \leq k$. But if the reflection σ_α acts in an nontrivial way on $\beta \in \Delta^+$, it changes the grading of β. Hence we get immediately the following corollary.

3.3. **Weight graphs and Hasse graphs for parabolic subalgebras.** In this part, we want to find a suitable characterization of sets Φ_w in the Hasse diagrams for parabolic subalgebras useful for practical applications. Let us first recall the definition of the Hasse diagram for parabolic subalgebras bigger than the Borel subalgebra.

Let $\mathfrak{p} = \mathfrak{g}_0 + \dots + \mathfrak{g}_k$ be a parabolic subalgebra of $\mathfrak{g}$. The Hasse diagram $H(\mathfrak{p}, \mathfrak{g})$ for $\mathfrak{p}$ is the subgraph of the Hasse diagram $H(\mathfrak{g})$ obtained from $H(\mathfrak{g})$ by deleting all vertices w with $\Phi_w \not\subset \Delta(\mathfrak{p}_+)$. The notion of the weight graphs will be a useful tool.

Definition 3.9. The weight graph for $\mathfrak{p}$ is defined in the following way:

- the set of its vertices is the set of all roots $\beta \in \Delta(\mathfrak{p}_+)$,
- there is an arrow $\beta_1 \xrightarrow{\alpha_i} \beta_2$ if and only if there exists a simple root α_i in $\Delta(\mathfrak{g}_0)$ such that $\beta_2 = \beta_1 - \alpha_i$.

Definition 3.10. A subset $\mathcal{V} \subset \Delta(\mathfrak{p}_+)$ is called acceptable if the following three conditions are satisfied:

(1) whenever $\gamma \in \mathcal{V}$, $\beta \in \Delta(\mathfrak{p}_+)$ and $\alpha \in \Delta^+$ such that $\gamma = \beta + \alpha$, then β is also in $\mathcal{V}$,
(2) whenever $\beta, \beta' \in \mathcal{V}$ and $\gamma = \beta + \beta' \in \Delta(\mathfrak{p}_+)$, then $\gamma \in \mathcal{V}$,
(3) whenever $\beta, \beta' \notin \mathcal{V}$ and $\gamma = \beta + \beta' \in \Delta(\mathfrak{p}_+)$, then $\gamma \notin \mathcal{V}$.

The property (1) implies that if the end point of an arrow in the weight graph belongs to an acceptable set $\mathcal{V}$, then the same is true for the beginning point of the arrow. By induction, if there is a path in the weight graph from β_1 to β_2 and $\beta_2 \in \mathcal{V}$, then also $\beta_1 \in \mathcal{V}$. The described property of acceptable sets makes it easy to construct them explicitly for low gradings.

Note that in $|1|$-graded case, the conditions (2) and (3) are vacuous. For $|2|$-graded cases, the condition (1) still plays a key role. On the other hand, for the Borel case, the condition (1) is superfluous.

The definition of acceptable sets has important practical consequences. The conditions (2) and (3) imply that whenever $\mathcal{V}$ is an acceptable subset of $\Delta(\mathfrak{p}_+)$ and β, β', γ is a triple of roots such that $\gamma = \beta + \beta'$, then the intersection $\mathcal{V} \cap \{\beta, \beta', \gamma\}$ can be only one of the six possible sets

$$\emptyset, \{\beta\}, \{\beta'\}, \{\beta, \gamma\}, \{\beta', \gamma\}, \{\beta, \beta', \gamma\},$$

while the sets $\{\beta, \beta'\}, \{\gamma\}$ are not possible. This causes the "diamond-like" subgraphs in Hasse graphs (see examples below). We can now prove the following useful characterization of the set of all vertices Φ_w in the Hasse diagram $H(\mathfrak{p}, \mathfrak{g})$.

Theorem 3.11. *A subgraph $\mathcal{V}$ of $\Delta(\mathfrak{p}_+)$ is acceptable if and only if there exists $w \in W$ such that $\mathcal{V} = \Phi_w$ and $\mathcal{V} \subset \Delta(\mathfrak{p}_+)$.*

Proof. We know that it is sufficient to prove that a subgraph $\mathcal{V}$ of $\mathfrak{p}_+$ is acceptable if and only if $\mathcal{V}$ is $(*)$-saturated.

Acceptability of a set was defined by condition (1), (2) and (3) of Definition 3.10. The definition of $(*)$-saturated sets can be written as:

(2') whenever $\beta, \beta' \in \mathcal{V}$ and $\gamma = \beta + \beta' \in \Delta^+$, then $\gamma \in \mathcal{V}$,
(3') whenever $\beta, \beta' \in (\Delta^+ - \mathcal{V})$ and $\gamma = \beta + \beta' \in \Delta^+$, then $\gamma \in (\Delta^+ - \mathcal{V})$.

