Toposym 4-B

Darrell W. Hajek
Wallman extendible functions with normal domains

In: Josef Novák (ed.): General topology and its relations to modern analysis and algebra IV, Proceedings of the fourth Prague topological symposium, 1976, Part B: Contributed Papers. Society of Czechoslovak Mathematicians and Physicist, Praha, 1977. pp. 154--155.

Persistent URL: http://dml.cz/dmlcz/700650

Terms of use:

© Society of Czechoslovak Mathematicians and Physicist, 1977

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.

This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* http://project.dml.cz

WALLMAN EXTENDIBLE FUNCTIONS WITH NORMAL DOMAINS

D. W. HAJEK

Mayaguez

It is known (see [1]) that every category containing all WI functions and all WK functions must contain functions with nonunique Wallman extensions. In this paper it is shown that for functions with normal domains, all WI functions are WK functions.

Recall that for a T_1 space X the Wallman compactification WX is the collection $\{\mu : \mu$ is an ultrafilter in the lattice of all closed subsets of $X\}$ with the topology generated by $\{C(A) = \{\mu \in WX : A \in \mu\} : A$ is a closed subset of $X\}$ as a base for the closed subsets. The space WX is a compact T_1 space which is T_2 if and only if X is normal. In this case WX is homeomorphic to the Stone-Čech compactification βX. The function $\varphi_X : X \to WX$ defined by $\varphi_X(a) = \{A : a \in A$ and A is a closed subset of $X\}$ is a dense embedding. When no ambiguity can result it is common practice to ignore the distinction between X and $\varphi_X[X]$ and to refer to X as a subspace of WX. A continuous function $g : WX \to WY$ is said to be a Wallman extension of a function $f : X \to Y$ provided that $g \circ \varphi_X = \varphi_Y \circ f$. A function $f : X \to Y$ is said to be a WK function if it has a Wallman extension $f^* : WX \to WY$ with the property that for any compact subspace A of WY, the inverse image $f^{*-1}[A]$ is a compact subspace of WX. A function $f : X \to Y$ is said to be a WI function if it has a Wallman extension and if for every indicative filter base $\mathcal{F}$ in X the collection $\{f[A] : A \in \mathcal{F}\}$ is indicative in Y. (A filter base in a space X is said to be indicative if $\bigcap_{A \in \mathcal{F}} cl_{WX}(A)$ is a singleton.)

We note that it is easily shown that for any closed subset A of a T_1 space X, the closure $cl_{WX}(A) = C(A)$, and that for a finite collection $A_1, A_2, \ldots, A_n$ of closed sets, $\bigcap_{i=1}^{n} C(A_i) = C(\bigcap_{i=1}^{n} A_i)$.

<u>Lemma</u>: If U is a neighborhood in WX of a point $\mu \in WX$ then $\mu \in cl_{WX}(U \cap X)$.

<u>Proof</u>: Since μ is in the interior of U, there is some closed

set A in X such that $\mu\in WX\sim C(A)\subseteq U$. Hence, since $A\in\mu$, there is some $B\in\mu$ such that $B\cap A=\emptyset$. Therefore $B\subseteq X\cap U$ implies $\mu\in C(B)\subseteq cl_{WX}(U\cap X)$.

<u>Theorem</u>: If X is a T_4 space then $f: X\longrightarrow Y$ a WI function implies f is a WK function.

<u>Proof</u>: Suppose f has a Wallman extension $f^*: WX\longrightarrow WY$ and that f is not a WK function. There must than be a compact subset $A\subseteq WY$ such that $f^{*-1}[A]$ is not compact. Since X is T_4, we know that WX is Hausdorff; so $f^{*-1}[A]$ is not closed. Hence there is some μ in the closure of $f^{*-1}[A]$ which is not in $f^{*-1}[A]$. Clearly $\mathcal{F}=\{X\cap U : U$ is a neighborhood of $\mu\}$ is indicative in X and $f^*(\mu)\in cl_{WY}(f[B])$. It is also clear that $\{A\cap cl_{WY}(f[B]) : B\in\mathcal{F}\}$ has the finite intersection property, and, hence, nonempty intersection. Thus f is not a WI function.

As a final note; in [2] is an example of a WK function with a normal domain which is not a WI function.

References

[1] D.W.Hajek: Categories of Wallman Extendible Functions. Compositio Math. (to appear).

[2] D.W.Hajek: Wallman Extendible Functions with Non-regular Ranges. (Submitted.)