Jerzy Mioduszewski; Marek Rochowski

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# REMARKS ON FIXED POINT THEOREM FOR INVERSE LIMIT SPACES

J. MIODUSZEWSKI and M. ROCHOWSKI

Wrocław

A topological space $X$ has the *fixed point property* (FPP) if for every continuous (single-valued) function $f : X \to X$ there exists a point $x \in X$ such that $f(x) = x$. Let us consider inverse systems $\{X_n, \pi_n^m, M\}$ of spaces and functions, where $\pi_n^m : X_m \to X_n$, $m \geqq n$, are continuous and onto, and $m, n \in M$, where $M$ is a directed set. The inverse limit $X = \lim \{X_n, \pi_n^m, M\}$ consists of all points $x = \{x_m\}$ $m \in M$, such that $\pi_n^m(x_m) = x_n$ for $m \geqq n$. Let $\pi_n(x) = X \to X_n$ be projections, i. e. functions defined by $\pi_n(x) = x_n$. The projections are assumed to be onto. We consider topological (not necessary metrizable) compact spaces $X$ only. It is known [1] that every compact space $X$ is an inverse limit of compact polyhedra. Hence we consider inverse systems of compact polyhedra only.

We shall say that the inverse system $\{X_n, \pi_n^m, M\}$ has the *special incidence point property* (SIPP) if for every continuous (single-valued) function $f : X_m \to X_n$, $m \geqq n$, there exists a point $x_m \in X_m$ such that $f(x_m) = \pi_n^m(x_m)$.

We consider the following question: under what conditions concerning the inverse system, the inverse limit has the FPP? For the inverse system described above we prove the following theorem.

**Theorem.** *If* $\{X_n, \pi_n^m, M\}$ *has the SIPP then the inverse limit of it has the FPP.*

In the proof are considered some multivalued functions $F_{mn} : X_m \to X_n$, induced by $f$, and their simplicial approximations.

The fixed point theorem for snake-like continua (see [2], and also [3] for a more general result) is an easy consequence of the Theorem.

**Corollary.** Let $\{X_m\}$ be an increasing system of compact polyhedra i. e. $X_n \subset X_m$ for every $m, n \in M$, $m \geqq n$. Let $\pi_n^m$ be retractions, i. e. $\pi_n^m \mid X_n$ is the identity. Then if all $X_n$ have the FPP then also the inverse limit $X$ has the FPP.

The following problem seems to be open: does the inverse limit have the FPP if all $X_n$ have the FPP and the projections are onto?

**References**

[1] *S. Eilenberg* and *N. Steenrod*: Foundations of Algebraic Topology. Princeton 1952.

[2] *O. H. Hamilton*: A fixed point theorem for pseudo-arcs and certain other metric continua. Proc. Amer. Math. Soc. *2* (1951), 173—174.

[3] *R. H. Rosen*: Fixed points for multi-valued functions on snake-like continua. Proc. Amer. Math. Soc. *10* (1959), 167—173.