Clearly (2) $\iff$ (2') and (3') $\implies$ (3).

To show that (1) and (3) $\implies$ (3'), we have to treat 4 subcases. If $\beta, \beta' \in \Delta(\mathfrak{g}_0)$, then also $\beta + \beta' \in \Delta(\mathfrak{g}_0) \subset \Delta^+ - \mathcal{V}$. If $\beta, \beta' \in (\Delta^+(\mathfrak{p}_+) - \mathcal{V})$, then it is sufficient to use (3). Suppose now that $\beta \in \Delta(\mathfrak{g}_0)$ and $\beta' \in (\Delta^+(\mathfrak{p}_+) - \mathcal{V})$, (the other case being similar). If $\gamma = \beta + \beta' \in \mathcal{V}$, then using (1), we get that β' belongs to $\mathcal{V}$, which is a contradiction. Hence $\gamma \notin \mathcal{V}$. □

3.4. **The $|1|$-graded case.** In the case that a parabolic subalgebra $\mathfrak{p}$ is given by a $|1|$-graded Lie algebra $\mathfrak{g}$, the description of the Hasse diagram is very easy.

Corollary 3.12. *Suppose that the algebra $\mathfrak{g}$ is $|1|$-graded. Then*

(1) *the set $\mathcal{V} \subset \Delta(p^+)$ is acceptable, if the condition (1) of the definition holds,*
(2) $w \xrightarrow{\alpha} w', \alpha \in \Delta^+$ *iff* $\Phi_{w'} = \Phi_w \cup \{\alpha\}$.

Proof. The conditions (2) and (3) of the definition are trivially satisfied for the $|1|$-graded Lie algebras. The second part follows from the fact that a sum of two roots from $\Delta(\mathfrak{p}^+)$ is from $\oplus_{j\geq 2}g_j$. □

4. Examples

4.1. The case ×—×.

In all examples, roots are denoted by $\alpha_{ij} = e_i - e_j$. The roots are α_{12}, α_{23} in $\mathfrak{g}_1$ and α_{13} in $\mathfrak{g}_2$ with the relation $\alpha_{13} = \alpha_{12} + \alpha_{23}$.

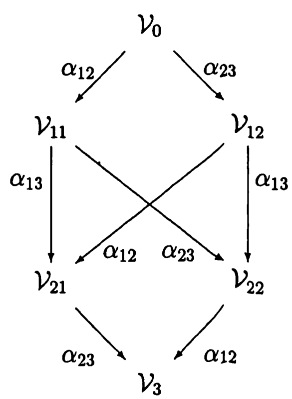
The weight graph has no arrows, it looks like

$$\alpha_{12} \qquad \alpha_{13} \qquad \alpha_{23}$$

The acceptable subgraphs are

$$\mathcal{V}_0 = \emptyset, \mathcal{V}_{11} = \{\alpha_{12}\}, \mathcal{V}_{12} = \{\alpha_{23}\},$$
$$\mathcal{V}_{21} = \{\alpha_{12}, \alpha_{13}\}, \mathcal{V}_{22} = \{\alpha_{23}, \alpha_{13}\}, \mathcal{V}_3 = \{\alpha_{12}, \alpha_{23}, \alpha_{13}\}.$$

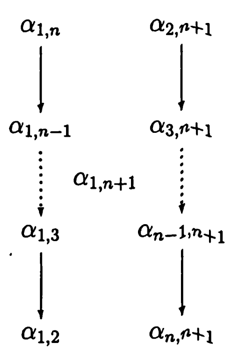
Hence the Hasse diagram is



4.2. The case ×—• ⋯ •—×.

The roots are $\alpha_{12}, \dots, \alpha_{1n}, \alpha_{2,n+1}, \dots, \alpha_{n,n+1}$ in $\mathfrak{g}_1$ and $\alpha_{1,n+1}$ in $\mathfrak{g}_2$ with the relations $\alpha_{1i} + \alpha_{i,n+1} = \alpha_{1,n+1}$ for all $i \in \{2, \dots, n\}$.

The weight graph is then



The acceptable subgraphs are of two types. Type 1 contains only roots of degree 1:

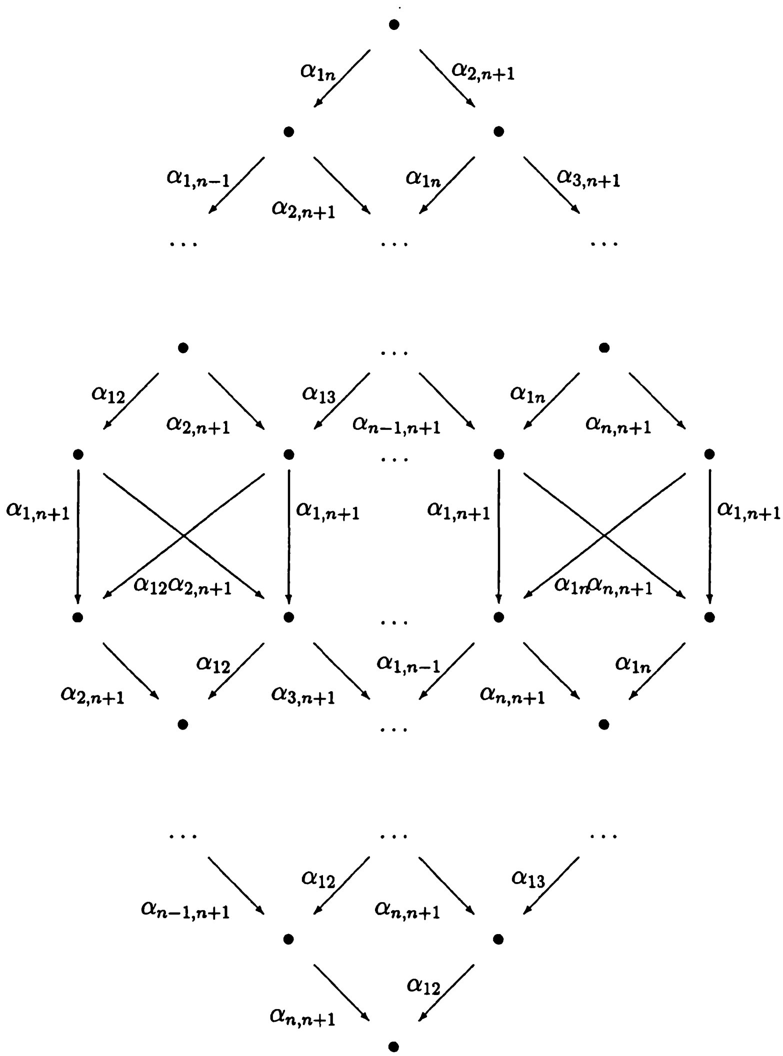
$$V = \{\alpha_{1n}, \dots, \alpha_{1k}\} \cup \{\alpha_{2,n+1}, \dots, \alpha_{n,n+1}\}$$

for $l < k$, while type 2 subgraphs contain the degree 2 root $\alpha_{1,n+1}$:

$$V = \{\alpha_{1n}, \dots, \alpha_{1k}\} \cup \{\alpha_{2,n+1}, \dots, \alpha_{n,n+1}\} \cup \{\alpha_{1,n+1}\}$$

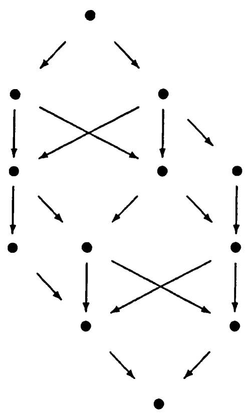
for $l \geq k$.

Hence the Hasse diagram is



4.3. The case ×—×—• .

Without any detailed analysis, we just show the resulting Hasse diagram:



4.4. The Grassmanian case $\overset{\alpha_{12}}{\bullet}$— $\cdots$ $\overset{\alpha_{k+1,k+2}}{\times}$ $\cdots$ $\overset{\alpha_{k+l+1,k+l+2}}{\bullet}$.

The algebra $\mathfrak{g} = A_{k+l+1} = \mathfrak{sl}(k+l+2, \mathbb{C})$, the module $\mathfrak{g}_1$ is isomorphic to $\mathbb{C}^{kl}$. Its roots are α_{ij}, $i \in \{1, \dots, k+1\}$, $j \in \{k+2, k+l+2\}$, ordering is given by

$$i < i' \implies \alpha_{ij} > \alpha_{i'j},$$

$$j > j' \implies \alpha_{ij} > \alpha_{ij'}.$$

The acceptable subgraphs are exactly the sets of roots of the form

$$\begin{aligned} V_{j_1,\dots,j_{k'+1}} = \{\alpha_{ij}; & i \in \{1, \dots, k'+1\} \text{ for some } k' \le k, \\ & \text{and for every such } i, j \in \{j_i, \dots, k+l+2\}, \\ & \text{where } j_1 \le \dots \le j_{k'+1}\}. \end{aligned}$$

The cardinality of $V_{j_1,\dots,j_{k'+1}}$ is $j_1 + \dots + j_{k'+1}$.

Hence, we can state the following

Proposition 4.1. *The level m of the Hasse diagram of the Grassmanian structure contains exactly all acceptable subgraphs $V_{j_1,\dots,j_{k'+1}}$ such that $j_1 + \dots + j_{k'+1} = m$.*

By the Corollary 3.12, all arrows in the Hasse diagram are given by inclusions. This gives the complete structure of the Hasse diagram.

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